Multiply and Divide Rational Numbers 1

In real life we often combine **fractions, decimals, ratios, and percents** — rational numbers in different forms — in the same situation. You need to be able to easily calculate with them in their different forms.

In this lesson, we will concentrate on multiplying and dividing *decimals* and *fractions* because percentages are usually rewritten as decimals and ratios as fractions before calculating with them.

Example 1. Solve $-0.2 \cdot 0.09 \cdot 40$.

First we multiply without any decimal points: $2 \cdot 9 \cdot 40 = 18 \cdot 40 = 720$. The shortcut for decimal multiplication says the answer will have as many decimal digits as are in the factors in total, which in this case means three decimal digits. Also, the answer will be negative (Why?). Therefore, the answer is -0.720, which simplifies to -0.72.

1. Multiply without a calculator.

a. $0.1 \cdot 6.5$	**b.** $-0.08 \cdot 0.006$	**c.** $-0.09 \cdot 0.02$
d. $-0.2 \cdot (-1.6)$	**e.** $-0.8 \cdot 1.1 \cdot (-0.02)$	**f.** 0.8^2
g. $(-0.5)^2$	**h.** $(-0.2)^3$	**i.** $(-0.1)^5$

2. Multiply. Look for the easiest order to multiply. You will also need the regular multiplication algorithm (writing one number under another).

a. $-3 \cdot 4 \cdot 12.5$	**b.** $-0.088 \cdot 4 \cdot (-0.04)$
c. $10 \cdot (-9.08) \cdot (-0.006) \cdot 3$	**d.** $(-0.5)^2 \cdot (-1.087)$

Example 2. Find the value of $-\frac{5}{9} \cdot 2 \cdot \left(-\frac{1}{8}\right) \cdot 9$.

Notice that we have both a fraction with a denominator 9, and a multiplication by 9. Since multiplication is both commutative and associative, we can rearrange these factors and put them in any order, like this:

$-\frac{5}{9} \cdot 9 \cdot 2 \cdot \left(-\frac{1}{8}\right)$. This expression can then be simplified like this: $-\frac{5}{\cancel{9}_{1}} \cdot \cancel{9}^{1} \cdot \cancel{2}^{1} \cdot \left(-\frac{1}{\cancel{8}_{4}}\right) = \frac{5}{4}$.

3. Multiply. Try to find shortcuts and arrangements that make the work easier.
Reminder: mixed numbers will usually need written as fractions before multiplying.

a. $-\frac{1}{7} \cdot \left(-\frac{3}{8}\right) \cdot 7 \cdot \frac{4}{5}$	**b.** $4 \cdot \frac{1}{5} \cdot \left(-2\frac{1}{2}\right)$
c. $-10 \cdot \frac{2}{9} \cdot \frac{5}{6} \cdot (-3) \cdot \frac{9}{10}$	**d.** $\frac{8}{7} \cdot \left(-\frac{3}{10}\right) \cdot 7 \cdot (-2) \cdot \left(1\frac{3}{4}\right)$

4. Explain how you can use the distributive property to find these products. (*Hint: write one factor as a sum.*)

a. $5 \cdot (-0.37)$	**b.** $-3 \cdot (2\frac{3}{4})$

Change divisions into multiplications

You have learned that a division by a fraction can be changed into a multiplication by the reciprocal of the divisor. We can apply this principle, not only with fractions, but any numbers. For example, we can change a division by 6 into a multiplication by 1/6.

Example 3. $\frac{4}{5} \div 3 \div \left(-2\frac{1}{2}\right)$ We will change both divisions into multiplications, step by step. Also, the mixed number needs to be written as a fraction.

$= \frac{4}{5} \cdot \frac{1}{3} \div \left(-\frac{5}{2}\right)$ One more division to change now.

$= \frac{4}{5} \cdot \frac{1}{3} \cdot \left(-\frac{2}{5}\right) = -\frac{8}{75}$

5. Match the equivalent expressions: each expression in the top row matches with one from the bottom row. Also, find the value of each expression.

a. $-\frac{5}{3} \div \frac{1}{4} \cdot 3 \div 5$ **b.** $-\frac{5}{3} \cdot \frac{1}{4} \div 3 \cdot 5$ **c.** $-\frac{5}{3} \div \frac{1}{4} \div 3 \cdot 5$

(i) $-\frac{5}{3} \cdot \frac{1}{4} \cdot \frac{1}{3} \cdot 5$ **(ii)** $-\frac{5}{3} \cdot 12 \cdot \frac{1}{5}$ **(iii)** $-\frac{5}{3} \cdot 20 \cdot \frac{1}{3}$

6. Find the value of the expressions. Give your answer as a mixed number, if applicable.

a. $-\frac{2}{9} \div 4 \div \frac{6}{7}$	**b.** $-3 \div \frac{9}{8} \div \left(-1\frac{1}{2}\right)$	**c.** $-2 \cdot 5 \div 8 \div \frac{5}{6}$

7. Solve using mental math.

a. $-0.88 \div 4$	**b.** $8.1 \div (-9)$	**c.** $72 \div (-10{,}000)$
d. $-1.6 \div (-0.2) \div 4$	**e.** $-8 \div 0.1 \cdot 5$	**f.** $2 \cdot 0.4 \div (-0.2)$

8. Find the value of the expressions. Give your answer as a mixed number, if applicable.

a. $-\frac{1}{9} \div \left(-\frac{1}{3}\right) \cdot 6 \div \frac{2}{3}$	**b.** $10\frac{1}{5} \cdot \frac{5}{3} \div \left(-2\frac{1}{3}\right)$	**c.** $-\frac{21}{6} \div \frac{1}{6} \cdot 4 \div 10$

9. Divide using long division. *Reminder: you will need a whole-number divisor. To obtain that, multiply both the dividend and the divisor repeatedly by 10 until the divisor becomes a whole number.*

a. $-27.5 \div (-0.3)$	**b.** $-28.9 \div 0.11$

Puzzle Corner Find the value of $\frac{(-2)^3}{5} + \frac{3}{4} \div \left(\frac{3}{4} \cdot \frac{10}{18}\right)$.