Two-Step Equations, Part 1

In two-step equations, we need to apply two different operations to both sides of the equation.

Example 1. On the side of the unknown (left), there is a multiplication by 2 and an addition of 3. To isolate the unknown, we need to undo those two operations, in two steps.

$$\begin{aligned} 3x + 2 &= 25 \quad &| -2 \\ 3x &= 23 \quad &| \div 3 \\ x &= 23/3 \end{aligned}$$

Check:

$$\begin{aligned} 3 \cdot (23/3) + 2 &\overset{?}{=} 25 \\ 23 + 2 &\overset{?}{=} 25 \\ 25 &= 25 \ \checkmark \end{aligned}$$

What if you divide first? That is possible:

$$\begin{aligned} 3x + 2 &= 25 \quad &| \div 3 \\ \frac{3x+2}{3} &= \frac{25}{3} \\ x + \frac{2}{3} &= \frac{25}{3} \quad &| -2/3 \\ x &= 23/3 \end{aligned}$$

Note that this leads to fractions in the middle of the solution process which is more error-prone. Then, the 2 on the left side also has to be divided by 3 (to become 2/3). This is something that is easy to forget and is therefore another reason why subtracting first is the "safer" way, in this case.

If this was a real-life application, we would probably give the answer as a decimal, rounded to a reasonable accuracy. Since it is a mathematical problem, we will leave the answer as a fraction. (Why not as a mixed number? It is not wrong, but fractions are less likely to be misread. The mixed number 7 2/3 can easily be misread as 72/3.)

1. Solve. Check your solutions (as always!).

a. $5x + 2 = 67$	**b.** $3y - 2 = 70$	**c.** $3x + 11 = 74$
d. $8z - 2 = 98$	**e.** $75 = 12x + 3$	**f.** $55 = 4z - 11$

Example 2. This equation has decimals. The solution process works the same way. However, the final answer is typically given as rounded. $1.2n - 9.7 = 0.45$	$+ 9.7$ $1.2n = 10.15$	$\div 1.2$ $n \approx 8.5$	When checking an equation with a rounded answer, we don't require precise equality. Near equality is taken as the equation checking. $1.2 \cdot 8.5 - 9.7 \stackrel{?}{=} 0.45$ $0.5 \approx 0.45$ ✓

In mathematics, the usage of fractions typically implies that the values are precise, whereas the usage of decimals implies that the numbers might be rounded, approximate numbers and not precise. As you know, real-life applications often use decimals. So, in the case of an equation like this, we give the final answer rounded. In 8th grade, you will learn precise rounding rules governing these situations. For now, unless otherwise stated, round the final answer to the same accuracy as the least accurate decimal in the original equation.

2. Solve. Give the solutions to two decimal digits. Check your solutions (as always!).

a. $6.3y - 0.4 = 3$	**b.** $5.5 = 0.4y - 2.8$	**c.** $0.77s - 0.12 = 0.43$
d. $62.4 + 10x = 72.78$	**e.** $0.825 = 0.25y + 0.3$	**f.** $2.27t - 3.12 = 3.098$

3. Check each solution below. If it is incorrect, find the error, and correct it.

a.

$$10x - 14 = 31$$
$$10x = 17$$
$$x = 1.7$$

b.

$$31 = 15 + 2x$$
$$46 = 2x$$
$$x = 23$$

These equations have negative numbers. The solution process works the same way. Just be careful to follow the rules of integer arithmetic.

Example 3a.

$$\begin{aligned} -2x + 7 &= -4 && \mid -7 \\ -2x &= -11 && \mid \div(-2) \\ x &= 11/2 \end{aligned}$$

Check:

$$-2(11/2) + 7 \stackrel{?}{=} -4$$
$$-11 + 7 \stackrel{?}{=} -4$$
$$-4 = -4 \ ✓$$

Example 3b.

$$\begin{aligned} -5x - 11 &= 40 && \mid +11 \\ -5x &= 51 && \mid \div(-5) \\ x &= -51/5 \end{aligned}$$

Check:

$$-5(-51/5) - 11 \stackrel{?}{=} 40$$
$$51 - 11 \stackrel{?}{=} 40$$
$$40 = 40 \ ✓$$

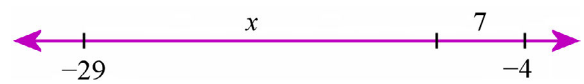
4. Solve. Check your solutions (as always!).

a. $-6x + 2 = -30$	**b.** $-y - 2 = 26$	**c.** $9x - 11 = -8$
d. $12z - 44 = -98$	**e.** $-100 = 30x - 15$	**f.** $-6 = -4t - 11$

5. Solve. Compare the two equations.

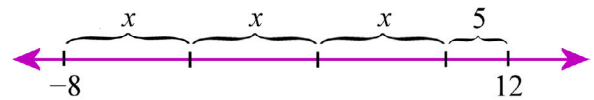
a. $-5y + 2 = -11$	**b.** $5y - 2 = -11$

6. The number line diagram on the right illustrates the equation $-29 + x + 7 = -4$. Figure out the solution using the diagram.

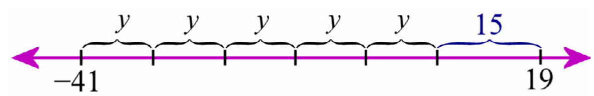


Next, write an equation to match each number line diagram below and find the value of the unknown.

a.



b.



7. Use these for more practice as needed. Use blank paper as necessary. Round the solutions for (b) and (c) to two decimal digits.

a. $3y - 20 = 65$	**b.** $6z + 5 = 2.2$	**c.** $5.2x + 6.25 = 108$
d. $-6s - 2 = 40$	**e.** $5a + 1 = -20$	**f.** $-3t + 2 = -9$

a. Choose from the expressions below to build an equation that has the root $x = 2$.

$2x - 10$ $2x + 10$ 12 $5x + 6$ $3x - 9$ 14 $3 \cdot 3$ $5x - 6$

b. Choose from the expressions below to build an equation that has the root $x = 5$.

$1 - 2x$ -4 $3x - 10$ $-8 \cdot 3$ -2 $-9 - 3x$ $-3x + 6$ $-2x - 1$

Puzzle Corner