

# MATH MAMMOTH

## Grade 5-A

### Complete Worktext

- The four operations
- Problem solving
- Large numbers and the calculator
- Decimals
- Graphing and statistics



By Maria Miller

[www.MathMammoth.com](http://www.MathMammoth.com)

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# Foreword

Math Mammoth Grade 5-A and Grade 5-B worktexts comprise a complete math curriculum for the fifth grade mathematics studies. It contains eight main chapters. Each chapter has an introduction, which contains notes to the teacher. Then follow the actual lessons with problems. Each chapter ends in a review lesson. The chapter tests are found in a separate folder and are printed separately. This product also includes an HTML page that you can use to make extra practice worksheets for computation.

The fifth grade is time for fractions and decimals, in particular. We study them in depth and with substantial detail. Students also deepen their understanding of whole numbers, learn much more problem solving, and study geometry, graphing, and other smaller topics.

The year starts out with a study of whole numbers, their operations, and problem solving. Students get to review multi-digit multiplication and long division. Then we study equations with the help of a balance and bar models (simple diagrams). The main idea is to get students used to the idea of equation and what it means to solve an equation.

In the second chapter the focus is on large numbers and using a calculator. This is the first time a calculator is introduced in Math Mammoth complete curriculum - thus far all calculations have been done mentally, or with paper and pencil. I want students to learn to be critical in their use of the calculator - use it with good judgment. Every exercise where calculator use is to be allowed is marked with a little calculator symbol.

The third chapter is about decimals and their operations. It is a long chapter because now is the time to learn decimal operations well. It is assumed that the student already has a solid foundation for decimal place value, as taught in Math Mammoth 4th grade curriculum. That is the true means of preventing common misconceptions, or students resorting to rote memorization of the decimal operations.

In chapter 4, we study graphing in coordinate grid, statistical graphs, and average and mode. Today's world has become increasingly complex, with lots of data in the media, so our children need a good grasp of statistical graphs to be able to make sense of all that information.

Part B starts out with chapter 5, which is about fractions - another focus topic for 5th grade. We go through all fraction operations, and compare those with the corresponding decimal operations.

Chapter 6 takes us to geometry and measuring. There are lots of topics to study, such as classifying triangles and quadrilaterals, angle calculations, area and volume, and transformations.

Chapter 7 is a short chapter and has to do with number theory: divisibility and primes. This is also tied in with certain fraction operations, acting as a review for fraction simplification and fraction addition.

The last main chapter (chapter 8) has to do with tying together fractions, decimals, ratios, and a little bit of percent. Here will we use lots of comparisons between these concepts in word problems, and introduce students to the concept of percent.

I wish you success in your math teaching!

*Maria Miller, the author*

## Concerning Challenging Word Problems

I would heartily recommend supplementing this program with regular practice of challenging word problems and puzzles from other sources. You could do that once a week to once every two weeks - just occasionally as it suits your schedule. The goal of challenging story problems and puzzles is to simply **develop children's logical and abstract thinking and mental discipline.**

I have made lots of word problems for the Math Mammoth curriculum. Those are for the most part multi-step word problems. I have included several lessons that utilize the bar model for solving problems and tried to vary the problems.

Even so, the problems I've created are usually tied to a specific concept or concepts. I feel children can also benefit from problem solving practice where the problems require "out of the box" thinking, or are puzzle-type in nature, or are just different from the ones I have made. Additionally, I feel others are more capable of making very different, very challenging problems.

So I'd like for you to use one or several of the resources below for some different problems and puzzles. Choose something that fits your budget (most of these are free) and that you will like using.

### **Math Kangaroo Problem Database**

Easily made worksheets of challenging math problems based on actual past Math Kangaroo competition problems.

<http://www.kangurusa.com/clark/pdb/>

### **Primary Grade Challenge Math by Edward Zaccaro**

The book is organized into chapters, with each chapter presenting a type of problem and the ways to think about that problem. And then there is a series of related story problems to solve, divided into 4 levels.

\$25, ISBN 978-0967991535

*You can find this at Amazon.com or various other bookstores.*

<http://www.amazon.com/dp/0967991536/?tag=homeschoolmath-20>

### **Problem Solving Decks from North Carolina public schools**

Includes a deck of problem cards for grades 1-8, student sheets, and solutions. Many of these problems are best solved with calculators. All of these problems lend themselves to students telling and writing about their thinking.

[http://community.learnnc.org/dpi/math/archives/2005/06/problem\\_solving.php](http://community.learnnc.org/dpi/math/archives/2005/06/problem_solving.php)

### **Math Stars Problem Solving Newsletter (grades 1-8)**

These newsletters are a fantastic, printable resource for problems to solve and their solutions.

[http://community.learnnc.org/dpi/math/archives/2005/06/math\\_stars\\_news.php](http://community.learnnc.org/dpi/math/archives/2005/06/math_stars_news.php)

### **Mathematics Enrichment - [nrich.maths.org](http://nrich.maths.org)**

Open-ended, investigative math challenges for all levels from the UK. Find the past issues box down in the left sidebar. Choose Stage 2 problems for 5th grade.

<http://nrich.maths.org/public/>

<http://nrich.maths.org/public/themes.php> lets you find problems organized by mathematical themes.

### **Figure This! Math Challenges for Families**

Word problems related to real life. They don't always have all the information but you have to estimate and think. For each problem, there is a hint, other related problems, and interesting trivia. Website supported by National Council of Teachers of Mathematics.

<http://www.figurethis.org/>

### **MathStories.com**

Over 12,000 interactive and non-interactive NCTM compliant math word problems, available in both English and Spanish. Helps elementary and middle school children boost their math problem solving and critical-thinking skills. A membership site.

<http://www.mathstories.com/>

### **"Problem of the Week" (POWs)**

Problem of the week contests are excellent for finding challenging problems and for motivation. There exist several:

- **Math Forum: Problem of the Week**

Five weekly problem projects for various levels of math. Mentoring available.

<http://mathforum.org/pow/>

- **Math Contest at Columbus State University**

Elementary, middle, algebra, and "general" levels.

<http://www.colstate.edu/mathcontest/>

- **Aunty Math**

Math challenges in a form of short stories for K-5 learners posted bi-weekly. Parent/Teacher Tips for the current challenge explains what kind of reasoning the problem requires and how to possibly help children solve it.

<http://www.auntymath.com/>

- **Grace Church School's ABACUS International Math Challenge**

This is open to any child in three different age groups.

<http://www.gcschool.org/pages/program/Abacus.html>

- **MathCounts Problem of the Week Archive**

Browse the archives to find problems to solve. You can find the link to the current problem on the home page.

<http://mathcounts.org/Page.aspx?pid=355>

- **Math League's Homeschool Contests**

Challenge your children with the same interesting math contests used in schools. Contests for grades 4, 5, 6, 7, 8, Algebra Course 1, and High School are available in a non-competitive format for the homeschoolers. The goal is to encourage student interest and confidence in mathematics through solving worthwhile problems and build important critical thinking skills. By subscription only.

<http://www.mathleague.com/homeschool.htm>

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# Chapter 1: The Four Operations

## Introduction

We start out fifth grade by studying: the order of operations, multiplication, long division, equations, problem solving, and ratios.

The main line of thought throughout the chapter is that of *equations* and *problem solving*. Students encounter the exact definition of an *equation* and an *expression*. They practice the order of operations with problems that also reinforce the idea of the equal sign (“=”) as denoting equality of the right and left sides of an equation. These kind of exercises are needed because children may think that an equal sign signifies *the act of finding the answer* to a problem (as in  $134 + 23 = ?$ , for example), which is not so.

Students solve addition and subtraction equations both with the help of diagrams (a.k.a. bar models) and also without. Diagrams are also used for simple multiplication and division equations and for mixture equations, such as  $4x + 38 = 128$ .

In the middle of the chapter, we present lessons on multi-digit multiplication (multiplying in columns). These lessons go farther than just reviewing the well-known algorithm. We study in detail: multiplying in parts (partial products), how those partial products can be seen in the algorithm itself, and how multi-digit multiplication can be visualized geometrically.

Students practice long division, including two-digit divisors, as a review from 4th grade. And just in case they haven’t already learned it, this review gives them a “second chance.”

The last lessons concentrate on problem solving with the help of diagrams, including a lesson on the concept of *ratio*. Word problems involving fractional parts should now gradually become easy, routine problems.

The “Introduction to Ratios” is an important lesson. It shows the connection between fractional parts, ratios, and bar diagrams. We also study ratios further in other chapters, such as the one on fractions, so they will not be forgotten.

Although the chapter is named, “The Four Operations,” please notice that the idea is not to practice each of the four operations separately, but rather to see how they are used together in solving problems and in simple equations. We are trying to develop students’ *algebraic thinking*, including the abilities to: translate problems into mathematical operations, comprehend the many operations needed to yield an answer to a problem, “undo” operations, and so on. Many of the ideas in this chapter are preparing them for algebra in advance.

### The Lessons in Chapter 1

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Warm Up: Mental Math .....	11	2 pages
The Order of Operations and Equations .....	13	2 pages
Addition and Subtraction Review.....	15	3 pages
Multiplication and Division .....	18	3 pages
Multiplying in Parts and the Multiplication Algorithm .....	21	4 pages
A Three-Digit Multiplier, Plus Zeros .....	25	3 pages
Multiplication and Area .....	28	2 pages

	page	span
Long Division .....	30	3 pages
Long Division Practice Puzzle .....	33	1 page
A Two-Digit Divisor .....	34	4 pages
Long Division and Remainder .....	38	1 pages
Balance Problems and Equations .....	39	5 pages
More Equations .....	44	3 pages
Problem Solving 1: Finding a Fractional Part of the Whole .....	47	2 pages
Problem Solving 2: Problem Solving with Diagrams .....	49	2 pages
Problem Solving 3: One part is a multiple of the other .....	51	2 pages
Problem Solving 4 .....	53	2 pages
Introduction to Ratios .....	55	4 pages
Chapter 1 Review .....	59	2 pages

## Helpful Resources on the Internet

### Rectangle Multiplication

An interactive tool that illustrates multiplying in parts using the area model. Choose the “common” option for multiplying in parts.

[nlvm.usu.edu/en/nav/frames\\_asid\\_192\\_g\\_2\\_t\\_1.html](http://nlvm.usu.edu/en/nav/frames_asid_192_g_2_t_1.html)

### Calculator Chaos

Most of the keys have fallen off the calculator but you have to make certain numbers using the keys that are left.

[http://www.mathplayground.com/calculator\\_chaos.html](http://www.mathplayground.com/calculator_chaos.html)

### ArithmeTiles

Use the four operations and numbers on neighboring tiles to make target numbers.

<http://www.primarygames.com/math/arithmetiles/index.htm>

### Choose Math Operation

Choose the mathematical operation(s) so that the number sentence is true. Practice the role of zero and one in basic operations or operations with negative numbers. Helps develop number sense and logical thinking.

<http://www.homeschoolmath.net/operation-game.php>

### MathCar Racing

Keep ahead of the computer car by thinking logically, and practice any of the four operations at the same time.

<http://www.funbrain.com/osa/index.html>

### SpeedMath Deluxe

Create an equation from the four given digits using addition, subtraction, multiplication and division. Make certain that you remember the order of operations.

<http://education.jlab.org/smdeluxe/index.html>



**Fill and Pour**

Fill and pour liquid with two containers until you get the target amount. A logical thinking puzzle.

[http://nlvm.usu.edu/en/nav/frames\\_asid\\_273\\_g\\_2\\_t\\_4.html](http://nlvm.usu.edu/en/nav/frames_asid_273_g_2_t_4.html)

**Thinking Blocks**

An interactive math tool developed to help students learn how to solve multistep word problems. Using brightly colored blocks, students model the relationships among the components of each word problem. The website has addition/subtraction problems, multiplication/division problems, and ratio problems. This block model corresponds to the bar model used in this book.

<http://www.thinkingblocks.com/>

**Algebraic Reasoning**

Find the value of an object based on two scales.

[http://www.mathplayground.com/algebraic\\_reasoning.html](http://www.mathplayground.com/algebraic_reasoning.html)

**Algebra Puzzle**

Find the value of each of the three objects presented in the puzzle. The numbers given represent the sum of the objects in each row or column.

[http://www.mathplayground.com/Algebra\\_Puzzle.html](http://www.mathplayground.com/Algebra_Puzzle.html)

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# Order of Operations and Equations

**Solve multiplications and divisions before additions and subtractions.**  
**Solve multiplications and divisions “on the same level,” from left to right.**  
**Solve additions and subtractions “on the same level,” from left to right.**  
**Parentheses ( ) change the order. *First* solve whatever is inside parentheses.**

1. Solve in the right order!

a. $12 \times 5 + 8 = \underline{\quad}$	b. $10 \times 2 + 9 \times 8 = \underline{\quad}$	c. $(8 + 16) \div 3 = \underline{\quad}$
$45 + 5 \times 7 = \underline{\quad}$	$10 + 2 \times 9 + 8 = \underline{\quad}$	$120 - 2 \times 11 = \underline{\quad}$
$8 \times 5 \div 2 = \underline{\quad}$	$10 + 2 \times (9 + 8) = \underline{\quad}$	$2 \times (100 - 80 + 20) = \underline{\quad}$

Which expression(s) match each problem?

2. Mark bought three light bulbs for \$8 each, and paid with \$50. What was his change?

- a.  $3 \times \$8 - \$50$       b.  $\$50 - \$8 + \$8 + \$8$   
 c.  $\$50 - 3 \times \$8$       d.  $\$50 - (\$8 - \$8 - \$8)$

3. Andy buys a salad for \$8 and a pizza for \$13, and shares them evenly with his friend. How many dollars is Andy's share of the cost?

- a.  $\$8 + \$13 \div 2$       b.  $\$2 \div (\$8 + \$13)$   
 c.  $2 \times \$8 + 2 \times \$13$       d.  $(\$8 + \$13) \div 2$

4. Melissa shares equally the cost of a new fence with three other neighbors and the cost of road repair with two other neighbors. The fence cost \$600 and the road repair cost \$1,200. What is Melissa's share of the costs?

- a.  $\$600 \div 4 + \$1,200 \div 3$   
 b.  $(\$600 + \$1,200) \div 3 \div 2$   
 c.  $\$600 \div 3 + \$1,200 \div 2$   
 d.  $(\$600 + \$1,200) \div 5$

5. Division can also be written with a line. Solve in the right order.

a.  $6 + \frac{24}{2} =$       b.  $\frac{32}{2} - 6 =$       c.  $\frac{54}{6} - 6 - 2 =$

In this case, what we do first is the operation that is *above* the line, as though it were written in parentheses:

d.  $\frac{6 + 24}{2} =$       e.  $\frac{32 - 6}{2} =$       f.  $\frac{54 - 6}{6} - 2 =$

An **equation** has numbers, letters, operation symbols, and one equal sign, “=”. It's called an *equation* because it contains an *equal* sign.

An **expression** only has numbers, letters, and operation symbols—but no equal sign. For example, “ $40 \times 2 + 6 \times 5$ ” is an expression.

6. Equation or expression? (You do not have to solve these.)

a.  $4t = 180$

b.  $2 + 60 \times 345 \div 9$

c.  $15 = x + y$

d.  $\frac{5.4 - 2.12}{0.4} = 8.2$

e.  $1,000 = 1,000$

f.  $12 - \frac{24 \div 0.8}{189}$

<p><math>120 - 75 = 3 \times 15</math></p> <p>This is the left side of the equation.      This is the right side of the equation.</p> <p>Do the left and right sides have the same value? Just calculate <math>120 - 75</math>, then calculate <math>3 \times 15</math>, and check.</p> <p>If yes, it's a <b>true equation</b>. If not, it's a <b>false equation</b>.</p>	<p><math>2 = 5</math></p> <p>left side      right side</p> <p>This is a very simple equation - but it's false!</p> <p><math>4 + 5 = 21 - 3</math></p> <p>left side      right side</p> <p>This is also a false equation!</p>	<p><math>18 = x - 3</math></p> <p>left side      right side</p> <p>Solving the equation means finding the value of x (the <b>unknown</b>) that makes it true.</p> <p>The value <math>x = 21</math> makes this equation true, so we say <math>x = 21</math> is the solution.</p>
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7. Place parenthesis into these equations to make them true.

a.  $10 + 40 + 40 \times 2 = 180$

b.  $144 = 3 \times 2 + 4 \times 8$

c.  $40 \times 3 = 80 - 50 \times 4$

8. Find a number to fit in the box so the equation is true.

a.  $40 = (\square + 9) \times 2$

b.  $4 \times 8 = 5 \times 6 + \square$

c.  $4 + 5 = (20 - \square) \div 2$

d.  $81 = 9 \times (2 + \square)$

e.  $\square \times 11 = 12 + 20 \times 6$

f.  $(4 + 5) \times 3 = \square \div 2$


9. Build at least three true equations using (only) the symbols and numbers given. You may use the same number or symbol many times.

11, 3, 1, -, +, ×, ( ), =

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# Problem Solving with Diagrams, Part 1

## Finding a Fractional Part of the Whole

Problem	Solution
<p>Jackie earns \$1,840 monthly and Jessie earns <math>\frac{3}{4}</math> as much. How much does Jessie earn?</p> 	<p>In the diagram, Jackie's salary is divided into four equal parts. To find <math>\frac{3}{4}</math> of it, <u>first find <math>\frac{1}{4}</math> of it</u>, and then multiply that by 3.</p> <p><math>\\$1,840 \div 4 = \\$460.</math></p> <p><math>3 \times \\$460 = \\$1,380.</math> Jessie earns \$1,380.</p>

Solve the problems. You may draw a diagram to help.

1. A pizza that weighs 680 g is divided into five equal pieces.  
How much do two pieces weigh?

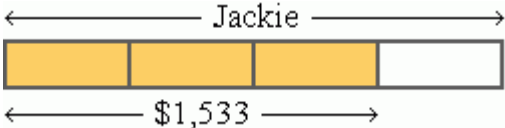
2. A \$125 camera was discounted by  $\frac{1}{5}$  of its price.  
What is its new price?



3. A bottle of water costs  $\frac{2}{3}$  as much as a \$1.50 juice.  
How much do two bottles of water and two juices cost?

4. A T-shirt is discounted by  $\frac{2}{5}$  of its price. The original price is \$10.50.  
How much would ten shirts cost with the discounted price?

Notice carefully what happens if we turn the earlier problem (about Jackie and Jessie) around so that we know how much *Jessie* earns and are asked how much Jackie earns.

Problem	Solution
<p>Jessie earns <math>\frac{3}{4}</math> as much as Jackie. Jessie earns \$1,533. How much does Jackie earn?</p> 	<p>1) First we divide Jessie's earnings by 3.  <math>\\$1,533 \div 3 = \\$511</math>.            That gives us <math>\frac{1}{4}</math> of Jackie's salary.</p> <p>2) Then we multiply that number by 4:  <math>4 \times \\$511 = \\$2,044</math>, which is Jackie's salary.</p>

Solve the problems. Drawing a bar diagram can help.

5. Mark's car weighs 840 lb, which is  $\frac{2}{3}$  of what Jake's car weighs. How much does Jake's car weigh?
  
6. Henry is  $\frac{5}{6}$  as tall as Mike, and Mike is 6 ft 6 in tall. How tall is Henry?
  
7. Jack is  $\frac{5}{8}$  as tall as his dad.  
 If Jack is 120 cm tall, then how tall is his dad?
  
8. Duckville has 3,687 inhabitants, which is  $\frac{3}{5}$  of the number of inhabitants in Eagleby.  
 How many people *total* live in Eagleby *and* Duckville?
  
9. Six-month subscriptions to an online math-practice website are discounted to  $\frac{3}{4}$  of the normal month-by-month price.  
 The discounted price for 6 months is \$54.  
  
 Find the cost of an undiscounted one-month subscription.

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# Chapter 2: Large Numbers and the Calculator

## Introduction

In this chapter we study place value up to the billions—that is, numbers of up to 12-digits, rounding them and using a calculator.

This is the first time the calculator is introduced in the Math Mammoth Light Blue Series books. I have delayed introducing the use of a calculator (as compared to most math curricula) for good reasons. On my HomeschoolMath.net web site I have received numerous comments on the harm that indiscriminate calculator usage can cause. In a nutshell, if children are allowed to use calculators freely, their minds get “lazy,” and they will start relying on calculators even for simple things. It’s just human nature!

As a result, students enter college without even knowing their multiplication tables by heart. Then they have tremendous trouble if they are required to use mental math to solve simple problems.

So we educators need to *limit* calculator usage until the students are much older. Children can *not* decide this for themselves, and definitely not in fifth grade.

However, I realize that the calculator is extremely useful, and students do need to learn to use it. In this curriculum, I strive to show the students not only *how* to use a calculator, but also *when* to use it and when *not* to use it.

This chapter includes many problems where calculator usage is appropriate. We also practice estimating the result before calculating it with calculator. In the last lesson, students need to choose whether mental math or a calculator is the best “tool” for the calculation.

### The Lessons in Chapter 2

	page	span
Place Value Up to Billions .....	63	3 pages
Counting and Adding Large Numbers .....	66	3 pages
Rounding .....	69	3 pages
Calculator .....	72	3 pages
Multiples, Estimation and Calculator .....	75	3 pages
Review .....	78	2 pages

## Helpful Resources on the Internet

### Naming Numbers

These pages teach number naming skills covered in K8 math courses. Each page has an explanation, interactive practice and challenge games about naming numbers.

<http://www.aaamath.com/B/nam.htm>

### Megapenny project

Visualizes big numbers with pictures of pennies.

<http://www.kokogiak.com/megapenny/default.asp>

### Powers of ten

Illustrates the dramatic changes of scale when zooming in or out is by powers of ten.

<http://microcosm.web.cern.ch/Microcosm/P10/english/welcome.html>

### Cookie dough

Practices naming big numbers.

[www.funbrain.com/numwords/index.html](http://www.funbrain.com/numwords/index.html)

### Keep My place

Fill in the big numbers to this cross-number puzzle.

<http://www.mathsyear2000.org/magnet/kaleidoscope2/Crossnumber/index.html>

### Estimation

Exercises about rounding whole numbers and decimals, front-end estimation, estimating sums and differences.

<http://www.aaamath.com/B/est.htm>

### Estimation at AAA Math

Exercises about rounding whole numbers and decimals, front-end estimation, estimating sums and differences. Each page has an explanation, interactive practice, and games.

<http://www.aaamath.com/B/est.htm>

### Place Value Game

Create the largest possible number from the digits the computer gives you.

Unfortunately, the computer will give you each digit one at a time and you won't know what the next number will be.

<http://education.jlab.org/placevalue/index.html>

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# Add and Subtract Decimals

This “trick” will help you a lot in adding or subtracting decimals:

**“Tag” zeros to the end of the decimal numbers so that all addends have the same amount of digits after the decimal. Then the answer will also have that same number of digits.**

Look how the problem  $0.2 + 0.05$  is done (on the right). We “tag” a zero onto the end of 0.2 to make it have *two* digits after the decimal point! That way *both* addends have hundredths. Notice how it’s just like adding fractions using a common denominator.

Note that  $0.2 + 0.05$  is *not* 0.7 or 0.07!

When adding in columns, write the numbers under each other and align the decimal points. You can write a zero in the empty “spot.” Then add.

$$\begin{array}{r}
 0.2 + 0.05 \\
 \downarrow \quad \downarrow \\
 0.20 + 0.05 = 0.25 \\
 \\
 \frac{2}{10} + \frac{5}{100} \\
 \downarrow \quad \downarrow \\
 \frac{20}{100} + \frac{5}{100} = \frac{25}{100} \\
 \\
 \begin{array}{r}
 0.20 \\
 + 0.05 \\
 \hline
 0.25
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 0.09 + 0.5 \\
 = 0.09 + 0.50 = 0.59 \\
 \begin{array}{r}
 0.09 \\
 + 0.50 \\
 \hline
 0.59
 \end{array}
 \end{array}$$

*Both addends have hundredths, and so does the answer.*

$$\begin{array}{r}
 1.007 + 2.02 \\
 = 1.007 + 2.020 = 3.027 \\
 \begin{array}{r}
 1.007 \\
 + 2.020 \\
 \hline
 3.027
 \end{array}
 \end{array}$$

*Both addends have thousandths, and so does the answer.*

1. Write the decimal that is one tenth, one hundredth, or one thousandth more than the given decimal.

a.

O	t	h	th
0	.	2	8 5

- 1 tenth more: \_\_\_\_\_  
 1 hundredth more: \_\_\_\_\_  
 1 thousandth more: \_\_\_\_\_

b.

O	t	h	th
0	.	0	1 6

- 2 tenths more: \_\_\_\_\_  
 2 hundredths more: \_\_\_\_\_  
 2 thousandths more: \_\_\_\_\_

c.

O	t	h	th
1	.	0	7

- 5 tenths more: \_\_\_\_\_  
 2 hundredths more: \_\_\_\_\_  
 6 thousandths more: \_\_\_\_\_

2. Add.

- |                    |                  |                    |
|--------------------|------------------|--------------------|
| a. $0.009 + 0.006$ | d. $0.8 + 0.6$   | g. $0.5 + 0.7$     |
| b. $0.009 + 0.06$  | e. $0.8 + 0.06$  | h. $0.05 + 0.07$   |
| c. $0.009 + 0.6$   | f. $0.8 + 0.006$ | i. $0.005 + 0.007$ |

3. a. Write a number that is 5 thousandths, 2 tenths, and 8 hundredths more than 1.004.

b. Write a number that is 3 thousandths and 3 tenths less than 3.411.

4. Add hundredths so that the next *whole tenth* is filled.

a. $0.47 + \underline{0.03} = \underline{0.5}$	d. $0.06 +$	l. $3.99 +$
b. $0.55 +$	f. $0.97 +$	i. $2.13 +$

5. These kids have some misconceptions. Correct their answers. And learn from these!

<p>a. Laura thinks: Both of these have two zeros after the decimal point so I'll put the same two zeros in the answer.</p> <p><math>0.008 + 0.003 = 0.0011</math></p>	<p>b. Jessie reasons: Just add the decimal part separately from the whole number part:</p> <p><math>0.7 + 0.7 = 0.14</math></p>
---	---

<p><i>10 tenths makes 1.</i></p> <p><i>100 hundredths makes 1.</i></p> <p><i>1000 thousandths makes 1.</i></p>	<p>Solve <math>0.42 + \underline{\hspace{2cm}} = 1</math>.</p> <p>42 hundredths and 58 hundredths makes 100 hundredths, so</p> <p><math>0.42 + 0.58 = 1</math></p>	<p>Solve <math>0.219 + \underline{\hspace{2cm}} = 1</math>.</p> <p>219 thousandths and 781 thousandths makes 1000 thousandths, so</p> <p><math>0.219 + 0.781 = 1</math></p>
--	--	---

6. Complete the addition so the sum is 1.

- |  |  |   |
|--|--|---|
| a. $0.6 + \underline{\hspace{2cm}} = 1$  | d. $0.99 + \underline{\hspace{2cm}} = 1$ | g. $0.999 + \underline{\hspace{2cm}} = 1$ |
| b. $0.60 + \underline{\hspace{2cm}} = 1$ | e. $0.87 + \underline{\hspace{2cm}} = 1$ | h. $0.002 + \underline{\hspace{2cm}} = 1$ |
| c. $0.61 + \underline{\hspace{2cm}} = 1$ | f. $0.22 + \underline{\hspace{2cm}} = 1$ | i. $0.304 + \underline{\hspace{2cm}} = 1$ |

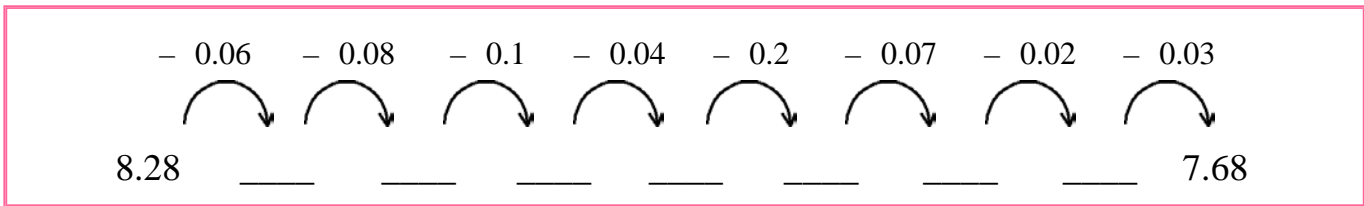
7. Subtract from 1.

- |  |   |   |
|--|---|---|
| a. $1 - 0.01 = \underline{\hspace{2cm}}$ | d. $1 - 0.001 = \underline{\hspace{2cm}}$ | g. $1 - 0.506 = \underline{\hspace{2cm}}$ |
| b. $1 - 0.04 = \underline{\hspace{2cm}}$ | e. $1 - 0.008 = \underline{\hspace{2cm}}$ | h. $1 - 0.56 = \underline{\hspace{2cm}}$  |
| c. $1 - 0.51 = \underline{\hspace{2cm}}$ | f. $1 - 0.021 = \underline{\hspace{2cm}}$ | i. $1 - 0.411 = \underline{\hspace{2cm}}$ |

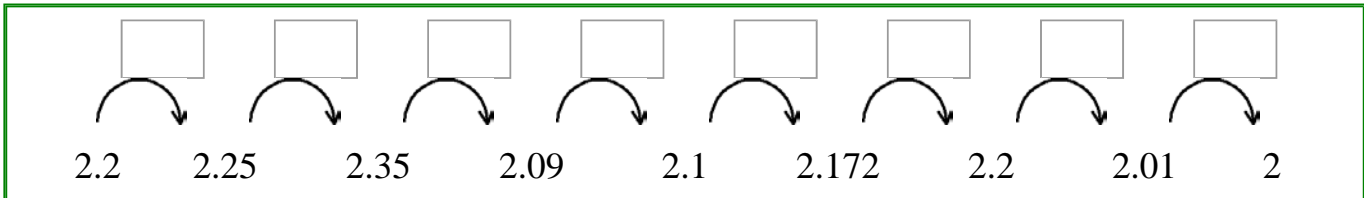
8. Solve the equations. Mental math should be enough!

a. $0.50 + x = 0.677$	b. $x + 1.52 = 2$	c. $1 - x = 0.378$	d. $x - 0.5 = 1.27$
-----------------------	-------------------	--------------------	---------------------

9. Find the missing numbers.



10. Figure out what was done in each step - either addition or subtraction!



11. Add or subtract. Remember to line up the decimal points.

a.  $145.5 - 24.93$

b.  $190 + 342.25 + 45.808$

c.  $2,530 - 451.55 + 231.2$

12. Mrs. Wood's precious puppy weighed 0.236 kg when it was born. At the next checkup it weighed 0.37 kg. How much weight had it gained?

13. Dad caught some fish that weighed 1.19 kg, 1.565 kg, 2.1 kg and 1.47 kg.

a. What was the total weight of the catch?

b. How much short of 7 kg was his catch?

14. Allison checked some distances using an online map service. Find the total distance for these routes:

a. From Allison's home to the library, then to the store, then home again.

b. From the library to Sheila's house, to the store, then home.

From	To	Distance (miles)
My home	library	2.3
home	store	0.67
library	store	1.055
home	Sheila's	1.08
store	Sheila's	1.508
library	Sheila's	1.25

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## Long Division with Decimals

<p>It is very easy to use long division to divide a decimal by a whole number.</p> <p>During the division process, divide as if there were no decimal point.</p> <p>Then, simply put the decimal point in the quotient <i>in the same place</i> as it was in the dividend.</p>	$  \begin{array}{r}  05\overline{)41.51} \\  \underline{35} \phantom{0} \\  65 \\  \underline{-63} \\  21 \\  \underline{-21} \\  0  \end{array}  $	<p><b>Check:</b></p> $  \begin{array}{r}  5.93 \\  \times \phantom{0}7 \\  \hline  \end{array}  $
--	---	---

1. Divide. Check each division result with multiplication.

a.  $5 \overline{)5.30}$

b.  $3 \overline{)0.72}$

c.  $7 \overline{)6.23}$

d.  $6 \overline{)2.388}$

e.  $19 \overline{)23.94}$

f.  $23 \overline{)57.638}$



You are used to dividing *whole numbers* with long division, and sometimes getting a remainder. For example,  $24 \div 5 = 4 \text{ R}4$ .

If we add decimal zeros (.0 or .00 or .000) to the dividend, we don't change its value, but sometimes the quotient comes out even!

For example, if we use long division to divide 24.0 by 5, the quotient is exactly 4.8! Multiplying  $5 \times 4.8 = 24$  verifies this.

$$\begin{array}{r} 04.8 \\ 5 \overline{)24.0} \\ \underline{20} \phantom{0} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

**Check:**

$$\begin{array}{r} 4 \\ 4.8 \\ \times 5 \\ \hline 24.0 \end{array}$$

2. Divide in two ways: first by indicating a remainder, then by long division.  
Check your result with multiplication.

<p>a. <math>31 \div 4 = \underline{\quad} \text{ R} \underline{\quad}</math></p> <p><math>31.00 \div 4 = \underline{\hspace{2cm}}</math></p> $\begin{array}{r} \phantom{0} \\ 4 \overline{)31.00} \end{array}$ <p>Check:</p>	<p>b. <math>56 \div 5 = \underline{\quad} \text{ R} \underline{\quad}</math></p> <p><math>56.0 \div 5 = \underline{\hspace{2cm}}</math></p> <p>Check:</p>	<p>c. <math>15 \div 8 = \underline{\quad} \text{ R} \underline{\quad}</math></p> <p><math>15.000 \div 8 = \underline{\hspace{2cm}}</math></p> <p>Check:</p>
--	---	---

3. Now you need both division and multiplication. Find the given parts of these quantities.

a. Find  $\frac{3}{4}$  of 0.130 kg.

b. Find  $\frac{3}{5}$  of 23 seconds.

Sometimes decimal division is not even. In that case, stop the division at some point, and *give the answer as a rounded decimal number*. Round to the place value just before the last decimal digit you found for the quotient. That way, the last digit will tell you whether to round up or down.

**Example:** Find  $2 \div 7$  to two decimal digits.

$$\begin{array}{r} 0.285 \\ 7 \overline{)2.000} \\ \underline{14} \phantom{00} \\ 60 \phantom{0} \\ \underline{-56} \phantom{0} \\ 40 \\ \underline{-35} \\ 5 \end{array}$$

It is enough to find three decimal digits for the quotient, and then we can round to two decimal digits. The third decimal digit is 5, so we round up and  $2 \div 7 \approx 0.29$ .

**Example:** Find  $8 \div 9$  to three decimal digits.

$$\begin{array}{r} 0.8888 \\ 9 \overline{)8.0000} \\ \underline{72} \phantom{000} \\ 80 \phantom{00} \\ \underline{-72} \phantom{00} \\ 80 \phantom{0} \\ \underline{-72} \phantom{0} \\ 8 \end{array}$$

We need to find *four* decimal digits for the quotient before we can round it to three decimal digits:  $8 \div 9 \approx 0.889$ .

4. Find

a.  $2 \div 9$  to two decimal digits;

b.  $1 \div 3$  to three decimal digits;

c.  $1 \div 6$  to three decimal digits.

### Fractions and division

Remember that the fraction line is *also* a division symbol. So  $\frac{1}{8}$  can mean both one-eighth (a fraction), and a division problem  $1 \div 8$ . This gives us a means of converting fractions to decimals!

5. Convert the following fractions into decimals using long division (use a notebook). If needed, round the decimal to three decimal digits.

a.  $\frac{5}{8} =$

b.  $\frac{6}{7} =$

c.  $\frac{7}{20} =$

6. a. Fill in the explanation on how to solve the problem, and find the price of one roll.

24 rolls and one loaf of rye bread cost \$10.70. If the bread costs \$2.30, find the cost of one roll.

First subtract \$\_\_\_\_\_ from \$\_\_\_\_\_. Then \_\_\_\_\_ that result by \_\_\_\_\_. One roll costs \$\_\_\_\_\_.

b. Write a *single* expression to match the explanation above.

7. Seven muffins and one drink cost \$7.11. If the drink costs \$1.23, find the cost of one muffin.

8. Three packs of transistors and seven packs of capacitors cost a total of \$8.70. What is the price of one transistor pack, if one capacitor pack costs \$0.60?

9. Solve in the right order:  $2 \times (68.43 \div 3 + 2.9)$ .

10. Write a word problem that is solved by the calculations below.

a.  $(\$50 - \$26) \div 3 = \$8$

b.  $(\$100 - 25 \times \$1.40) \div 2 = \$32.50$

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# Chapter 4: Statistics and Graphing

## Introduction

The fourth chapter starts out with a study of the coordinate grid, but only the first quadrant. I have also included a very gentle *Introduction to Functions* lesson, where students plot ordered pairs from number rules.

Practicing the use of the coordinate grid is a natural “prelude” to the study of line graphs, which follow next. The goals are that the student will be able to:

- read line graphs, including double line graphs, and answer questions about data already plotted;
- draw line graphs from a given set of data.

To achieve these goals I have provided plenty of exercises, with a lot of variety in topics.

The goals for the study of bar graphs are similar to those for the study of line graphs, in that the student will need to both:

- read bar graphs, including double bar graphs, and answer questions about data already plotted; and
- draw bar graphs and histograms from a given set of data.

In order to make histograms, it is necessary to understand how to group the data into categories. The lesson *Making Histograms* explains the method we use to make categories if the numerical data is not already categorized.

Toward the end of the chapter, we study the mean and the mode and how these two concepts relate to line and bar graphs. Other math curricula commonly introduce the median, too, but I decided to omit it from 5th grade. There is still plenty of time to learn that concept in 6th, 7th, and 8th grades. Introducing all three concepts at the same time tends to jumble the concepts together and confuse them—and all that many students are able to grasp out of that jumble is often just fairly meaningless calculation procedures. I feel it is better to introduce and contrast initially just the two concepts, the mean and the mode, in order to give the student a solid foundation to study them later in more depth when the median is introduced and compared and contrasted with them.

This chapter also includes an optional statistics project, in which the student can develop investigative skills.

### The Lessons in Chapter 4

	page	span
Coordinate Grid .....	141	3 pages
Introduction to Functions .....	144	4 pages
Lines Graphs .....	148	4 pages
Reading Line Graphs .....	152	2 pages
Double and Triple Line Graphs .....	154	2 pages
Making Bar Graphs .....	156	2 pages
Making Histograms .....	158	2 pages

Double Bar Graphs .....	160	2 pages
Average (Mean) .....	162	3 pages
Mean, Mode and Bar Graphs .....	165	2 pages
Statistics Project (optional) .....	167	1 page
Review .....	168	2 pages

## Helpful Resources on the Internet

### Bar Chart Virtual Manipulative

Build your bar chart online using this interactive tool:

[nlvm.usu.edu/en/nav/frames\\_asid\\_190\\_g\\_1\\_t\\_1.html?from=category\\_g\\_1\\_t\\_1.html](http://nlvm.usu.edu/en/nav/frames_asid_190_g_1_t_1.html?from=category_g_1_t_1.html)

### An Interactive Bar Grapher

Graph data sets in bar graphs. The color, thickness, and scale of the graph are adjustable. You can input your own data, or you can use or alter pre-made data sets.

[illuminations.nctm.org/ActivityDetail.aspx?ID=63](http://illuminations.nctm.org/ActivityDetail.aspx?ID=63)

### Create a Graph

Create bar graphs, line graphs, pie graphs, area graphs, and xyz graphs to view, print, and save.

[nces.ed.gov/nceskids/createagraph/default.aspx](http://nces.ed.gov/nceskids/createagraph/default.aspx)

### Mode of a Set of Data

This is a very simple and clear lesson with examples and interactive quiz questions.

[www.mathgoodies.com/lessons/vol8/mode.html](http://www.mathgoodies.com/lessons/vol8/mode.html)

### Using and Handling Data

Simple explanations for finding the mean, the median, or the mode.

[www.mathsisfun.com/probability](http://www.mathsisfun.com/probability)

### Finding the Mean, Median, and Mode

This is a great lesson, with interactive quiz questions at the end. It also explains briefly the different uses for mean, median, and mode. After all, why do we have three different numbers to describe the central tendency of a data set?

[www.algebra.org/lessons/lesson.aspx?file=Algebra\\_StatMeanMedianMode.xml](http://www.algebra.org/lessons/lesson.aspx?file=Algebra_StatMeanMedianMode.xml)

### Mean, Median, and Mode

How to calculate the mean, the median, and the mode for sets of data given in different ways. There are also interactive exercises.

[www.cimt.plymouth.ac.uk/projects/mepres/book8/bk8i5/bk8\\_5i2.htm](http://www.cimt.plymouth.ac.uk/projects/mepres/book8/bk8i5/bk8_5i2.htm)

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# Reading Line Graphs

The graph shows how many people were living on farms in the United States during 1900-1990. You can see how dramatically the number has dropped!

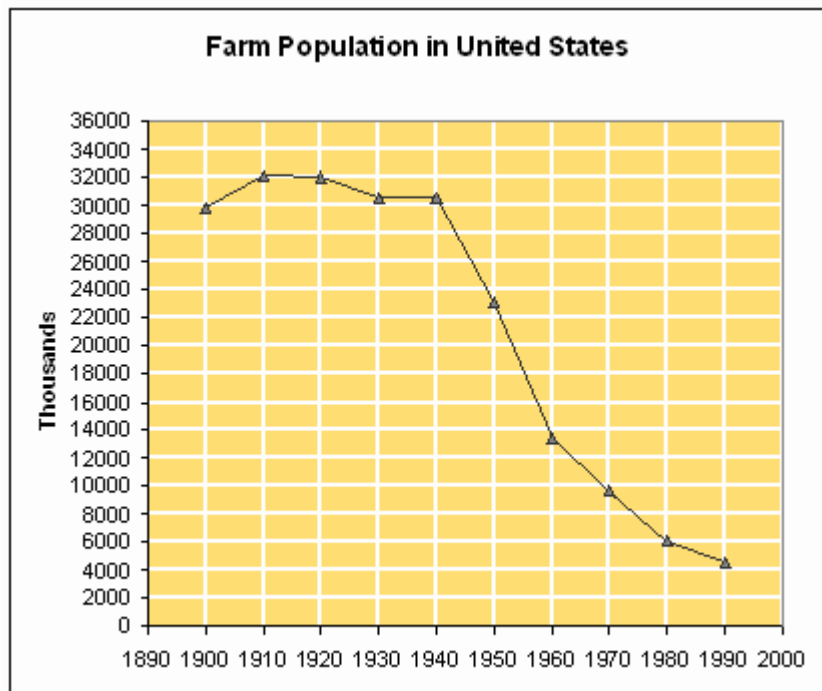
The question (a) in exercise 1 asks you to *estimate* the farm population in year 2000. Do it by tracing over the graph and continuing the graph in a natural way till the year 2000. The plain numbers listed in the table do not really help with estimation (without further mathematical tools).

Notice that the table lists the farm population in *thousands of people*. For example, in year 1970 there were 9712 thousand people—or 9,712,000 people—living on farms. In other words, you need to tag three zeros onto each of those numbers.

Note also that these numbers are actually *rounded* to the nearest thousand—no population remains an exact number of so many thousand people, year after year.

Year	Farm Population (thousands of people)
1900	29875
1910	32077
1920	31974
1930	30529
1940	30547
1950	23048
1960	13445
1970	9712
1980	6051
1990	4591

Source: Census of Agriculture



- Consider the graph above. Estimate the U.S. farm population in the year 2000.
  - In which two decades were the greatest drops in farm population?
  - How many people did the farm population decrease during those two decades (separately)?
  - What was the first year when the farm population dropped below 10 million?
  - When approximately did the farm population drop below 5,000,000?



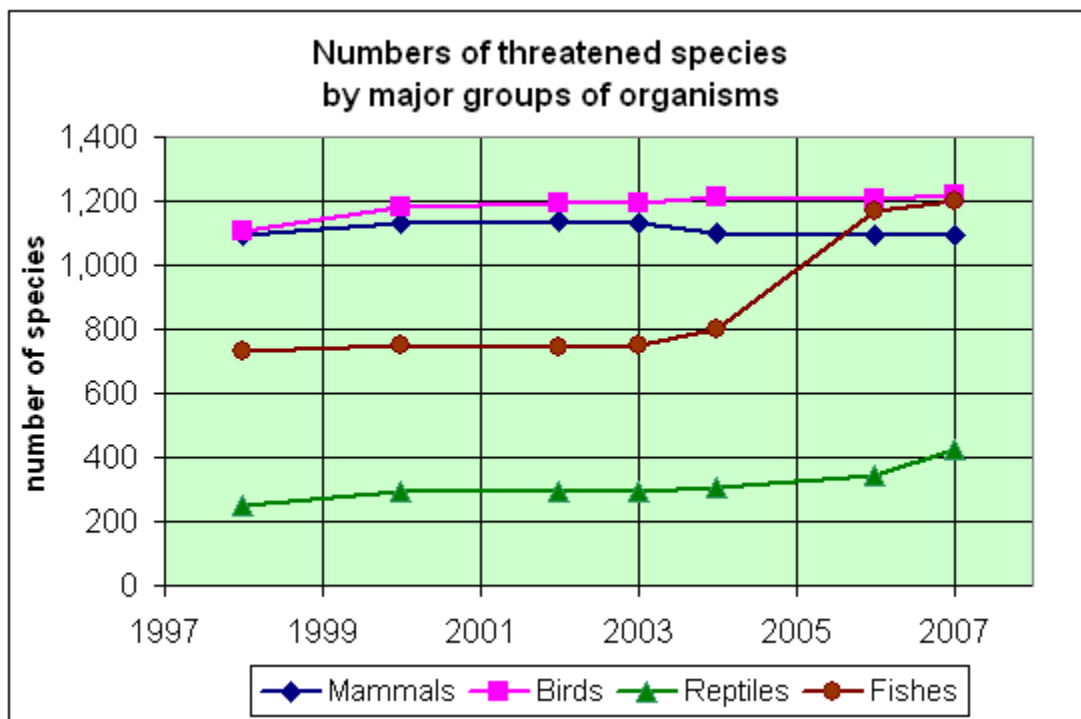
2. The International Union for Conservation of Nature (IUCN) produces a report every few years called *IUCN Red List of Threatened Species*. This report lists the number of animal and plant species that are considered endangered and extinct. The term “threatened” actually means the species can either be considered “Critically Endangered,” “Endangered,” or “Vulnerable.”

Study the data and the graph below, and answer the questions.

**Table 1: Numbers of threatened species by major groups of organisms (1996–2007)**

Number of threatened species →	in 1996/98	in 2000	in 2002	in 2003	in 2004	in 2006	in 2007
Mammals	1,096	1,130	1,137	1,130	1,101	1,093	1,094
Birds	1,107	1,183	1,192	1,194	1,213	1,206	1,217
Reptiles	253	296	293	293	304	341	422
Fishes	734	752	742	750	800	1,171	1,201

(Data from 2007 IUCN Red List: Table 1)



- How many reptile species were considered “threatened” in 2003? In 2007?
- How many fish species were considered “threatened” in 2003? In 2007?
- In which major animal group has the number of threatened species stayed approximately the same over this period?
- In which major animal group has the number of threatened species increased substantially over this period?
- In which major animal groups has the number of threatened species increased only slightly over this period?

# MATH MAMMOTH

## Grade 5-B

### Complete Worktext

- Fractions: add and subtract
- Fractions: multiply and divide
- Geometry
- Integers
- Introduction to percent



By Maria Miller

[www.MathMammoth.com](http://www.MathMammoth.com)

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# Foreword

In part B of the Math Mammoth Grade 5 complete curriculum we study several important topics: fractions, geometry, integers, and a little about percent.

Each chapter has an introduction, which contains notes to the teacher; then follow the actual lessons with problems. Each chapter ends in a review lesson. The chapter tests and cumulative reviews are found in a separate folder and are printed separately. This product also includes an HTML page that you can use to make extra practice worksheets for computation.

In chapter 5, we start the study of fractions, especially focusing on learning to add and subtract unlike fractions. The next chapter continues with fraction topics such as multiplication, simplifying, and the division of fractions. The conversion of fractions to decimals is also included. This essentially completes the main parts of fraction arithmetic, and what is left for 6th grade is mainly review and some deepening of the fraction topics studied here.

Chapter 7 has to do with geometry. The main focus areas are calculating areas of common shapes, including surface areas of some solids, and the volume of a rectangular prism. The chapter also covers classifying quadrilaterals and triangles, similar figures in a coordinate grid, nets of solids, and includes lots of drawing problems.

What follows is a short chapter on integers (chapter 8). We study adding and subtracting integers using several different models as “stepping stones” to the common shortcuts. Then students encounter again the coordinate grid, this time with all four quadrants, and have some fun with moving figures and plotting simple functions.

Lastly, the book includes a short chapter on percent (chapter 9). This is an introduction to the concept of percent with some easy calculations. We will delve into the concept of percent much more in 6th grade.

## Concerning Challenging Word Problems

I would heartily recommend supplementing this program with regular practice of challenging word problems and puzzles from other sources. You could do that once a week to once every two weeks - just occasionally as it suits your schedule. The goal of challenging story problems and puzzles is to simply **develop children's logical and abstract thinking and mental discipline.**

I have made lots of word problems for the Math Mammoth curriculum. Those are for the most part multi-step word problems. I have included several lessons that utilize the bar model for solving problems and tried to vary the problems.

Even so, the problems I've created are usually tied to a specific concept or concepts. I feel children can also benefit from problem solving practice where the problems require "out of the box" thinking, or are puzzle-type in nature, or are just different from the ones I have made. Additionally, I feel others are more capable of making very different, very challenging problems.

So I'd like for you to use one or several of the resources below for some different problems and puzzles. Choose something that fits your budget (most of these are free) and that you will like using.

### **Math Kangaroo Problem Database**

Easily made worksheets of challenging math problems based on actual past Math Kangaroo competition problems.

<http://www.kangurusa.com/clark/pdb/>

### **Primary Grade Challenge Math by Edward Zaccaro**

The book is organized into chapters, with each chapter presenting a type of problem and the ways to think about that problem. Then, there is a series of related story problems to solve, divided into 4 levels. \$25, ISBN 978-0967991535

*You can find this at Amazon.com or various other bookstores.*

<http://www.amazon.com/dp/0967991536/?tag=homeschoolmath-20>

### **Problem Solving Decks from North Carolina Public Schools**

Includes a deck of problem cards for grades 1-8, student sheets, and solutions. Many of these problems are best solved with calculators. All of these problems lend themselves to students telling and writing about their thinking.

[http://community.learnnc.org/dpi/math/archives/2005/06/problem\\_solving.php](http://community.learnnc.org/dpi/math/archives/2005/06/problem_solving.php)

### **Math Stars Problem Solving Newsletter (grades 1-8)**

These newsletters are a fantastic, printable resource for problems to solve and their solutions.

[http://community.learnnc.org/dpi/math/archives/2005/06/math\\_stars\\_news.php](http://community.learnnc.org/dpi/math/archives/2005/06/math_stars_news.php)

### **Mathematics Enrichment - nrich.maths.org**

Open-ended, investigative math challenges for all levels from the UK. Find the past issues box down in the left sidebar. Choose Stage 2 problems for 5th grade.

<http://nrich.maths.org/public/>

<http://nrich.maths.org/public/themes.php> lets you find problems organized by mathematical themes.

### **Figure This! Math Challenges for Families**

Word problems related to real life. They don't always have all of the information but you have to estimate and think. For each problem, there is a hint, other related problems, and interesting trivia. Website supported by the National Council of Teachers of Mathematics.

<http://www.figurethis.org/>

### **MathStories.com**

Over 12,000 interactive and non-interactive NCTM compliant math word problems, available in both English and Spanish. Helps elementary and middle school children boost their math problem solving and critical-thinking skills. It is a membership site.

<http://www.mathstories.com/>

### **“Problem of the Week” (POWs)**

Problem of the week contests are excellent for finding challenging problems and for motivation. There exist several:

- **Math Forum: Problem of the Week**

Five weekly problem projects for various levels of math. Mentoring available.

<http://mathforum.org/pow/>

- **Math Contest at Columbus State University**

Elementary, middle, algebra, and “general” levels.

<http://www.colstate.edu/mathcontest/>

- **Aunty Math**

Math challenges in a form of short stories for K-5 learners posted bi-weekly. Parent/Teacher Tips for the current challenge explains what kind of reasoning the problem requires and how to possibly help children solve it.

<http://www.auntymath.com/>

- **Grace Church School’s ABACUS International Math Challenge**

This is open to any child in three different age groups.

<http://www.gcschool.org/pages/program/Abacus.html>

- **MathCounts Problem of the Week Archive**

Browse the archives to find problems to solve. You can find the link to the current problem on the home page.

<http://mathcounts.org/Page.aspx?pid=355>

- **Math League’s Homeschool Contests**

Challenge your children with the same interesting math contests used in schools. Contests for grades 4, 5, 6, 7, 8, Algebra Course 1, and High School are available in a non-competitive format for the homeschoolers. The goal is to encourage student interest and confidence in mathematics through solving worthwhile problems and build important critical thinking skills. By subscription only.

<http://www.mathleague.com/homeschool.htm>



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# Chapter 5: Fractions: Add and Subtract

## Introduction

In fifth grade students study all aspects of fraction arithmetic. What students have learned in previous years hopefully has built a solid conceptual understanding in their minds, on which we can now build more.

The chapter starts out with lessons on various ways to add and subtract mixed numbers. These are meant partially to review and partially to develop speed in fraction calculations.

The focus of this chapter is on adding and subtracting unlike fractions. Students have already added and subtracted *like* fractions in previous grades. Now it's time to "tackle" the more complex situation.

For that purpose, students need to learn first how to convert fractions into other equivalent fractions. We begin with a visual model of splitting pieces of pie, and from that we develop the common procedure. This skill is then used immediately in the next lessons about adding unlike fractions.

In learning how to add unlike fractions, we begin similarly by using visual models. From the visual and concrete we gradually advance toward the abstract. Several lessons are devoted to understanding the concepts and also to applying this new skill to subtraction as well as to addition and to mixed numbers as well as to fractions.

Next come two lessons on comparing fractions. In these, students learn both mental math methods for comparing fractions and a "brute force" method based on converting to equivalent fractions.

This chapter ends with a lesson on measuring in inches, this year using units as small as  $1/16^{\text{th}}$  of an inch.

### The Lessons in Chapter 5

	<i>page</i>	<i>span</i>
Fraction Terminology .....	12	1 page
Review: Mixed Numbers .....	13	4 pages
Adding Mixed Numbers 1 .....	17	3 pages
Subtracting Mixed Numbers 1 .....	20	3 pages
Subtracting Mixed Numbers 2 - Renaming .....	23	3 pages
Subtracting Mixed Numbers - Extra Practice .....	26	2 pages
Equivalent Fractions .....	28	4 pages
Equivalent Fractions 2 .....	32	2 pages
Adding Unlike Fractions 1 .....	34	3 pages
Adding Unlike Fractions 2: Finding the Common Denominator .....	37	3 pages
Add and Subtract Unlike Fractions .....	40	2 pages
Mixed Numbers with Unlike Fractional Parts .....	42	4 pages
Add and Subtract Several Unlike Fractions .....	46	2 pages

Comparing Fractions 1 .....	48	2 pages
Comparing Fractions 2 .....	50	3 pages
Measuring in Inches .....	53	4 pages
Review: Fractions .....	57	1 page

## Helpful Resources on the Internet

### Visual Fractions

A great site for studying all aspects of fractions, including: identifying, renaming, comparing, addition, subtraction, multiplication, division. Each topic is illustrated by a Java applet with either a number line or a circle. There are also a couple of games, for example: make cookies for Grampy.

<http://www.visualfractions.com>

### Conceptua Math

Conceptua Math has free, interactive fraction tools and activities that are very well made. The activities include identifying fractions, adding and subtracting, estimating, finding common denominators and more. Each activity uses several fraction models such as fraction circles, horizontal and vertical bars, number lines, etc. that allow students to develop conceptual understanding of fractions.

[www.conceptuamath.com](http://www.conceptuamath.com)

### Who Wants Pizza?

Explains the concept of fractions, presents addition and multiplication with a pizza example, and then has some interactive exercises.

<http://math.rice.edu/~lanius/fractions/index.html>

### Fraction lessons at MathExpression.com

Tutorials, examples, and videos explaining all the basic fraction math topics. Look for the lesson links in the left sidebar.

<http://www.mathexpression.com/understanding-fractions.html>

### Clara Fraction's Ice Cream Shop

A game in which you convert improper fractions to mixed numbers and scoop the right amount of ice cream flavors into the cone.

<http://www.mrnussbaum.com/icecream/index.html>

### Equivalent Fractions from National Library of Virtual Manipulatives (NLVM)

See how two fractions are equivalent as the applet divides the whole into more pieces.

[http://nlvm.usu.edu/en/nav/frames\\_asid\\_105\\_g\\_2\\_t\\_1.html](http://nlvm.usu.edu/en/nav/frames_asid_105_g_2_t_1.html)

### Equivalent Fractions

Draw two fractions equivalent to the given fraction. Choose either a square or a circle for the basic shape.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=80>

### Fresh Baked Fractions

Practice equivalent fractions by clicking on a fraction that is *not* equal to the others.

<http://www.funbrain.com/fract/index.html>

**Adding fractions**

Uses pictures to illustrate finding the common denominator.

[matti.usu.edu/nlvm/nav/frames\\_asid\\_106\\_g\\_2\\_t\\_1.html](http://matti.usu.edu/nlvm/nav/frames_asid_106_g_2_t_1.html)

**Fraction Frenzy**

Click on pairs of equivalent fractions as fast as you can. See how many levels you can finish!

<http://www.learningplanet.com/sam/ff/index.asp>

**Fractioncity**

Make “fraction streets” and help children with comparing fractions, equivalent fractions, addition of fractions of like and unlike denominators while they drive toy cars on the streets. This is not an online activity but has instructions of how to do it at home or at school.

<http://www.teachnet.com/lesson/math/fractioncity.html>

**Visual Math Learning**

Free tutorials with some interactivity about all the fraction operations. Emphasizes visual models and lets the student interact with them.

[http://www.visualmathlearning.com/pre\\_algebra/chapter\\_9/chap\\_9.html](http://www.visualmathlearning.com/pre_algebra/chapter_9/chap_9.html)

**Comparison Shoot Out**

Choose level 2 or 3 to compare fractions and shoot the soccer ball to the goal.

<http://www.fuelthebrain.com/Game/play.php?ID=47>

**Old Egyptian Fractions**

Puzzles to solve: add fractions like a true Old Egyptian Math Cat!

<http://www.mathcats.com/explore/oldegyptianfractions.html>

**Online Fraction Calculator**

Add, subtract, multiply, or divide fractions and mixed numbers.

[http://www.homeschoolmath.net/worksheets/fraction\\_calculator.php](http://www.homeschoolmath.net/worksheets/fraction_calculator.php)

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# Adding Unlike Fractions 1

Cover the page below the black line. Then try to figure out additions below.

$\frac{1}{3} + \frac{1}{2} =$  What fraction would this be?

$\frac{1}{3} + \frac{1}{4} =$  What fraction would this be?

Did you solve the problems above? Study the pictures below for solutions. Discuss them with your teacher.

<p style="text-align: center;"> <math>\frac{1}{3} + \frac{1}{2}</math>  <math>\downarrow \quad \downarrow</math>  <math>\frac{2}{6} + \frac{3}{6} = \frac{5}{6}</math> </p>	<p style="text-align: center;"> <math>\frac{1}{3} + \frac{1}{4}</math>  <math>\downarrow \quad \downarrow</math>  <math>\frac{4}{12} + \frac{3}{12} = \frac{7}{12}</math> </p>	<p style="text-align: center;"> <math>\frac{1}{6} + \frac{1}{2}</math>  <math>\downarrow \quad \downarrow</math>  <math>\frac{1}{6} + \frac{3}{6} = \frac{4}{6}</math> </p>
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To add unlike fractions, first convert them to \_\_\_\_\_ fractions. Then add.

1. Write the fractions, convert them into equivalent fractions, and then add them. Color the missing parts.

<p style="text-align: center;"> <b>a.</b> <math>\frac{1}{2} + \frac{1}{4} = \frac{3}{4}</math> </p>	<p style="text-align: center;"> <b>b.</b> <math>\frac{2}{5} + \frac{1}{2} = \frac{9}{10}</math> </p>	<p style="text-align: center;"> <b>c.</b> <math>\frac{1}{5} + \frac{1}{3} = \frac{7}{15}</math> </p>
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Let's write the addition all on the same line now (horizontally).

$$\frac{3}{10} + \frac{2}{5} = \frac{3}{10} + \frac{4}{10} = \frac{7}{10}$$

2. Change these to equivalent fractions first and then add them. Each box below has TWO problems. In the bottom problem, you need to figure out what kind of pieces to use, but the *top* problem will help you do that!

**a.**

$$\frac{1}{2} + \frac{1}{6} = \frac{3}{6} + \frac{1}{6} =$$

$$+ = + =$$

**b.**

$$+ = + =$$

$$+ = + =$$

**c.**

$$+ = + =$$

$$+ = + =$$

**d.**

$$+ = + =$$

$$+ = + =$$

3. Look at the problems above in exercise (2). What kind of parts did the fractions have? What kind of parts did you use in the final addition? Fill in the table.

Types of parts to add:	Converted to:
a. 2nd parts + 6th parts	<u>6th</u> parts
c. 8th parts + 4th parts	___ parts

Types of parts to add:	Converted to:
b. 4th parts + 6th parts	___ parts
d. 2nd parts + 8th parts	___ parts

4. In the problems below, split the parts (as in the example at the right) so that both fractions will have the same kind of parts.

Write an addition sentence.

These pictures change into...      ...these.

**Example:**

$\frac{1}{4} + \frac{3}{8}$        $\frac{2}{8} + \frac{3}{8} = \frac{5}{8}$

 <b>a.</b> $\square + \square =$	 <b>b.</b> $\square + \square =$	 <b>c.</b> $\square + \square =$
 <b>d.</b> $\square + \square =$	 <b>e.</b> $\square + \square =$	 <b>f.</b> $\square + \square =$

5. Fill in the table based on the problems above in exercise (4).

Types of parts to add:	Converted to:	Types of parts to add:	Converted to:
a. 2nd parts + 8th parts	___ parts	d. 2nd parts + 5th parts	___ parts
b. 2nd parts + 4th parts	___ parts	e. 3rd parts + 5th parts	___ parts
c. 3rd parts + 6th parts	___ parts	f. 3rd parts + 2nd parts	___ parts

6. Now think: How can you know into what kind of parts to convert the fractions that you are adding? Can you see any patterns or rules in the table?

7. **Challenge:** If you think you know what kind of parts to convert these fractions to, then try these problems. Don't worry if you don't know how to do them—we will study this more in the next lesson.

<b>a.</b> $\frac{1}{2} + \frac{2}{3} =$	<b>b.</b> $\frac{2}{5} + \frac{2}{3} =$	<b>c.</b> $\frac{1}{3} + \frac{1}{4} =$
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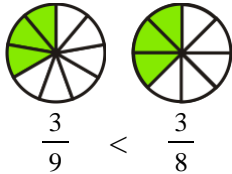
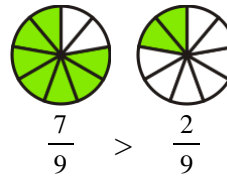
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# Comparing Fractions 1

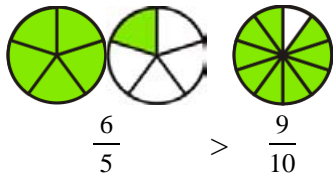
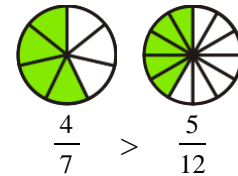
Sometimes it is easy to know which fraction is greater.

With like fractions, all you have to do is to check which fraction has more “slices,” and that fraction is greater.

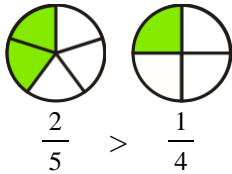


If both fractions have the same number of pieces, then the one with bigger pieces is greater.

Sometimes you can see that one fraction is less than  $1/2$  and the other is more than  $1/2$ . Here,  $4/7$  is clearly more than  $1/2$ , and  $5/12$  is clearly less than  $1/2$ .



Any fraction that is bigger than one must also be bigger than any fraction that is less than one. Here,  $6/5$  is more than 1, and  $9/10$  is less than 1.



If you can imagine the pie pictures in your mind, then you can “see” which fraction is bigger. For example, it’s easy to see that  $2/5$  is more than  $1/4$ .

1. These are like fractions. Compare them, and write  $>$  or  $<$ .

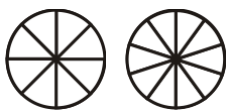
a.  $\frac{8}{11}$     $\frac{4}{11}$

b.  $\frac{21}{16}$     $\frac{25}{16}$

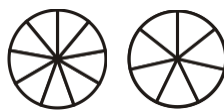
c.  $\frac{4}{20}$     $\frac{2}{20}$

d.  $\frac{49}{100}$     $\frac{61}{100}$

2. These fractions have the same number of pieces. Compare them, and write  $>$  or  $<$ .



a.  $\frac{1}{8}$     $\frac{1}{10}$



b.  $\frac{3}{9}$     $\frac{3}{7}$

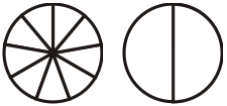
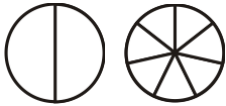
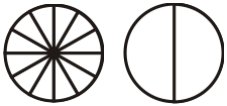
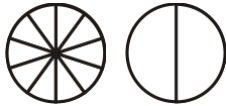
c.  $\frac{2}{11}$     $\frac{2}{5}$

e.  $\frac{7}{4}$     $\frac{7}{6}$

d.  $\frac{5}{14}$     $\frac{5}{9}$

f.  $\frac{1}{20}$     $\frac{1}{8}$

3. Compare these fractions to one half. Then write  $>$ ,  $<$ , or  $=$ .

 <b>a.</b> $\frac{4}{9}$ $\frac{1}{2}$	 <b>b.</b> $\frac{1}{2}$ $\frac{4}{7}$	 <b>c.</b> $\frac{7}{12}$ $\frac{1}{2}$	 <b>d.</b> $\frac{6}{10}$ $\frac{1}{2}$
<b>e.</b> $\frac{1}{2}$ $\frac{3}{4}$	<b>f.</b> $\frac{3}{6}$ $\frac{1}{2}$	<b>g.</b> $\frac{1}{2}$ $\frac{5}{8}$	<b>h.</b> $\frac{5}{11}$ $\frac{1}{2}$

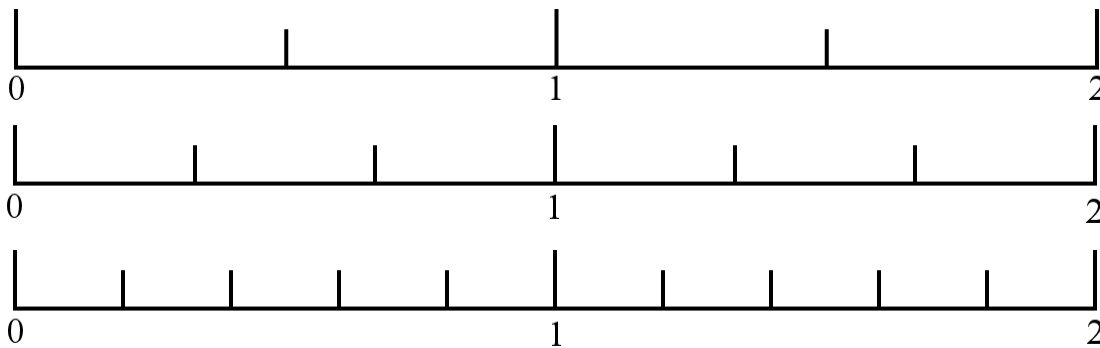
4. Compare each fraction to one. Then write  $>$ ,  $<$ , or  $=$  in the box.

**a.**  $\frac{8}{7}$    $\frac{3}{3}$      
 **b.**  $\frac{4}{4}$    $\frac{9}{11}$      
 **c.**  $\frac{6}{5}$    $\frac{3}{4}$      
 **d.**  $\frac{7}{8}$    $\frac{8}{7}$      
 **e.**  $\frac{12}{9}$    $\frac{8}{11}$

5. Compare these fractions by imagining the pies in your mind.

**a.**  $\frac{3}{4}$    $\frac{5}{6}$      
 **b.**  $\frac{1}{3}$    $\frac{2}{8}$      
 **c.**  $\frac{1}{3}$    $\frac{3}{9}$      
 **d.**  $\frac{5}{6}$    $\frac{7}{8}$      
 **e.**  $\frac{5}{8}$    $\frac{3}{4}$

6. Here are three number lines that are divided respectively into halves, thirds, and fifths. Use them to help you put the given fractions in order, from the least to the greatest.



**a.**  $\frac{1}{3}, \frac{2}{5}, \frac{2}{3}, \frac{1}{5}, \frac{1}{2}$

**b.**  $\frac{7}{5}, \frac{3}{2}, \frac{4}{3}, \frac{6}{5}, \frac{2}{2}$

\_\_\_\_\_ < \_\_\_\_\_ < \_\_\_\_\_ < \_\_\_\_\_ < \_\_\_\_\_

\_\_\_\_\_ < \_\_\_\_\_ < \_\_\_\_\_ < \_\_\_\_\_ < \_\_\_\_\_

7. For each pair of fractions, find one that is between them. Any such fraction will do!

(Hint: You can visualize pies in your mind, or convert the fractions into like fractions.)

**a.**  $\frac{1}{6} < \text{---} < \frac{1}{3}$

**b.**  $\frac{2}{3} < \text{---} < \frac{7}{8}$

**c.**  $\frac{3}{8} < \text{---} < \frac{1}{2}$

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## Chapter 6: Fractions: Multiply and Divide

### Introduction

This is another long chapter devoted solely to fractions. It rounds out our study of fraction arithmetic. (If you feel that your student would benefit from taking a break from fractions for a while, then you can optionally have him or her study chapter 7 on geometry in between chapters 5 and 6.)

We start out by simplifying fractions. Since this process is the opposite to making equivalent fractions, studied in chapter 5, it should be relatively simple for students to understand. We also use the same visual model, just backwards: This time the pie pieces are joined together instead of split apart.

Next comes multiplying a fraction by a whole number. Since this can be solved by repeated addition, it is not a difficult concept at all. In order to teach the concept, multiplying a fraction by a fraction is first explained as taking a certain part of a fraction. After that, students are shown the usual shortcut for multiplication of fractions.

Simplifying before multiplying is a process that is actually not absolutely necessary for 5th graders, but I have included it here because it prepares students for the same process in future algebra studies and because it makes fraction multiplication easier. Also, I have tried to include explanations of *why* the methods work. Most of these explanations are actually *proofs*. I feel it is a great advantage for students to get used to mathematical reasoning and proof methods well before they start high school geometry.

Then we apply fraction multiplication to calculating the area of rectangles and to multiplying mixed numbers.

Next comes the division of fractions. Fraction division is presented gradually in *five* separate lessons to totally avoid just “announcing” it as one simple shortcut rule. This gradual approach in several steps leads to understanding the concept involved and provides justification for the shortcut.

The first step is dividing a fraction by a whole number, which is easy to understand with the model of dividing pieces of pie among a certain number of people. The next step is “measurement division,” in which we consider, “How many times does the divisor fit into the dividend?” For example,  $3 \div (1/2)$  can be solved that way. Students will notice a shortcut for divisions of the type  $n \div (1/m)$  where  $n$  and  $m$  are whole numbers. This shortcut is of course leading them towards the ultimate shortcut that works with any kind of fraction division.

The lesson about reciprocal numbers is crucial for understanding why the shortcut for fraction division works. It explains, for example, why  $3/4$  fits exactly  $4/3$  times into 1. After this, we generalize the process learned in this lesson to apply to *any* fraction. This also is a common way mathematical proofs are often built: First a specific case is established, and then the general case is derived from it. So the lessons actually illustrate a common technique for mathematical reasoning. It may be slightly disguised, since it is spread over several lessons, but nevertheless students are exposed to the thinking processes used in mathematics.

Some students may not grasp the logical ideas behind the proof. If that happens, make sure they at least get the procedural understanding of fraction division (“To divide by a fraction, multiply by its reciprocal”) and are able to perform the calculations. Then come back to these lessons next year and try again.

The last major topic in this chapter is converting fractions into decimals, and it will be studied again in 6th grade. Students need to understand that sometimes we can perform this conversion with mental math and that division (either long division or dividing with a calculator) will *always* do the trick. The skill of converting fractions to decimals is then applied to some problems with measuring units.

Lastly, we present a comparison between ratios and fractions. Ratios is a topic that will be studied a lot in 6th and 7th grades, especially in connection with proportions. We are laying the groundwork for that here.

## The Lessons in Chapter 6

	<i>page</i>	<i>span</i>
Simplifying Fractions .....	61	4 pages
Simplifying Fractions 2 .....	65	4 pages
Multiplying Fractions by Whole Numbers, 1 .....	69	2 pages
Multiplying Fractions by Whole Numbers, 2 .....	71	2 pages
Multiplying Fractions by Fractions .....	73	4 pages
Simplifying Before Multiplying 1 .....	77	3 pages
Comparing Fractions and Decimal Multiplication .....	80	3 pages
Fraction Multiplication and Area .....	83	3 pages
Multiply Mixed Numbers .....	86	4 pages
Dividing Fractions 1: Divide a Fraction by a Whole Number .....	90	3 pages
Dividing Fractions 2a: How Many Times Does It Fit? .....	93	2 pages
Dividing Fractions 2b: Reciprocal Numbers .....	95	4 pages
Dividing Fractions 3a: The Shortcut .....	99	3 pages
Dividing Fractions 3b: Using the Shortcut .....	102	3 pages
Dividing Mixed Numbers .....	104	4 pages
Fractions to Decimals 1 .....	108	4 pages
Fractions to Decimals 2 - With a Calculator .....	112	1 page
Fractions and Decimals in Measuring Units .....	113	2 pages
Ratios and Fractions .....	115	3 pages
Review 1 .....	118	3 pages
Review 2 .....	121	2 pages

## Helpful Resources on the Internet

### Visual Fractions

A great site for studying all aspects of fractions, including: identifying, renaming, comparing, addition, subtraction, multiplication, division. Each topic is illustrated by a Java applet with either a number line or a circle. There are also a couple of games; for example: make cookies for Grampy.

<http://www.visualfractions.com/>

### Conceptua Math

Conceptua Math has free, interactive fraction tools and activities that are very well made. The activities include identifying fractions, adding and subtracting, estimating, finding common denominators and more. Each activity uses several fraction models such as fraction circles, horizontal and vertical bars, number lines, etc. that allow students to develop conceptual understanding of fractions.

[www.conceptuamath.com](http://www.conceptuamath.com)

### Who Wants Pizza?

These fun tutorials use a pizza example to explain the concept of a fraction and addition and multiplication of fractions. Some interactive exercises are included.

<http://math.rice.edu/~lanius/fractions/index.html>

### Fraction lessons at MathExpression.com

Tutorials, examples, and videos explain all of the basic topics of math with fractions. Look for the lesson links in the left sidebar.

<http://www.mathexpression.com/understanding-fractions.html>

### Fraction Model and Fraction Pie

Select the numerator and the denominator, and the applet shows the fraction as a pie/rectangle/set model, as a decimal, and as a percent.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=44>

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=45>

### Comparing Fractions, Decimals, and Percentages

This site has fact sheets, a nice matching pairs game, online quiz, and printable worksheets.

<http://www.bbc.co.uk/skillswise/numbers/fractiondecimalpercentage/comparing/comparingall3/index.shtml>

### Fraction Decimal Conversion

Practice fractions and decimal numbers online with a matching game, concentration, and flash cards.

<http://www.quia.com/jg/65724.html>

### Online Fraction Calculator

Add, subtract, multiply or divide fractions and mixed numbers.

[http://www.homeschoolmath.net/worksheets/fraction\\_calculator.php](http://www.homeschoolmath.net/worksheets/fraction_calculator.php)

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# Multiplying Fractions by Fractions

Most textbooks simply “announce” the rule for multiplying fractions by fractions. This lesson and the exercises in it will let you think and discover WHY the rule works. So please follow all of the explanation and do all of the exercises.

We have studied how to find  $\frac{1}{2}$  of a whole number. For example  $\frac{1}{2}$  of 24 is  $\frac{1}{2} \times 24 = 12$ .

- **NOTE:** The word *OF* translates into **MULTIPLICATION**.



Finding  $\frac{1}{2}$  of any fraction also means multiplying  $\frac{1}{2}$  times that fraction!

$\frac{1}{2} \times \frac{1}{3}$  means  $\frac{1}{2}$  of  $\frac{1}{3}$ .

Half of the  $\frac{1}{3}$ -piece  is .













$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

$\frac{1}{4} \times \frac{1}{3}$  means  $\frac{1}{4}$  of  $\frac{1}{3}$ .

A fourth part of the  $\frac{1}{3}$ -piece  is .

$$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

1. The pictures show how much pizza is left, and you must share it equally with one, two, or three other people. Divide the pizza. What kind of part do you get? Write a multiplication sentence.

<p>a. Find <math>\frac{1}{2}</math> of </p> $\frac{1}{2} \times \frac{1}{4} =$	<p>b. Find <math>\frac{1}{2}</math> of </p> $\times =$	<p>c. Find <math>\frac{1}{2}</math> of </p> $\times =$	<p>d. Find <math>\frac{1}{2}</math> of </p> $\times =$
<p>e. Find <math>\frac{1}{3}</math> of </p> $\frac{1}{3} \times \frac{1}{2} =$	<p>f. Find <math>\frac{1}{3}</math> of </p> $\times =$	<p>g. Find <math>\frac{1}{3}</math> of </p> $\times =$	<p>h. Find <math>\frac{1}{3}</math> of </p> $\times =$
<p>i. Find <math>\frac{1}{4}</math> of </p> $\times =$	<p>j. Find <math>\frac{1}{4}</math> of </p> $\times =$	<p>k. Find <math>\frac{1}{4}</math> of </p> $\times =$	<p>l. Find <math>\frac{1}{4}</math> of </p> $\times =$

## Shortcut - multiply fractions of the type $\frac{1}{n}$

You might have noticed that in the above exercises, all of our fractions

were of the form  $\frac{1}{n}$  (where  $n$  is a whole number),

and that we could have just multiplied the denominators to get the new denominator.

$$\frac{1}{4} \times \frac{1}{5} = \frac{1}{20} \quad \text{or} \quad \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$



We have now studied how to find  $\frac{1}{2}$  or  $\frac{1}{3}$  or  $\frac{1}{5}$  of some fractions. But how about finding some other kind of fractional part? Let's again compare this to finding fractional parts of whole numbers.

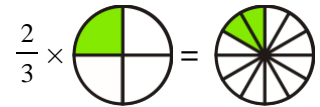
Remember? To find  $\frac{3}{4}$  of 16, or in other words  $\frac{3}{4} \times 16$ , you can first find  $\frac{1}{4}$  of 16, which is 4.

Then just take that three times, which is 12. In other words,  $\frac{3}{4} \times 16 = 12$ .

We can use the same exact idea when finding a fractional part of another fraction.

**Example. Find  $\frac{2}{3}$  of  $\frac{1}{4}$ .** First we find  $\frac{1}{3}$  of  $\frac{1}{4}$ , which is  $\frac{1}{12}$ .

Then,  $\frac{2}{3}$  of  $\frac{1}{4}$  is double that much, or  $\frac{2}{12}$ .



**Example. Find  $\frac{4}{5}$  of  $\frac{1}{7}$**

First we find  $\frac{1}{5}$  of  $\frac{1}{7}$ , which is  $\frac{1}{35}$ . Then,  $\frac{4}{5}$  of  $\frac{1}{7}$  is four times that much, or  $\frac{4}{35}$ .

Multiplying a fraction by a fraction means taking that fractional part of the fraction. It is just like taking a certain part of what is left over when that leftover is already a fraction.

2. The pictures show how much pizza is left, and you get a certain part of the leftovers. How much will you get? Write a multiplication sentence. Color in an answer picture.

a.  $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$  (First find  $\frac{1}{4}$  of  $\frac{1}{2}$ , then multiply the result by 3.)

$$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8} \quad \text{and} \quad \frac{1}{8} \times 3 = \frac{3}{8}$$

b.  $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$  (First find  $\frac{1}{3}$  of  $\frac{1}{2}$ , then multiply the result by 2.)

$$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6} \quad \text{and} \quad \frac{1}{6} \times 2 = \frac{1}{3}$$

c.  $\frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$  (First find  $\frac{1}{4}$  of  $\frac{1}{3}$ , then multiply the result by 3.)

$$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12} \quad \text{and} \quad \frac{1}{12} \times 3 = \frac{1}{4}$$

d.  $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$  (First find  $\frac{1}{3}$  of  $\frac{1}{3}$ , then multiply the result by 2.)

$$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9} \quad \text{and} \quad \frac{1}{9} \times 2 = \frac{2}{9}$$

e.  $\frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$  (First find  $\frac{1}{5}$  of  $\frac{1}{2}$ , then multiply the result by 2.)

$$\frac{1}{5} \times \frac{1}{2} = \frac{1}{10} \quad \text{and} \quad \frac{1}{10} \times 2 = \frac{1}{5}$$

f.  $\frac{4}{5} \times \frac{1}{5} = \frac{4}{25}$  (First find  $\frac{1}{5}$  of  $\frac{1}{5}$ , then multiply the result by 4.)

$$\frac{1}{5} \times \frac{1}{5} = \frac{1}{25} \quad \text{and} \quad \frac{1}{25} \times 4 = \frac{4}{25}$$

**A shortcut for multiplying fractions:**

Multiply the numerators to get the numerator for the answer.  
 Multiply the denominators to get the denominator for the answer.

Let's compare.

The roundabout way	The shortcut
$\frac{5}{6} \times \frac{1}{2} = ?$ (First find $\frac{1}{6}$ of $\frac{1}{2}$ , then multiply the result by 5.) $\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$ and $\frac{1}{12} \times 5 = \frac{5}{12}$	$\frac{5}{6} \times \frac{1}{2} = \frac{5 \times 1}{6 \times 2} = \frac{5}{12}$
$\frac{2}{8} \times \frac{3}{5} = ?$ (Find $\frac{1}{8}$ of $\frac{3}{5}$ , then multiply that result by 2. And to find $\frac{1}{8}$ of $\frac{3}{5}$ , first find $\frac{1}{8}$ of $\frac{1}{5}$ , and then multiply that by 3.) $\frac{1}{8} \times \frac{1}{5} = \frac{1}{40}$ Multiplied by 3 that's $\frac{1}{40} \times 3 = \frac{3}{40}$ Then, multiplied by 2 that's $\frac{3}{40} \times 2 = \frac{6}{40} = \frac{3}{20}$ (simplified)	$\frac{2}{8} \times \frac{3}{5} = \frac{2 \times 3}{8 \times 5} = \frac{6}{40} = \frac{3}{20}$
In the "roundabout way," we do each multiplication separately. In the shortcut, we can just do them all at once.	

**Study the examples on the right.**

$$\Rightarrow \frac{3}{7} \times \frac{4}{9} = \frac{3 \times 4}{7 \times 9} = \frac{12}{63} = \frac{4}{21}$$

Remember always to give your final answer as a mixed number in simplified form.

$$\Rightarrow \frac{12}{5} \times \frac{9}{8} = \frac{12 \times 9}{5 \times 8} = \frac{108}{40} = \frac{27}{10} = 2\frac{7}{10}$$

3. Multiply. Give your answers in the lowest terms (simplified) and as mixed numbers, if possible.

a.  $\frac{3}{9} \times \frac{2}{9}$

b.  $\frac{11}{12} \times \frac{1}{6}$

c.  $8 \times \frac{3}{13}$

d.  $9 \times \frac{2}{3}$

e.  $\frac{2}{9} \times 8$

f.  $10 \times \frac{5}{7}$

4. Multiply. Give your answers in the lowest terms (simplified) and as mixed numbers, if possible.

a.  $\frac{3}{4} \times \frac{7}{8}$

b.  $\frac{7}{10} \times \frac{6}{5}$

c.  $\frac{9}{20} \times \frac{4}{5}$

d.  $\frac{2}{5} \times \frac{1}{3} =$

e.  $\frac{1}{4} \times \frac{2}{7} =$

f.  $\frac{5}{4} \times \frac{1}{3} =$

g.  $\frac{2}{3} \times \frac{3}{4} =$

h.  $\frac{2}{9} \times \frac{2}{3} =$

i.  $\frac{3}{5} \times \frac{1}{10} =$

5. There was  $\frac{1}{4}$  of the pizza left. Marie ate  $\frac{2}{3}$  of that.  
What part of the *original* pizza did she eat?

What part of the *original* pizza is left now?

6. Mary jogs  $\frac{7}{12}$  miles each day, five days a week.  
Calculate how many miles she jogs during a 52-week year.

7. Sally wants to make  $\frac{1}{3}$  of the recipe at the right.  $\Rightarrow$   
How much does she need of each ingredient?

Brownies

- 3 cups sweetened carob chips
- 8 tablespoons extra virgin olive oil
- 2 eggs
- $\frac{1}{2}$  cup honey
- 1 teaspoon vanilla
- $\frac{3}{4}$  cup whole wheat flour
- $\frac{3}{4}$  teaspoon baking powder
- 1 cup walnuts or other nuts

**Puzzle Corner**

Find the missing factors.

a.  $\times \frac{6}{7} = \frac{1}{7}$

b.  $\times \frac{1}{4} = \frac{5}{16}$

c.  $\times \frac{3}{8} = \frac{1}{16}$

d.  $\times \frac{2}{5} = \frac{3}{10}$

e.  $\frac{1}{5} \times = \frac{1}{20}$

f.  $\frac{1}{5} \times = 1$

g.  $\frac{3}{8} \times = 1$

h.  $\frac{5}{6} \times = \frac{1}{3}$


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# Chapter 7: Geometry

## Introduction

The problems in this chapter involve lots of drawing. Geometry is a “hands-on” subject, and many children like that. Moreover, drawing is an excellent means of achieving the conceptual understanding that geometry requires.

Exercises marked with the symbol “” are meant to be done in the student’s notebook or on blank paper.

This geometry chapter starts out with several lessons that review topics studied in previous grades, such as measuring angles, the vocabulary of basic shapes, and using a compass to draw circles. These review lessons also give those children who haven’t previously used *Math Mammoth* an opportunity to “catch up.”

After the review lessons we study equilateral and isosceles triangles. Students are now able to classify triangles both in terms of their sides and also in terms of their angles. The lesson has several drawing problems and one easy compass-and-ruler construction of an equilateral triangle.

Then we go on to study the seven different terms used for classifying quadrilaterals. Many textbooks concentrate on learning the vocabulary only, but I have also included several problems that require some thought and even one construction, that of a rhombus. Although many state standards dictate that these words for various quadrilaterals be learned during some earlier grade (3<sup>rd</sup> or 4<sup>th</sup>), just learning the words—“rhombus,” “trapezoid,” “kite,” and so on—is meaningless until students can also do something with the figures, such as calculate their areas, find their angles, and reason about their properties. For this reason these terms are introduced here in 5<sup>th</sup> grade instead.

This chapter focuses on calculating areas, beginning with the area of a right triangle, which is always half of the area of the corresponding rectangle. Once students learn to calculate the area of a parallelogram and realize that the principle applies not just to right triangles and rectangles, but that the area of *any* triangle is always half of the area of the corresponding parallelogram, then they can split any polygon into triangles and thus find its area.

*Converting Between Area Units* is an introductory lesson. This topic will be studied more in higher grades.

Then we study certain aspects of common solids: the volume of a rectangular prism, surface areas of some solids, and nets of common solids. (Printable pages of some of the nets are included at the very end of the book rather than at the end of the chapter.)

The volumes of round-shaped solids, such as cylinders, cones, and spheres, are not studied until middle school, as students first need to study the concept of  $\pi$  (3.1416...) and its relation to the area of a circle. Another limitation is that students cannot yet find the altitude of the triangle from only the lengths of its sides because that requires the Pythagorean Theorem. Because of these limitations, students cannot yet calculate the surface area or volume of most solids, and these calculations are therefore limited here to certain solids only.

The lesson, *Similar Figures in a Coordinate Grid*, gives students an opportunity to review the coordinate grid and familiarize themselves with similar figures. This lesson is introductory in the sense that similar figures will be studied in much more depth when students understand how to solve proportions.

## The Lessons in Chapter 7

	<i>page</i>	<i>span</i>
Review: Angles .....	127	2 pages
Review: Drawing Figures .....	129	2 pages
Review: Circles .....	131	1 page
Quadrilaterals .....	132	4 pages
Equilateral, Isosceles, and Scalene Triangles .....	136	4 pages
Area of Right Triangles .....	140	3 pages
Area of Parallelograms .....	143	3 pages
Area of Triangles ..	146	3 pages
Area of Polygons .....	149	2 pages
Area and Perimeter Problems .....	151	3 pages
Converting Between Area Units .....	154	2 page
Review: Volume .....	156	1 pages
Printable Cutouts for Common Solids .....	157	6 pages
Volume of Rectangular Prisms (Cuboids) .....	163	3 pages
Surface Area of a Rectangular Prism .....	166	3 pages
Pyramids, Prisms, Cylinders, and Cones .....	169	3 pages
Similar Figures in a Coordinate Plane .....	172	4 pages
A Little Bit of Problem Solving .....	176	1 page
Geometry Review.....	177	3 pages

## Helpful Resources on the Internet

### Turtle Pond

Guide a turtle to a pond using commands that include turning him through certain angles and moving him specific distances.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=83>

### Banana Hunt at *Primary Games*

Help the monkey find bananas and learn to estimate angles.

<http://www.primarygames.co.uk/pg2/bhunt/bhunt.html>

### Ladybug Leaf

Guide the ladybug to turn right or left,  $90^\circ$  or  $45^\circ$ , or to move forward or backward.

[http://nlvm.usu.edu/en/nav/frames\\_asid\\_287\\_g\\_2\\_t\\_3.html](http://nlvm.usu.edu/en/nav/frames_asid_287_g_2_t_3.html)

### Ladybug Mazes

Similar to the Ladybug Leaf, but this time you guide the ladybug through a maze.

[http://nlvm.usu.edu/en/nav/frames\\_asid\\_141\\_g\\_2\\_t\\_3.html](http://nlvm.usu.edu/en/nav/frames_asid_141_g_2_t_3.html)

### Polygon Matching Game

[http://www.mathplayground.com/matching\\_shapes.html](http://www.mathplayground.com/matching_shapes.html)

### Polygon Vocabulary

Another matching game.

<http://www.quia.com/cc/2758.html>

### Shape Explorer

Find the perimeter and area of odd shapes on a rectangular grid.

<http://www.shodor.org/interactivate/activities/perimeter/index.html>

### Patch Tool

An online activity where the student designs a pattern using geometric shapes.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=27>

### Interactive Tangram Puzzle

Place the tangram pieces so that they form a given shape.

[http://nlvm.usu.edu/en/nav/frames\\_asid\\_112\\_g\\_2\\_t\\_1.html](http://nlvm.usu.edu/en/nav/frames_asid_112_g_2_t_1.html)

### Tangram Set

Cut out your tangram set by folding paper.

<http://tangrams.ca/inner/foldtan.htm>

### Geometry - Math Warehouse

Detailed lessons about angles, triangles, quadrilaterals, circles, similar triangles, parallelograms, polygons, and trapezoids.

[www.mathwarehouse.com/geometry](http://www.mathwarehouse.com/geometry)

### National Library of Virtual Manipulatives for Interactive Mathematics: Geometry

A collection of interactive activities: Congruent triangles, fractals, geoboard activities, golden rectangle, ladybug leaf, ladybug mazes, platonic solids, tangrams, tessellations, transformations, and more.

[http://nlvm.usu.edu/en/nav/category\\_g\\_3\\_t\\_3.html](http://nlvm.usu.edu/en/nav/category_g_3_t_3.html)

## **Interactive Geometry**

at Mathsnet.net. This is by far the most comprehensive and magnificent interactive geometry resource I've found on the Internet! The link above is to the main page. You can use these quick links to access some of the tutorials (there are lots more):

[www.mathsnet.net/geometry/index.html](http://www.mathsnet.net/geometry/index.html)

- **Interactive Shape**

Get a clear and complete understanding of the patterns and properties of shapes with the help of interactive pictures.

[www.mathsnet.net/shape/index.html](http://www.mathsnet.net/shape/index.html)

- **Three Dimensions**

View 3-D interactive pictures of Platonic solids, prisms, and pyramids; practice your ability to visualize 3-dimensional objects from all viewpoints; and more.

[www.mathsnet.net/geometry/solid/index.html](http://www.mathsnet.net/geometry/solid/index.html)

## **Triangle Explorer**

Practice finding the area of triangles. NICE!

<http://www.shodor.org/interactivate/activities/triangle/index.html>

## **Interior Angles**

Nice lesson and explanation about interior angles of polygons.

<http://www.coolmath4kids.com/interior.html>

## **Angles**

Practice obtuse/right/acute angles and vertical/corresponding/alternate interior or exterior angle concepts.

<http://www.shodor.org/interactivate/activities/angles/index.html>

## **Area Of Parallelogram**

This Java applet and its explanation illustrate where the formula for the area of a parallelogram comes from.

<http://www.cut-the-not.com/Curriculum/Geometry/AreaOfParallelogram.shtml>

## **Dynamic Rectangle and Parallelogram**

Drag the sides of a dynamic parallelogram or a rectangle to explore these concepts.

<http://standards.nctm.org/document/eexamples/chap5/5.3/index.htm>

## **Length, Perimeter, Area, and Volume of Similar Figures**

Use this interactive figure to explore how the scale factor affects the size and the area of similar figures. The discussion provided helps the teacher somewhat, but a specific lesson plan would be more helpful.

<http://standards.nctm.org/document/eexamples/chap6/6.3/index.htm>

## **Dynamic Geometry® Explorations**

A bunch of interactive geometry activities, along with instructions and questions to guide your exploration. You need a Java-enabled browser.

<http://www.keymath.com/DG/dynamic/index.html>

## **Tim's Triangular Page**

Lists all the basic facts about triangles, followed by some problems.

<http://sakharov.net/triangle.html>

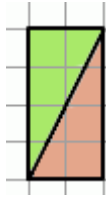


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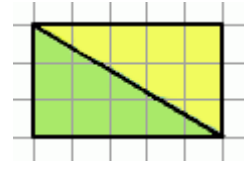
# Area of Right Triangles

This rectangle is divided into two right triangles that are *congruent*. This means that if you could flip one of them and move it on top of the other, they would match exactly.

The rectangle has an area of  $2 \times 4 = 8$  square units.  
Can you figure out what the area of just *one* of the triangles is?

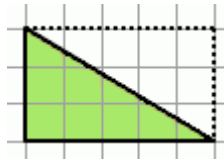


Here the area of the whole rectangle is  $3 \times 5 = 15$  square units.  
How could you figure out the area of just one of the triangles?

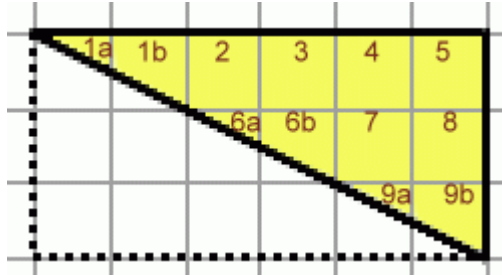


Here the sides of the triangle are 4 and 3 units. The other two sides of the rectangle are drawn with dotted lines.

The area of the *rectangle* is 12 square units. The area of just the triangle is half of that, or 6 square units.



Can you figure out the area of this triangle?

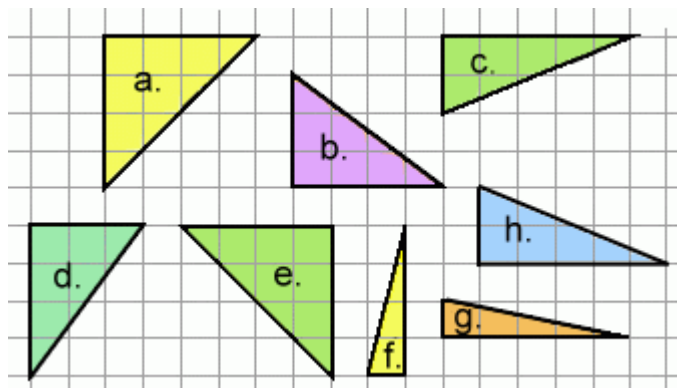


Let's look closer at the last triangle above. Hopefully you figured out that its area is 9. To confirm that, we can actually count the little squares in the triangle.

Notice that some of the parts don't cover a complete square, but by combining those we can make whole squares and then count them.

1. Trace the "helping rectangle" for these right triangles. Then find the area of the triangles.

- a. \_\_\_\_\_ square units
- b. \_\_\_\_\_ square units
- c. \_\_\_\_\_ square units
- d. \_\_\_\_\_ square units
- e. \_\_\_\_\_ square units
- f. \_\_\_\_\_ square units
- g. \_\_\_\_\_ square units
- h. \_\_\_\_\_ square units



To find the area of a right triangle, **multiply the lengths of the two sides** that are *perpendicular* to each other (in other words, the two that form the right angle). Then take **half of that**.

This works because the area of a right triangle is exactly \_\_\_\_\_ of the area of a certain rectangle.

2. Draw a right triangle whose two perpendicular sides are given below, and then find its area.

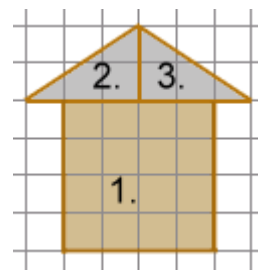


- a. 1 inch and 3 inches
- b. 1.2 cm and 5 cm
- c. 2 1/2 inches and 1 1/2 inches.

We can find the area of this house-shape in three parts.

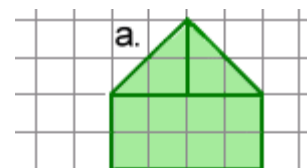
- 1. The square has an area of  $4 \times 4 = 16$  square units.
- 2. Triangle 2 has perpendicular sides of 3 and 2 units, so its area is  $(1/2) \times 2 \times 3 = 3$  square units.
- 3. Triangle 3 is the same shape and size as triangle 2, so its area is also 3 square units.

Lastly, add the areas:  $16 + 3 + 3 = 22$  square units in total.

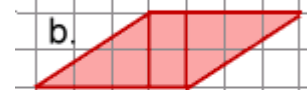


3. Find the area of these compound shapes. The shapes have already been divided into rectangles and right triangles. Just add the areas of the individual pieces!

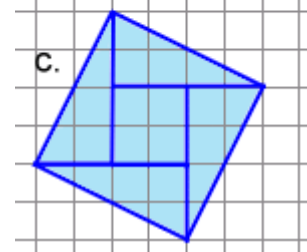
a.



b.

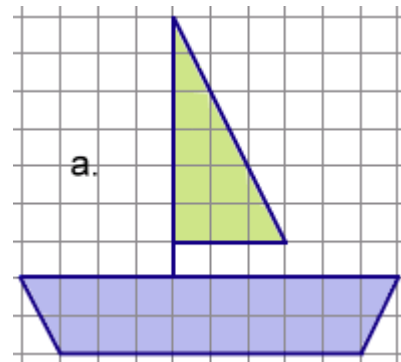


c.

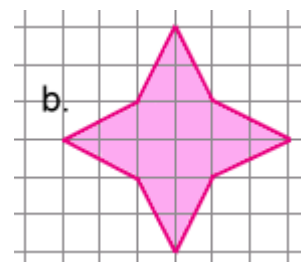


4. Divide the colored shapes into rectangles and right triangles and find their area.

a.



b.

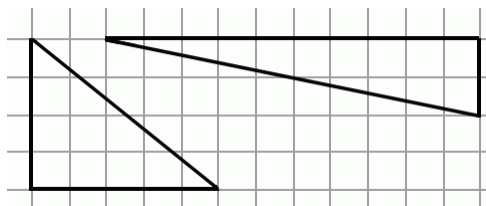


5. a. Draw a right triangle whose *area* is 8 square inches.  
What can the lengths of the two perpendicular sides be?



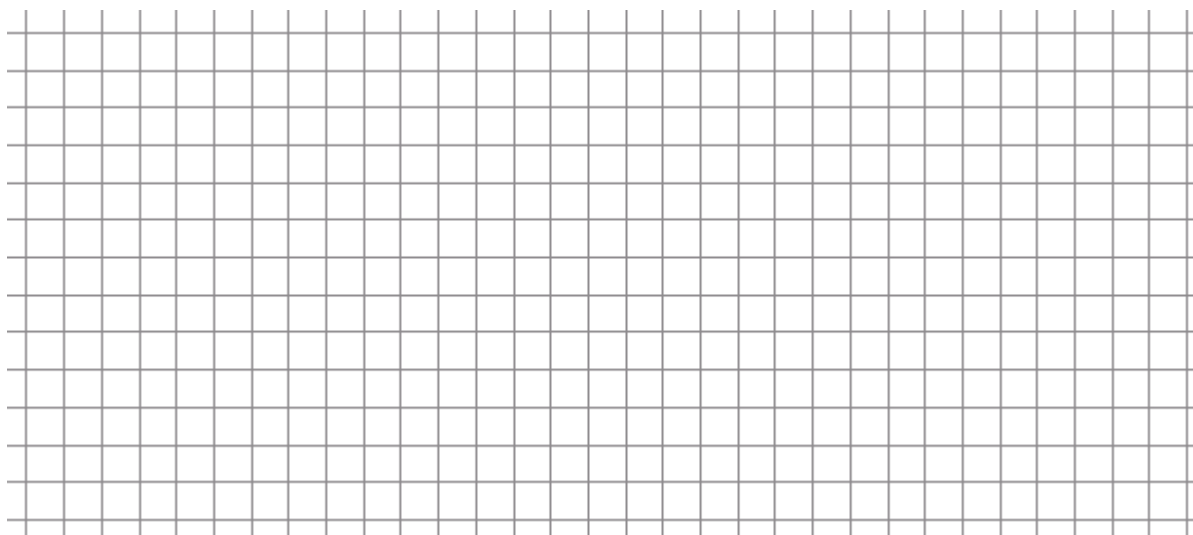
b. Draw a right triangle whose *area* is 13 square centimeters.  
What can the lengths of the two perpendicular sides be?

6. Here you see two different-shaped right triangles that both have the area of \_\_\_\_\_ square units.



On the grid below, draw 2 different right triangles that each have an area of:

a. 6    b. 9 square units.



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# Chapter 8: Integers

## Introduction

This short chapter is intended as an introduction to integers. We won't study all the aspects of arithmetic with integers yet—that will happen in 6<sup>th</sup> and 7<sup>th</sup> grades.

Integers are introduced using the number line to relate them to the concepts of temperature, elevation, and money. Adding and subtracting integers is presented through several models. The two main models are (1) movements along the number line and (2) positive and negative counters. With the help of these models, students should not only learn the shortcuts, or “rules,” for adding and subtracting integers, but also understand *why* these shortcuts work.

This new knowledge is then applied to the coordinate grid. Students learn to locate points in all four quadrants and associate movements in the coordinate grid with addition and subtraction. Lastly, we once again touch on the topic of *functions*. Not only is this last topic preparing students for pre-algebra and algebra courses, but it's also a really nice application of integer addition and subtraction.

This chapter does not cover absolute value or division of integers. Multiplication of integers is only touched on in one of the coordinate grid lessons, and there only the case of a positive integer times a negative integer. The topics left out are studied in 6<sup>th</sup> grade.

### The Lessons in Chapter 8

	<i>page</i>	<i>span</i>
Integers .....	182	2 pages
Addition and Subtraction as Movements .....	184	3 pages
Adding Integers 1 .....	187	3 pages
Adding Integers 2 .....	190	2 pages
Subtracting Integers .....	192	3 pages
Coordinate Grid .....	195	2 pages
Movements in the Coordinate Grid .....	197	2 pages
Functions Again .....	199	3 pages
Review .....	202	2 pages

### Helpful Resources on the Internet

#### Color Chips Addition

The user drags positive or negative chips to the working area, then combines them in pairs to see the sum.  
[http://matti.usu.edu/nlvm/nav/frames\\_asid\\_161\\_g\\_2\\_t\\_1.html](http://matti.usu.edu/nlvm/nav/frames_asid_161_g_2_t_1.html)

#### Number Balls Game

Click on the rotating number balls in ascending order.  
<http://themathgames.com/arithmetic-games/integers/order-positive-negative-integers.php>

### **MathCar Racing**

Outdrive Funbrain's math car by getting the highest score and practice integer operations.

<http://www.funbrain.com/osa/index.html>

### **Choose Operation Game**

Choose the range to include negative numbers in this fun game to get practice in how to add, subtract, multiply, and divide integers.

<http://www.homeschoolmath.net/operation-game.php>

### **Color Chips Subtraction**

Drag individual positive or negative chips or positive-negative chip pairs into working area as instructed, then subtract them.

[http://www.matti.usu.edu/nlvm/nav/frames\\_asid\\_162\\_g\\_2\\_t\\_1.html](http://www.matti.usu.edu/nlvm/nav/frames_asid_162_g_2_t_1.html)

### **Integers: Operations with Signed Numbers**

Clear and cute presentations of the rules for integer operations, including "The Party in the Mathland" for determining the sign of multiplication and division results. There is a link to practice exercises with instant feedback.

<http://amby.com/educate/math/integer.html>

### **Space Coupe to the Rescue**

By choosing a positive or negative number, the player controls the vertical position of a spaceship. If the spaceship reaches the same vertical position as a virus pod, then the pod is destroyed.

<http://pbskids.org/cyberchase/games/negativenumbers/index.html>

### **Flashcards with Negative Numbers**

Interactive flashcards at APlusMath.com.

<http://www.aplusmath.com/Flashcards/sub-nflash.html>

### **Adding Real Numbers at Explorelearning.com**

An interactive "gizmo" that illustrates the addition of integers on a number line. It has an exploration guide and assessment questions. It's by subscription, but it has a free 30-day trial account.

<http://www.explorelearning.com/index.cfm?method=cResource.dspResourcesForCourse&CourseID=211>

### **General Coordinates Game**

Either type in the coordinates of a displayed point or enter coordinates and the applet will plot the point.

<http://www.terragon.com/tkobrien/algebra/topics/orderpairs/op.html>

### **Graph Mole**

A fun game about plotting points in the coordinate plane. Plot the points before the mole eats the vegetables.

<http://funbasedlearning.com/algebra/graphing/default.htm>

### **Graphit**

A graphing tool that plots both functions and ordered pairs.

<http://www.shodor.org/interactivate/activities/graphit/index.html>

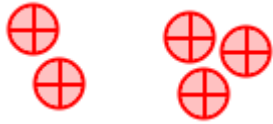
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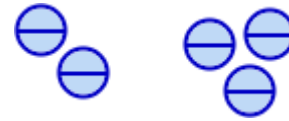
# Adding Integers 1

Addition of integers can be modeled using counters.

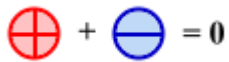
We'll use red counters with a "+" sign for positives and blue counters with a "-" sign for negatives.



This picture shows the addition,  $2 + 3$ . There is one group of 2 positives and another group of 3 positives. The sum is simply 5.



This picture shows the addition,  $(-2) + (-3)$ . We *add* negatives and negatives. In total there are five negatives, so the sum is  $-5$ .



$$1 + (-1) = 0$$

One positive counter and one negative counter *cancel* each other because their sum is zero!



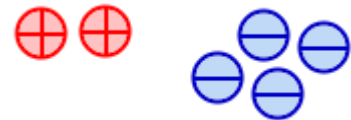
$$2 + (-2) = 0$$

Two negatives and two positives also cancel each other.



$$3 + (-1) = 2$$

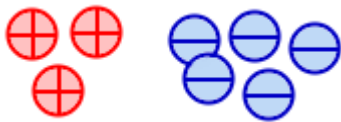
Here, one "positive-negative" pair is canceled, and we are left with 2 positives.



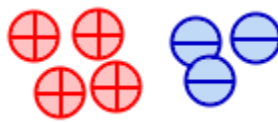
$$2 + (-4) = -2$$

Now the negatives outweigh the positives. Pair up two of each, and there are still two negatives left.

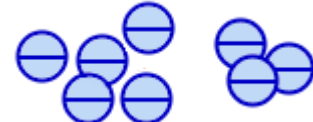
1. Refer to the pictures to solve the equations.



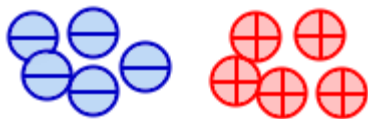
a.  $3 + (-5) = \underline{\hspace{2cm}}$



b.  $4 + (-3) = \underline{\hspace{2cm}}$



c.  $(-5) + (-3) = \underline{\hspace{2cm}}$



d.  $(-5) + 5 = \underline{\hspace{2cm}}$



e.  $(-4) + 3 = \underline{\hspace{2cm}}$



f.  $(-8) + 3 = \underline{\hspace{2cm}}$


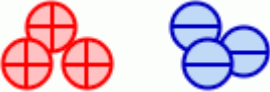
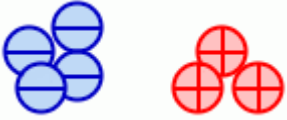
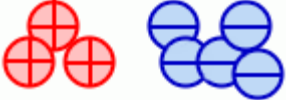


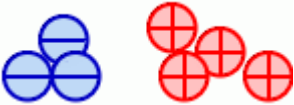
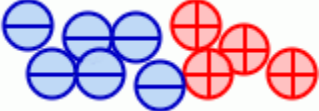
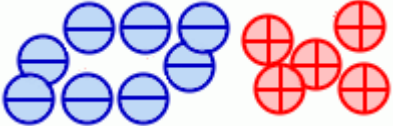
2. These equations contain all negative numbers. Solve them.

a.  $(-5) + (-5) = \underline{\hspace{2cm}}$

b.  $(-2) + (-7) = \underline{\hspace{2cm}}$

c.  $(-18) + (-5) = \underline{\hspace{2cm}}$

3. Write addition sentences (equations) to match the pictures.

<p>a. </p>	<p>b. </p>	<p>c. </p>
<p>d. </p>	<p>e. </p>	<p>f. </p>
<p>g. </p>	<p>h. </p>	<p>i. </p>

4. Rewrite these math sentences in symbols, and solve the resulting addition problems.

- The sum of five negatives and ten positives.
- Add  $-2$  and  $-11$
- Positive 20 and negative 15 added together.

5. Think of the counters. Add.

<p>a. <math>2 + (-6) =</math> <math>(-2) + 6 =</math></p>	<p>b. <math>(-2) + (-6) =</math> <math>2 + 6 =</math></p>	<p>c. <math>4 + (-9) =</math> <math>9 + (-4) =</math></p>	<p>d. <math>10 + (-20) =</math> <math>20 + (-10) =</math></p>
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6. Find the number that is missing from the equations. Think of the counters.

- $2 + \underline{\quad} = -5$
- $-4 + \underline{\quad} = 3$
- $-5 + \underline{\quad} = -11$

7. You can form four different *addition* expressions (sums) using 3 and 7 (in that order), and the plus and minus symbols.

- Write those four expressions.
- Write the expressions in order from the one with greatest value to the one with least value.

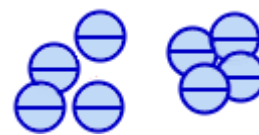
### A note on notation

We can write an elevated minus sign to indicate a negative number:  $^{-}4$ .

Or we can write it with a minus sign and parentheses:  $(-4)$ .

We can even write it without the parentheses if the meaning is clear:  $-4$ .

So  $^{-}4 + ^{-}4 = ^{-}8$  is the same as  $(-4) + (-4) = (-8)$ , which is the same as  $-4 + (-4) = -8$



You *should* write the parentheses if you have  $+$  and  $-$ , or two  $-$  signs, next to each other. So don't write " $8 + - 4$ "; write " $8 + (-4)$ ." And don't write " $3 - -3$ "; write " $3 - (-3)$ ."

8. Think of the counters. Add.

a. $5 + 7 =$ $(-5) + (-7) =$	b. $(-5) + 7 =$ $5 + (-7) =$	c. $^{-}4 + ^{-}4 =$ $^{-}4 + 4 =$	d. $12 + ^{-}1 =$ $^{-}12 + ^{-}1 =$
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9. Solve the problems, and observe the patterns.

a. $5 - 4 =$ $5 - 5 =$ $5 - 6 =$ $5 - 7 =$ $5 - 8 =$	b. $^{-}4 - 0 =$ $^{-}4 - 1 =$ $^{-}4 - 2 =$ $^{-}4 - 3 =$ $^{-}4 - 4 =$	c. $^{-}3 + 0 =$ $^{-}3 + 1 =$ $^{-}3 + 2 =$ $^{-}3 + 3 =$ $^{-}3 + 4 =$	d. $^{-}2 + 2 =$ $^{-}2 + 3 =$ $^{-}2 + 4 =$ $^{-}2 + 5 =$ $^{-}2 + 6 =$
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10. Complete the addition sentences, using one positive integer and one negative integer.

a. $\underline{\quad} + \underline{\quad} = (-1)$ $\underline{\quad} + \underline{\quad} = (-1)$ $\underline{\quad} + \underline{\quad} = (-1)$	b. $\underline{\quad} + \underline{\quad} = 0$ $\underline{\quad} + \underline{\quad} = 0$ $\underline{\quad} + \underline{\quad} = 0$	c. $\underline{\quad} + \underline{\quad} = 2$ $\underline{\quad} + \underline{\quad} = 2$ $\underline{\quad} + \underline{\quad} = 2$
---	--	--

11. Write an *addition or subtraction* sentence to match the temperature change.

a. The temperature was  $-3^{\circ}\text{C}$ . Then it rose 4 degrees. Now it is \_\_\_\_\_.

b. The temperature was  $-5^{\circ}\text{C}$ . Then it rose 2 degrees. Now it is \_\_\_\_\_.

c. The temperature was  $-1^{\circ}\text{C}$ . Then it rose 7 degrees. Now it is \_\_\_\_\_.

d. The temperature was  $3^{\circ}\text{C}$ . It dropped 4 degrees. Now it is \_\_\_\_\_.

e. The temperature was  $5^{\circ}\text{C}$ . It dropped 8 degrees. Now it is \_\_\_\_\_.

f. The temperature was  $-1^{\circ}\text{C}$ . It dropped 5 degrees. Now it is \_\_\_\_\_.

$$^{-}3 + 4 = 1$$

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## Chapter 9: Percent Introduction

In this chapter we study the basics of percent. The whole chapter is intended as an introduction to the concept of percent. These topics will be studied again, and in more depth, in the sixth grade.

We start out by learning what percent means, by writing percents as decimals and as fractions, and by writing fractions as percents. These conversions tie in very closely with earlier lessons about converting fractions to decimals.

Next we learn how to find a certain percentage of a number. The emphasis is on mental math methods; otherwise the student might start relying on a calculator or on a memorized rule without understanding the underlying concept.

Another important topic is “What Percent?”, or finding the percentage in word problems when other information is given. After some more practice with the basics of percent, students also get to study a comparison of ratios, fractions, and percent.

Although a calculator is certainly handy for calculating percent problems, in this chapter the use of a calculator is allowed only in a few problems. That is because the student needs to understand the concepts first before being allowed to rely on a calculator.

I hope the lessons here will give a good overview of the basics of percent, allowing for an understanding of the concepts. Percent will be an important topic in middle school and in real life, so it will pay to understand it well.

### The Lessons in Chapter 9

	<i>page</i>	<i>span</i>
Percent .....	206	3 pages
Percent of a Number .....	209	3 pages
Percent and Fractions with a Diagram .....	212	1 page
What Percent...? .....	213	2 page
Practice with Percent .....	215	2 pages
Ratios, Fractions, and Percents .....	217	3 pages
Review .....	220	1 page

## Helpful Resources on the Internet

### Comparing Fractions, Decimals, and Percentages

Fact sheets, a nice pair-matching game, an online quiz, and printable worksheets.

<http://www.bbc.co.uk/skillswise/numbers/fractiondecimalpercentage/comparing/comparingall3/index.shtml>

### Virtual Manipulatives: Percentages

An interactive tool where the student can fill in any two of three “boxes” (for the whole, the part, and the percent), and the app will calculate what should be in the remaining box and show the result visually in two different ways.

[http://matti.usu.edu/nlvm/nav/frames\\_asid\\_160\\_g\\_2\\_t\\_1.html](http://matti.usu.edu/nlvm/nav/frames_asid_160_g_2_t_1.html)

### Meaning of Percent; Writing Fractions as Percents; Percents and Proportions

Free percent lessons from Math Goodies. Their CD has many more.

[http://www.mathgoodies.com/lessons/vol4/meaning\\_percent.html](http://www.mathgoodies.com/lessons/vol4/meaning_percent.html)

[http://www.mathgoodies.com/lessons/vol4/fractions\\_to\\_percents.html](http://www.mathgoodies.com/lessons/vol4/fractions_to_percents.html)

<http://www.mathgoodies.com/lessons/percent/proportions.html>

### A Conceptual Model for Solving Percent Problems

An explanation of how to use a  $10 \times 10$  grid to explain the basic concept of percent and also to solve various types of percent problems.

[http://illuminations.nctm.org/LessonDetail.aspx?id=L249" target=](http://illuminations.nctm.org/LessonDetail.aspx?id=L249)

### Penguin Waiter

A simple game to calculate the correct tip to leave the penguin waiter.

<http://www.funbrain.com/penguin/>

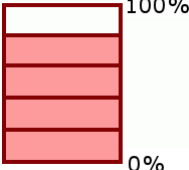
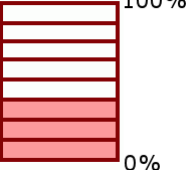
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# Review

1. Write as percentages, fractions, and decimals.

<b>a.</b> $44\% = \frac{\text{yellow}}{\text{yellow}} = \underline{\hspace{2cm}}$	<b>b.</b> $\underline{\hspace{1cm}}\% = \frac{7}{100} = \underline{\hspace{2cm}}$	<b>c.</b> $\underline{\hspace{1cm}}\% = \frac{\text{yellow}}{\text{yellow}} = 0.21$
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2. Write the fractions as percents. Use long division. Round the answers to the nearest tenth of a percent.

<b>a.</b>  Shaded: $\frac{\text{yellow}}{\text{yellow}} = \underline{\hspace{2cm}}\%$ Unshaded: $\frac{\text{yellow}}{\text{yellow}} = \underline{\hspace{2cm}}\%$	<b>b.</b>  Shaded: $\frac{\text{yellow}}{\text{yellow}} = \underline{\hspace{2cm}}\%$ Unshaded: $\frac{\text{yellow}}{\text{yellow}} = \underline{\hspace{2cm}}\%$
---	--

3. Fill in the table. Use mental math.

Percentage / Number →	1,300	700	80	48	2.4
1% of the number					
3% of the number					
10% of the number					
25% of the number					

4. **a.** If  $\frac{11}{20}$  of a certain parcel of land is wasteland, then what percentage of that land is wasteland?  
  
**b.** The area of the parcel is  $4,500 \text{ m}^2$ . Calculate how many square meters of the land is wasteland.
  
5. What is the discount price if a \$16 wall calendar is discounted by 20% ?
  
6. John weighs 27 kg and Matthew weighs 45 kg.  
What percentage of Matthew's weight is John's weight?
  
7. A chain store bought a shipment of 12,000 kg of red, yellow, and green apples. The apples were in a ratio of 2 : 1 : 2 (red to yellow to green).
  - a.** How many kilograms of apples are green?
  - b.** How many percent (by weight) of the apples are green?