End-of-Year Test

Use your judgment in grading. You can give points or partial points for partial answers.

Question #	Max. points	Student score
Exponents and Scientific Notation		
1	8 points	
2	9 points	
3	4 points	
4	2 points	
5	2 points	
	subtotal	/ 25
Irrational Numbers		
6	5 points	
7	5 points	
8	3 points	
9	2 points	
	subtotal	/ 15
Geometry		
10	3 points	
11	2 points	
12	3 points	
13	2 points	
14a	3 points	
14b	3 points	
15	3 points	
16	3 points	
	subtotal	/ 22
Linear Equations		
17	4 points	
18	4 points	
19	6 points	
20	2 points	
21	2 points	
22	3 points	
	subtotal	/21
Functions		
23	2 points	
24a	1 point	
24b	2 points	
24c	2 points	

Question #	Max. points	Student score
Functions		
25a	1 point	
25b	1 point	
25c	1 point	
25d	1 point	
25e	1 point	
26a	2 points	
26b	1 point	
26c	1 point	
26d	1 point	
	subtotal	/17
Graphing Linear Equations		
27a	1 point	
27b	1 point	
27c	2 points	
28	3 points	
29	3 points	
30	3 points	
	subtotal	/13
The Pythagorean Theorem		
31	4 points	
32	3 points	
33	3 points	
	subtotal	/10
Systems of Linear Equations		
34	6 points	
35	3 points	
36	3 points	
37	3 points	
	subtotal	/15
Bivariate Data		
38	3 points	
39	3 points	
40	3 points	
41	5 points	
	subtotal	/14
	TOTAL	/152

Exponents and Scientific Notation

1. a. -16 b. 16 c. $1/49$ d. 36
   e. 0.031 f. $110\ 000$ g. $-8/27$ h. 64

2.

a. $-8s^3$	b. $144x^2$	c. y^{15}
d. $-6x^8$	e. $\dfrac{1}{y^6}$	f. $\dfrac{1}{4v^2}$
g. $\dfrac{49x^2}{9y^2}$	h. $\dfrac{-x^3}{125}$	i. $\dfrac{81b^4}{c^{20}}$

3. a. $1.93 \cdot 10^8$ b. $3.0805 \cdot 10^{12}$
   c. $4.6 \cdot 10^{-4}$ d. $9 \cdot 10^{-7}$

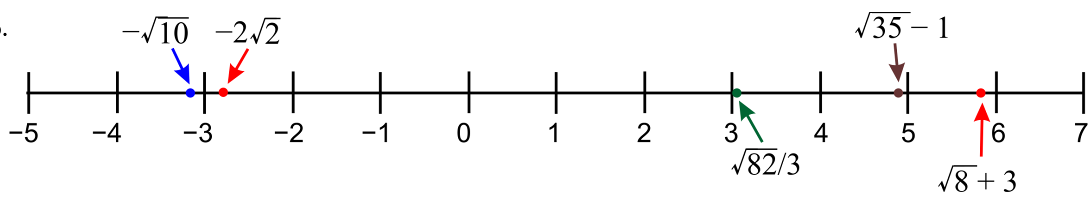
4. $\dfrac{6.0 \cdot 10^{24}\text{ kg}}{1.0 \cdot 10^{26}\text{ kg}} = \dfrac{6.0}{10^2}$ = 6/100 = 3/50. The earth's mass is (about) 3/50 of Neptune's mass.

5. We need to divide to find out how many gold atoms "fit" into 99 grams of gold:

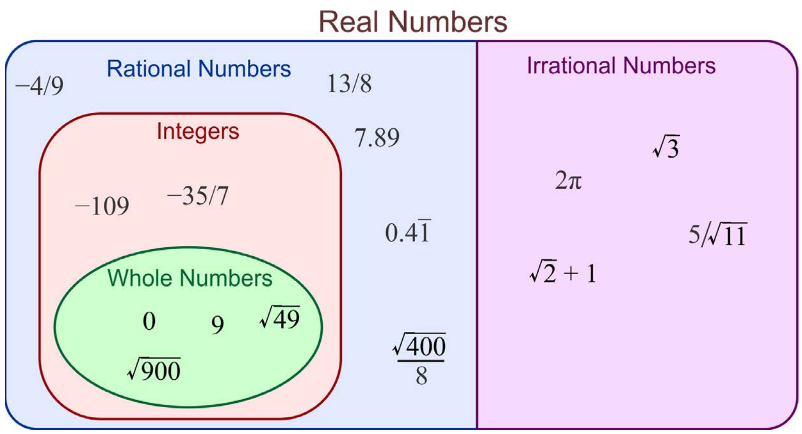
$\dfrac{9.9 \cdot 10^1\text{ g}}{3.3 \cdot 10^{-22}\text{ g}} = 3 \cdot 10^{23}$. There are about $3 \cdot 10^{23}$ gold atoms in 99 grams of gold.

Irrational Numbers

6.



7.



8. a. $x = \sqrt{54}$ or $x = -\sqrt{54}$ b. $n = 7$ or $n = -7$ c. $z = 4$

9. Let x = $0.\overline{71}$. Then $100x = 71.\overline{71}$. Subtracting those, we get:

$$\begin{aligned} 100x &= 71.717171\ldots \\ -\ x &= 0.717171\ldots \\ \hline 99x &= 71 \\ x &= \underline{71/99} \end{aligned}$$

Geometry

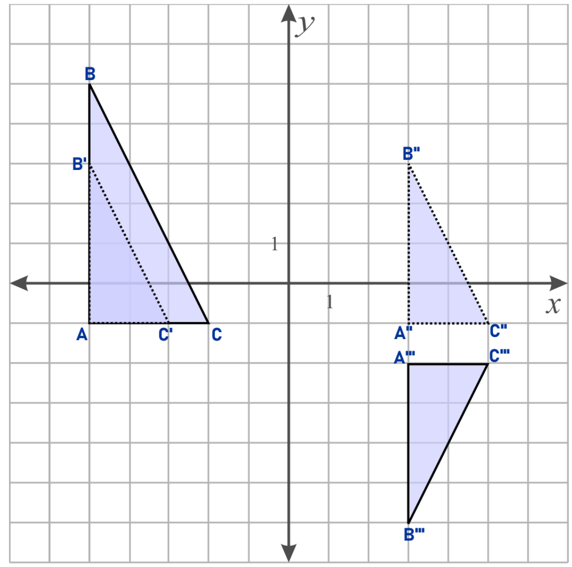
10. Answers will vary. Check the student's answer. For example:

    First, dilate triangle ABC from point A with scale factor 2/3.
    Then translate it 8 units to the right.
    Lastly, reflect it in the horizontal line $y = -1.5$.
    (See the image on the right.)

    But there are many possible answers. Here is another one.

    First, reflect the triangle ABC in the horizontal line $y = -1.5$.
    Then, translate it 8 units to the right.
    Lastly, dilate it from point A″ with scale factor 2/3.

    Another one:
    First, translate the triangle ABC 8 units to the right.
    Then dilate it from point A′ with scale factor 2/3.
    Lastly, reflect it in the horizontal line $y = -1.5$.

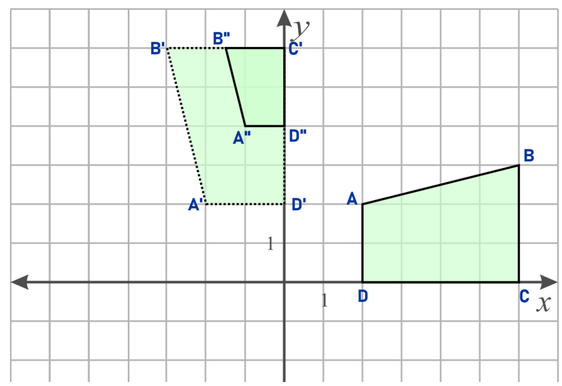


11. Answers will vary. Check the student's answer. For example:

    First, rotate trapezoid ABCD 90° counterclockwise around the origin.
    Then, dilate it from point C′ with scale factor 1/2.
    (See the image on the right.)

    Another way:

    First, dilate trapezoid ABCD from point C with scale factor 1/2.
    Then rotate it 90° counterclockwise around the origin.

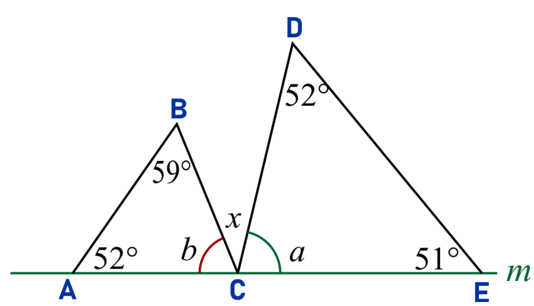


12. The bottom row of the table gives the coordinates after all the transformations.

Position	**Vertices**		
Original	A(−2, 5)	B(−5, 4)	C(−3, 0)
After reflection in y-axis	A'(2, 5)	B'(5, 4)	C'(3, 0)
After translation (2 units down, 1 to the left)	A"(1, 3)	B"(4, 2)	C"(2, −2)
After rotation (90° clockwise around the origin)	A'"(3, −1)	B'"(2, −4)	C'"(−2, −2)

13. Since the sum of the angles in a triangle is 180°, in triangle CDE, angle $a = 180° - 52° - 51° = 77°$.
    Similarly, in triangle ABC, angle $b = 180° - 52° - 59° = 69°$.

    Angles a, x, and b form a straight angle, so, $x = 180° - a - b$ $= 180° - 77° - 69° = \underline{34°}$.



14. a. Angle BAC = 180 − (x + 26) = 154 − x. Now, the angle sum of triangle ABC is 180°, so, we can write an equation using that fact, and then solve for x:

$$\begin{aligned} \angle BAC + \angle ABC + \angle BCA &= 180 \\ 154 - x + x - 48 + x - 7 &= 180 \\ x + 99 &= 180 \\ x &= 81 \end{aligned}$$

b. Since m and n are parallel, angles y and BAC are corresponding angles, thus congruent. So, y = 154° − x = 154° − 81° = <u>73°</u>.

15. V = (4/3) · π (7.5 cm)^3 · (2/3) ≈ 1180 cm^3. In millilitres, this is 1180 ml.

16. Let h be the height of the cup. Then, the volume is given by V = π (3.1 cm)^2 · h = 340 ml. This is an equation that we can use to solve for h. Since 1 ml = 1 cubic centimetre, the equation becomes π (3.1 cm)^2 · h = 340 cm^3, from which h = (340 cm^3)/(π · 3.1^2 cm^2) ≈ <u>11.3 cm</u>.

Linear Equations

17.

a.	b.
$$\begin{aligned} 10s + 8 &= 7s - 2(s - 5) \\ 10s + 8 &= 7s - 2s + 10 \\ 10s + 8 &= 5s + 10 \\ 5s &= 2 \\ s &= 2/5 \end{aligned}$$	$$\begin{aligned} 20 - 3(x + 4) &= 14 - 5x \\ 20 - 3x - 12 &= 14 - 5x \\ 8 - 3x &= 14 - 5x \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

18.

a.	b.
$$\begin{aligned} \frac{2x - 3}{5} - x &= 2 \quad \Big\vert \cdot 5 \\ 2x - 3 - 5x &= 10 \\ -3x &= 13 \\ x &= -13/3 \end{aligned}$$	$$\begin{aligned} \frac{y - 3}{4} &= \frac{1 - y}{5} \quad \Big\vert \cdot 20 \\ 4(1 - y) &= 5(y - 3) \\ 4 - 4y &= 5y - 15 \\ 4 - 9y &= -15 \\ -9y &= -19 \\ y &= 19/9 \end{aligned}$$ **or cross-multiply**

19.

a.	b.	c.
$$\begin{aligned} 6x - 1 &= 6(x - 1) \\ 6x - 1 &= 6x - 6 \\ -1 &= -6 \end{aligned}$$ No solutions.	$$\begin{aligned} -5x + 1 &= 6(x - 1) - 5 \\ -5x + 1 &= 6x - 6 - 5 \\ -5x + 1 &= 6x - 11 \\ -11x &= -12 \\ x &= 12/11 \end{aligned}$$ One solution.	$$\begin{aligned} 6x - 12 &= 6(x - 2) \\ 6x - 12 &= 6x - 12 \\ 0 &= 0 \end{aligned}$$ An infinite number of solutions. Any value of x is a solution.

20. Let d be the amount of discount. The non-discounted blocks cost 3000($1.35) = $4050.
The discounted blocks cost $1500(1.35 - d)$. The total of these equals $5775.

$$\begin{aligned} 4050 + 1500(1.35 - d) &= 5775 \\ 4050 + 2025 - 1500d &= 5775 \\ 6075 - 1500d &= 5775 \\ -1500d &= -300 \\ d &= 3/15 = 1/5 = 0.2 \end{aligned}$$

The discount was $0.20 per block. In other words, he paid $1.15 each for the 1500 blocks.

21. Let x be the first one of the four consecutive numbers. Then:

$$\begin{aligned} x + (x+1) + (x+2) + (x+3) &= 2342 \\ 4x + 6 &= 2342 \\ 4x &= 2336 \\ x &= 584 \end{aligned}$$

The numbers are <u>584, 585, 586, and 587</u>.

22. Let p be the original price of the item. Then:

$$\begin{aligned} 1.06(0.73p) &= 34.82 \\ 0.7738p &= 34.82 \\ p &\approx 44.9987 \end{aligned}$$

The item cost $45.00 originally.

Functions

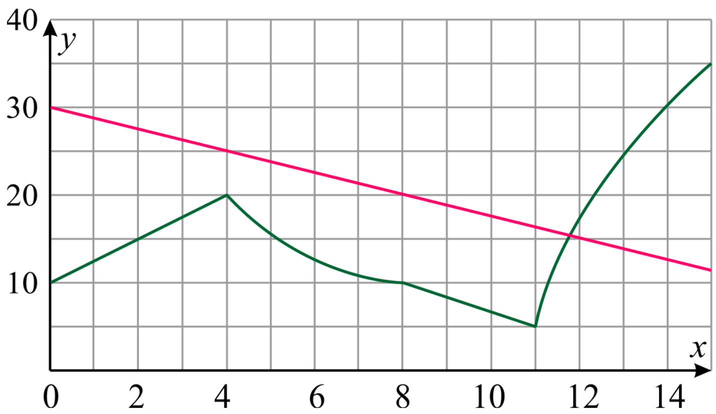
23. a. Because 3 is mapped to two different outputs: to 0 and to 3.
b. The number 6 works. If you place either 3 or 9 there, then you will have the same input mapping to two distinct outputs, which would make it not a function.

24. a. Farm B's pricing system is a linear function. $C = 6.25w$.
b. For Farm A, the rate of change is $(21.5 - 15)/(3 - 2) = 6.5$, or $6.50 per kg.
For Farm B, the change of rate is 6.25, or $6.25 per kg.
c. At Farm A, 4 kg will cost about $27, and at Farm B, $25. So, Farm B has the better deal.
For 7 kg, Farm B charges you $40 and Farm B $43.75, so, Farm A has the better deal.

25. a. $10.
b. That there is an initial fee of $10 just to get to go riding.
c. $1 per minute.
d. Horse riding will cost you $1 per minute, on top of the $10 initial fee.
e. cost = 10 + t, where t is the number of minutes you will go riding.

26. a. From $x = 0$ to $x = 4$: linear and increasing
From $x = 4$ to $x = 8$: nonlinear and decreasing
From $x = 8$ to $x = 11$: linear and decreasing
From $x = 11$ to $x = 15$: nonlinear and increasing

b.



c. $y = -(5/4)x + 30$

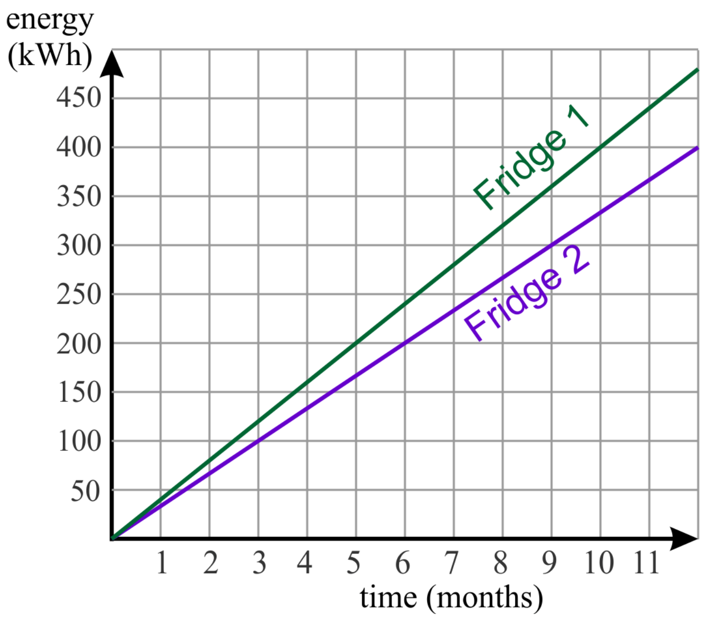
d. For the function in green: the rate of change is −5/3. For the function in red, it is −5/4.

Graphing Linear Equations

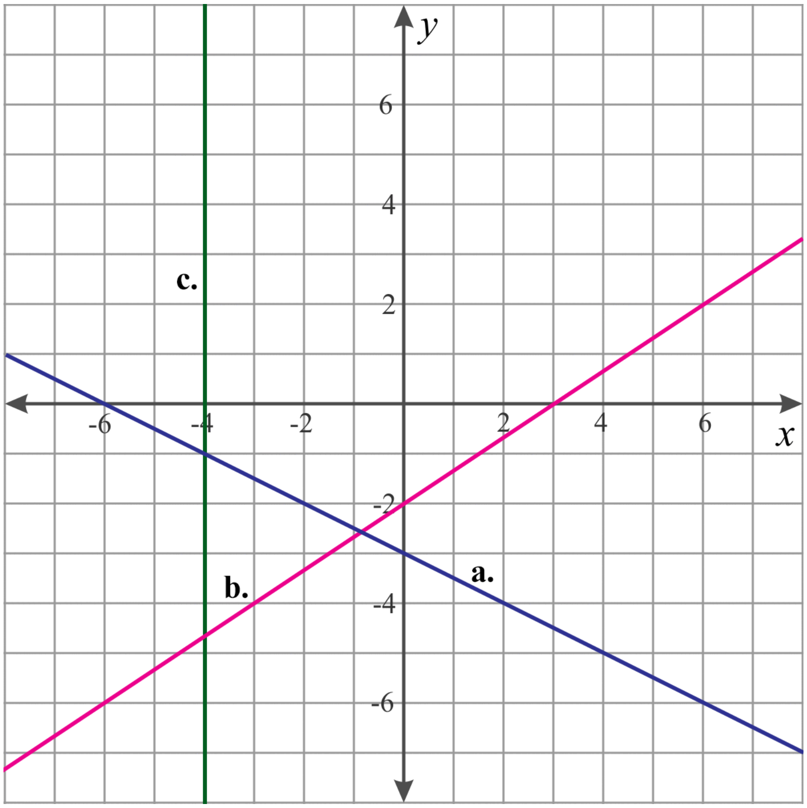
27. a. $y = (-2/3)x + 4$
b. $y = -3$
c. $y = 5x - 25$

28. a. Fridge 1 (120 kWh versus 100 kWh). It consumes 20 kWh more than Fridge 2.
b. Fridge 1: E = 40t. Fridge 2: E = (100/3)t. It is also acceptable to write it with a rounded decimal, as E = 33.3t.

c.



29.



30. The slope of this line can be calculated using the two given points. It is (−6 − 14)/(−7 − 3) = −20/(−10) = 2. The equation of this line is therefore of the form $y = 2x + b$. Substituting (3, 14) into it, we get 14 = 2(3) + b, from which b = 8. So, the equation is $y = 2x + 8$. Since point (a, 2) is on this line, let's substitute those values into the equation of the line:

$$\begin{aligned} 2 &= 2a + 8 \\ -6 &= 2a \\ a &= -3 \end{aligned}$$

So, $\underline{a = -3}$. There are also other ways to arrive to the final answer, such as using the formula for the slope.

The Pythagorean Theorem

31. Using the Pythagorean Theorem, we get:

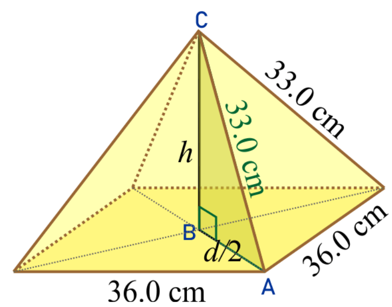
a. $r^2 + 17.5^2 = 26.6^2$ $r^2 = 26.6^2 - 17.5^2$ $r^2 = 401.31$ $r = \sqrt{401.31} \approx 20.0$ We ignore the negative root since this is a length of a side. The unknown side measures 20.0 units.	b. $x^2 + x^2 = (\sqrt{70})^2$ $2x^2 = 70$ $x^2 = 35$ $x = \sqrt{35}$ We ignore the negative root since this is a length of a side. The unknown side measures $\sqrt{35}$ units.

32. The rafter, the height of 22 in, and half of the 80-in span form a right triangle. In this triangle, the two legs measure 22 in and 40 in. Now, let r be the length of the rafter. According to the Pythagorean Theorem:

$$\begin{aligned} r^2 &= 22^2 + 40^2 \\ r^2 &= 2084 \\ r &= \sqrt{2084} \approx 45.651 \end{aligned}$$

The rafter measures about <u>45.7 inches</u>. (In real life, the decimal portion, 0.651 inches, would be converted into 16th parts of an inch by multiplying 0.7(16) = 10.416. So, the rafter measures 45 10/16 in.)

33. a. To find the height, we will use the right triangle ABC. First, we need to find the length of the diagonal of the bottom square (d). From the Pythagorean Theorem:



$$\begin{aligned} d^2 &= 36.0^2 + 36.0^2 \\ d^2 &= 2592 \\ d &= \sqrt{2592} \end{aligned}$$

Next, we apply the Pythagorean Theorem to triangle ABC. Note that one of its legs is $d/2$ (half of the diagonal), the other leg is the height of the pyramid (h), and the hypotenuse is the 33-cm edge of the pyramid.

$$\begin{aligned} h^2 + (\sqrt{2592}/2)^2 &= 33.0^2 \\ h^2 + 2592/4 &= 1089 \\ h^2 &= 1089 - 648 \\ h &= \sqrt{441} = 21 \end{aligned}$$

The height of the pyramid is <u>21.0 cm</u>.

b. The volume is V = 36.0 cm · 36.0 cm · 21.0 cm / 3 = <u>9072 cm</u>^{3}.

Systems of Linear Equations

34.

a. $\begin{cases} 2x - 3y = 8 \\ 3x + 4y = -5 \end{cases} \quad \begin{vmatrix} \cdot\, \mathbf{3} \\ \cdot\, \mathbf{(-2)} \end{vmatrix}$ $\downarrow$ $\begin{array}{rrcl} & 6x - 9y & = & 24 \\ + & -6x - 8y & = & 10 \\ \hline & -17y & = & 34 \\ & y & = & -2 \end{array}$ Substituting $y = -2$ in the first equation, we get: $\begin{aligned} 2x - 3(-2) &= 8 \\ 2x + 6 &= 8 \\ 2x &= 2 \\ x &= 1 \end{aligned}$ Solution: $(1, -2)$	b. $\begin{cases} -x = 4(y + 5) \\ 2x = -12y - 10 \end{cases}$ Solving for x from the top equation, we get that $x = -4(y + 5)$ which simplifies to $-4y - 20$. Now, substituting that for x in the bottom equation, we get: $\begin{aligned} 2(-4y - 20) &= -12y - 10 \\ -8y - 40 &= -12y - 10 \\ 4y - 40 &= -10 \\ 4y &= 30 \\ y &= 30/4 = 15/2 \end{aligned}$ Substituting $y = 15/2$ in the first equation, we get: $\begin{aligned} -x &= 4(15/2 + 5) \\ -x &= 4(25/2) \\ -x &= 50 \\ x &= -50 \end{aligned}$ Solution: $(-50, 15/2)$

35. a. No solutions. b. One solution. c. An infinite number of solutions.

36. Let x be the number of tables that seat 4, and y be the number of tables that seat 6.

We can write this system of equations: $\begin{cases} x + y = 106 \\ 4x + 6y = 500 \end{cases}$

Solving for y from the top equation, we get $y = 106 - x$. Substituting that in the bottom equation, we get:

$$\begin{aligned} 4x + 6(106 - x) &= 500 \\ 4x + 636 - 6x &= 500 \\ 636 - 2x &= 500 \\ -2x &= -136 \\ x &= 68 \end{aligned}$$

Then, $y = 106 - x = 106 - 68 = 38$. The restaurant has <u>68 tables that seat 4, and 38 tables that seat 6</u>.

37. Let G be Greta's age and S be Susan's age. Then: $\begin{cases} G + 10 = (3/4)(S + 10) \\ G + S = 127 \end{cases}$

From the bottom equation, we can solve that $G = 127 - S$. Substituting that in the top equation, we get:

$$\begin{aligned} 127 - S + 10 &= (3/4)(S + 10) & & \\ 137 - S &= (3/4)S + 7.5 & & \cdot\, \mathbf{4} \\ 548 - 4S &= 3S + 30 & & +\, \mathbf{4S} \\ 548 &= 7S + 30 & & -\, \mathbf{30} \\ 518 &= 7S & & \div\, \mathbf{7} \\ S &= 74 & & \end{aligned}$$

Then, $G = 127 - 74 = 53$. <u>Greta is 53 and Susan is 74</u>.

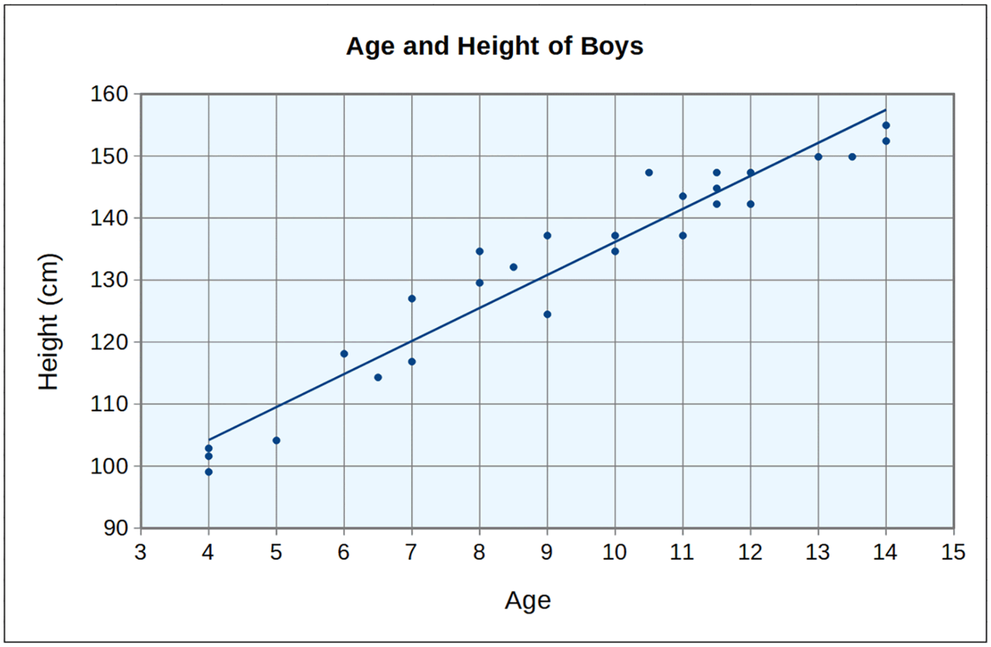
Bivariate Data

38. a. Nonlinear and decreasing association.   b. No association.   c. Linear and increasing association.

39. There is no association between the variables. For each age group, there is about an equal number of people who exercise and who do not exercise. (In other words, in each age group the relative frequencies for "Exercises" and "Does not exercise" would be close to 50%.).

40. a. 6   b. 24   c. 3

41. a. Answers will vary. Check the student's answer. For example:



b. Answers will vary. Check the student's answer. The line above goes approximately through (7, 120), and (10, 136). Therefore, its slope is 16/3 ≈ 5.33, and its equation is of the form $y = 5.33x + b$. Substituting (10, 136) into this allows us to solve for b: $136 = 5.33(10) + b$, from which $b = 82.7$. So, the equation is $y = 5.33x + 82.7$.

c. It means that each 1-year increment of age is associated with a 5.3-cm increment in height. In other words, boys tend to grow 5.3 cm per year.

d. The y-intercept of 82.9 cm means that this equation predicts a newborn baby to be 82.9 cm tall. However, we know newborns are not that tall; they are typically between 45-56 cm tall. They grow very fast during the first year. Then from about age 2 onward, the growth follows a linear pattern fairly closely. This shows us that we cannot extrapolate backwards all the way to zero years using this data and this equation.

e. We solve the equation $128 = 5.3x + 82.9$ for x:

$$\begin{aligned} 128 &= 5.3x + 82.9 \\ 45.1 &= 5.3x \\ x &\approx 8.509 \end{aligned}$$

The equation predicts the age of about 8.5 years for a boy that is 128 cm tall.