Inequalities

An **inequality** contains two expressions that are separated by one of these signs: $<$, $>$, $\leq$ or $\geq$.

(expression 1) $<$ (expression 2)

The sign $\leq$ is read "less than or equal to." It is the $<$ sign and $=$ sign combined.
The sign $\geq$ is read "greater than or equal to." It is the $>$ sign and $=$ sign combined.

Examples.	$\mathbf{6y \geq -2}$	$\mathbf{14 < 2x - 2}$	$\mathbf{w/4 > 200}$
Read:	"6*y* is greater than or equal to a negative two."	"14 is less than 2*x* minus two."	"*w* divided by 4 is greater than 200."

Example 1. Mom said, "Don't spend more than $100."

"Not more than" means "less than or equal to." You can spend $100 or any amount less than $100, but you cannot spend $100.29. We can represent this statement as

$$\textit{money spent} \leq \$100$$

Using the variable *m* for the "money spent," we can write the inequality $m \leq \$100$.

Example 2. The symbol $\geq$ (greater than or equal to) often corresponds to the phrase "at least."

"Could you get me at least 20 apples?" Using *n* for the number of apples, we can write $n \geq 20$.

1. Write an inequality for each phrase. Choose a variable to represent the quantity in question.

a. You have to be at least 21 years of age.

b. Citizens older than 59 years get a free entry.

c. We saw more than 12 birds.

d. I need at least 50 screws for the project.

e. You can spend $80 at the most.

2. Make up a situation from real life that could be described by the given inequality.

a. $a < 30$

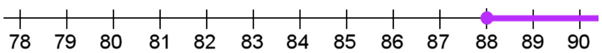
b. $p > 100$

c. $b \geq 8$

d. $x \leq 60$

Typically, an inequality has an infinite number of solutions. For example, the solution to the inequality $s > 6$ is any number that is greater than 6, which there are multitudes of such numbers.

The set of all the possible solutions to an inequality is its **solution set**. We can **plot the solution set of an inequality on a number line.**



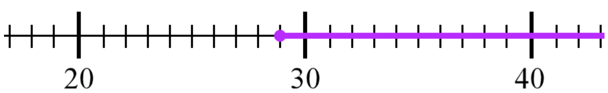
$x \geq 88$

We draw a *closed* circle at 88 because 88 fulfills the inequality. Then we color the number line solid after that point, because any number that is greater than 88 (such as 88.1 or 90.5) also fulfills the inequality.

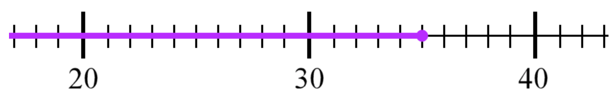
$x < 84$

We draw an *open* circle at 84, because 84 *does not* fulfill the inequality (84 < 84 is a false statement). Yet, any number that is less than 84 works fine, so, we color the number line solid up to 84.

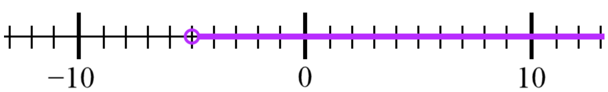
3. Write the inequality illustrated by the number line plot.



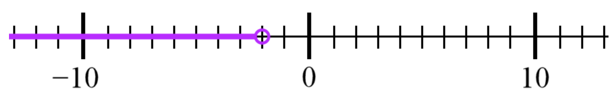
a.



b.

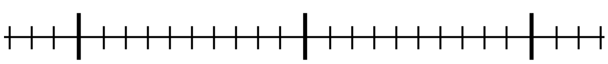


c.

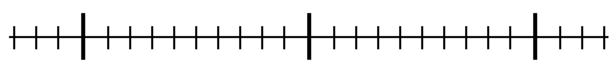


d.

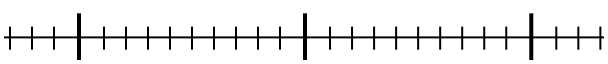
4. Plot each inequality on the number line. Write the appropriate multiples of ten under the bolded tick marks (for example, 30, 40, and 50).



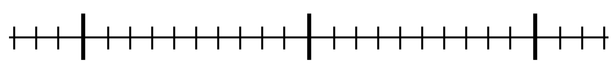
a. $x > 14$



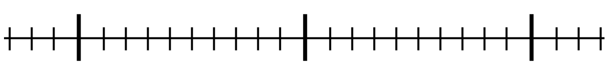
b. $x \geq 78$



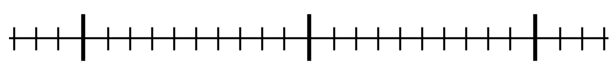
c. $t \leq 52$



d. $t < -7$



e. $t > -13$



f. $t \geq -1$

Inequalities can be solved using algebraic methods. We can:

- add the same number to both sides of an inequality;
- subtract the same number from both sides of an inequality;
- multiply or divide both sides of an inequality by the same positive number.

Each of these operations will preserve the inequality, just like they would preserve the equality in an equation.

Example 3. The inequality $2 < 8$ is a true inequality.

- If we add 5 to both sides, we get $7 < 13$, which is true.
- If we subtract 3 from both sides, we get $-1 < 5$, which is true also.
- If we multiply both sides by 7, we get $14 < 56$, which is true.
- If we divide both sides by 4, we get $1/2 < 2$, which is true.

In each of those operations, the inequality was preserved.

More formally speaking, if $\boldsymbol{a < b}$, then $\boldsymbol{a + 2 < b + 2}$. Also, $\boldsymbol{a - 2 < b - 2}$.

In fact, if $\boldsymbol{a < b}$, then $\boldsymbol{a + n < b + n}$ for any number n.

So, we can add any number to both sides of an inequality, and the inequality is preserved. It is also true that we can subtract any number from both sides of an inequality, and the inequality will be preserved.

Similarly for multiplication: if $\boldsymbol{a < b}$, then $\boldsymbol{3a < 3b}$. It is also true that if $\boldsymbol{a < b}$, then $\boldsymbol{a/8 < b/8}$.

In general, if $\boldsymbol{a < b}$, then $\boldsymbol{na < nb}$ for any positive number n.

There is *one* exception though, and that is if you divide or multiply an inequality by a *negative* number.

Example 4. If we multiply both sides of the inequality $\boldsymbol{3 < 7}$ by -1, we get $\boldsymbol{-3 < -7}$ which is *not* true.

However, if we *reverse* the sign of the inequality from $<$ to $>$ at the same time as multiplying by -1, then we get a true inequality: $\boldsymbol{-3 > -7}$.

However, we will not be dealing with this exception in this course. You will be solving only inequalities where you multiply or divide the inequality by a *positive* number.

Essentially then, solving an inequality works the same as solving an equation (except that one exception of multiplying or dividing by a negative number). We can also notate the solution steps in the same way.

Example 5.

$$\begin{aligned} 5x + 3 &< 6 && \vert\; -3 \\ 5x &< 3 && \vert\; \div 5 \\ x &< 3/5 \end{aligned}$$

Example 6.

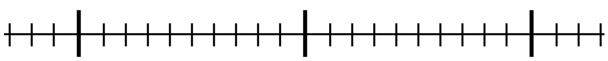
$$\begin{aligned} \tfrac{1}{7}t - 1 &\geq 7 && \vert\; +1 \\ \tfrac{1}{7}t &\geq 8 && \vert\; \cdot 7 \\ t &\geq 56 \end{aligned}$$

5. Solve these inequalities by applying the same operation to both sides. Notice that the inequality symbol (whether $<$, $>$, $\leq$ or $\geq$) does not change.

a. $3y < 48$ $<$	**b.** $y - 8 > 59$ $>$	**c.** $2c - 5 \geq 23$ $\geq$ $\geq$

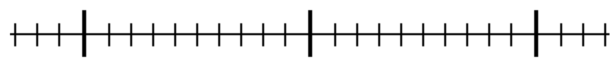
6. Solve each inequality and plot its solution set on the number line. Write the appropriate multiples of ten under the bolded tick marks (for example, 30, 40, and 50).

a. $2x + 12 < 30$



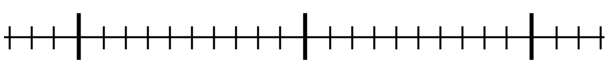
b. $3x - 5 > 83$

$>$



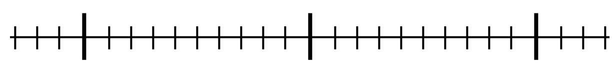
c. $6x - 25 \leq 47$

$\leq$

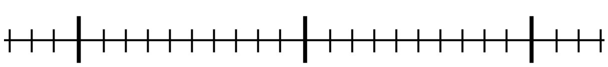


d. $171 \geq 20x - 9$

$\geq$



e. $9a + 5 \leq 12$



f. $-9 \leq 15y + 6$