Contents

Introduction

Math Mammoth Introduction to Fractions contains lessons for fraction arithmetic for grades 1-4. The bulk of the material here is for third and fourth grades, and only a few lessons are intended for grades 1-2.

The topics covered are on a simple level, illustrated with visual models, and with small denominators. The presentation avoids spelling out specific rules for manipulating fractions, but instead relies on the usage of pictures on a very concrete level. Children easily confuse the various rules for fraction arithmetic, because there are so many. There is a place for the rules, as *shortcuts* for ideas that are already understood, but we do not start with them. The goal is to let the big ideas sink in conceptually first, followed by some shortcuts.

The topics covered are

- one-half and one-fourth
- concept of a fraction
- concept of a mixed number
- comparing fractions
- equivalent fractions
- adding and subtracting like fractions
- adding and subtracting mixed numbers with like fractional parts
- adding one fraction that has tenths and another that has hundredths (such as 3/10 + 7/100)
- multiplying a fraction by a whole number

The book does not cover division or multiplication of fractions, nor adding unlike fractions, which are topics for fifth and sixth grades.

The lessons are organized by topic, not by increasing difficulty. For reference, first grade students study only the concept of one-half and one-fourth. In second grade, they study the concept of a fraction and optionally the easiest (first) lesson on comparing fractions. In third grade, students study the concept of a fraction, fractions on a number line, comparing fractions, and equivalent fractions. Then in fourth, they study mixed numbers, comparing fractions, equivalent fractions, adding and subtracting fractions and mixed numbers, and multiplying fractions by whole numbers.

The answers are appended.

I wish you success in teaching math!

Maria Miller, the author

Order Fractions on a Number Line
In this activity, you will place three given fractions on a number line. The fractions have the same denominator.
https://www.mathmammoth.com/practice/order-fractions-number-line#questions=5&number=propFrac,impropFrac&sameDen=1

Mixed Numbers with Visual Models
Explore mixed numbers and improper fractions with the help of visual models and the number line. You can also play a game where you make mixed numbers from visual models.
https://www.mathmammoth.com/practice/mixed-numbers

Equivalent Fractions Hidden Picture Matching Game
Practice simplifying fractions to find equivalent fractions while also uncovering a hidden picture in this fun matching game!
https://www.mathmammoth.com/practice/equivalent-fractions

Compare Fractions
Practice comparing fractions. In the link below, you will compare fractions that have the same denominator, same numerator, or one of the fractions is 1/2 or equal to 1.
https://www.mathmammoth.com/practice/comparing-fractions#q=10&dens=2,3,4,5,6,8&pie=0&improper=0&types=0,1,2,3

Add and Subtract Fractions
Practice addition and subtraction of fractions.
https://www.mathmammoth.com/practice/add-fractions#like=1&proper=1&simplified=0¬-lowest=1&denominators=2,3,4,5,6,8

Multiply Fractions
On this page, you can practice multiplying fractions by whole numbers.
https://www.mathmammoth.com/practice/multiply-fractions#proper=0¬-lowest=1&level=easy&multiply=fbwn

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this book. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

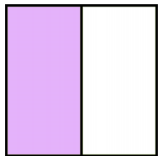
We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better, and even just for some fun. Enjoy!

https://l.mathmammoth.com/blue/introductiontofractions

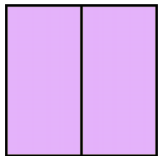
Halves and Quarters, Part 1

Each square below is divided into *two* equal parts or two equal shares. The parts are called ***halves***. Each part is <u>one half</u>.

Here, **one-half** of the square is colored. The other half is white.

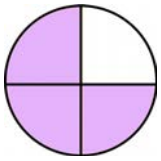


Here, **both halves** are colored. The two halves make up the whole square.

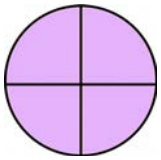


Each circle below is divided into *four* equal parts or four equal shares. The parts are called ***fourths*** or ***quarters***. Each part is <u>one quarter</u> or <u>one fourth</u>.

Here, three-fourths of the circle is colored. One quarter is white.

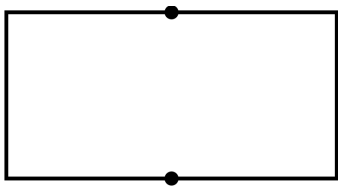


Here, all four quarters are colored. The four quarters make up the whole circle.



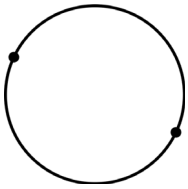
1. Divide these shapes into halves by drawing a straight line from dot to dot. Then color them as the instructions say. Lastly name the colored portion.

a. Color one part.



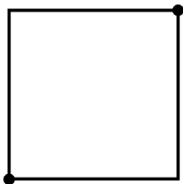
one half

b. Color two parts.



_____

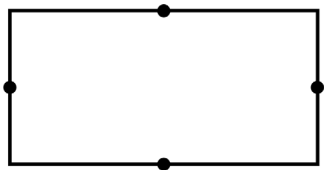
c. Color one part.



_____

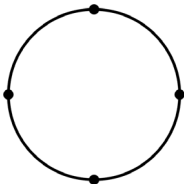
2. Divide these shapes into fourths by drawing two straight lines from dot to dot. Then color them as the instructions say. Lastly name the colored portion.

a. Color one part.



_____

b. Color three parts.



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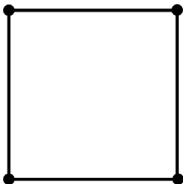
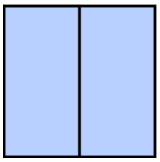
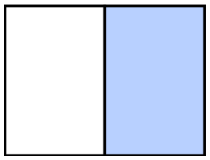
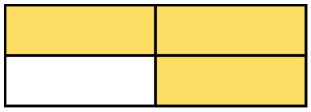
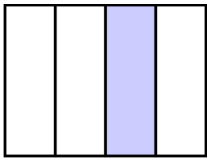
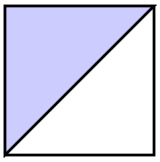
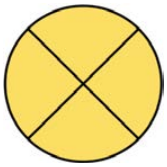
c. Color two parts.

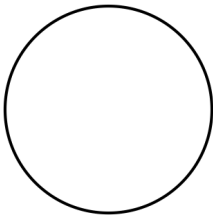
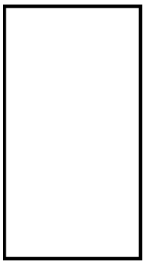
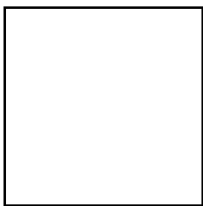
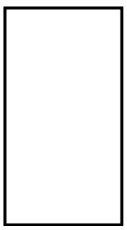
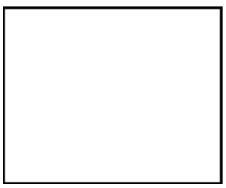
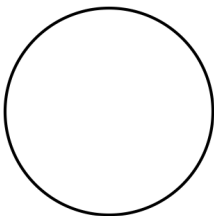


_____

3. Name the colored portion.

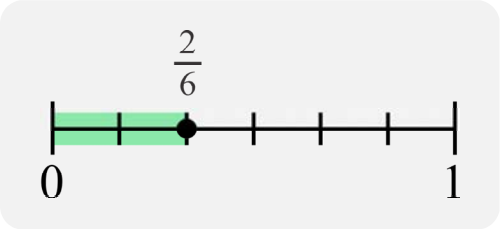
a.  _____	**b.**  _____
c.  _____	**d.**  _____
e.  _____	**f.**  _____

4. First divide the shape into equal shares, then color.

a. Color 1 fourth. 	**b.** Color 3 fourths. 	**c.** Color 2 halves. 	**d.** Color 4 fourths. 
e. Color 1 half. 	**f.** Color 1 fourth. 	**g.** Color 2 fourths. 	**h.** Color 2 halves. 

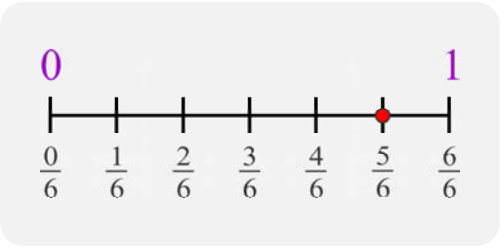
Fractions on the Number Line 1

This is a number line from 0 to 1. It is partitioned or divided into *six* parts. Each sixth, or one part, is this much: ⟼. The green rectangle shows the two parts out of six parts in total.
The point marks the fraction $\frac{2}{6}$ on the number line.



Note: the point for 2/6 is at the right end of the rectangle. Normally, we simply draw a *point* (not a rectangle) on the number line to indicate a number (and fractions *are* numbers). The rectangle is drawn here just to help you understand the concept.

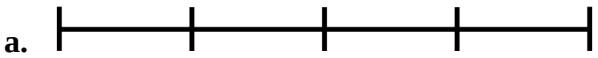
Here you see all the fractions from $\frac{0}{6}$ to $\frac{6}{6}$ on the number line. Note that $\frac{0}{6}$ is the same as 0, and $\frac{6}{6}$ is the same as 1.



The fraction $\frac{5}{6}$ is marked with a dot. Imagine or draw a rectangle extending from zero to 5/6. The length of that rectangle would be five times ⟼.

1. Write the fractions under every tick mark, including under 0 and 1.

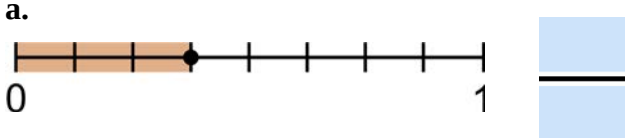
a.



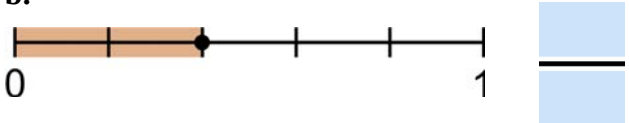
b.

2. Write the fraction marked by the dot on the number line.

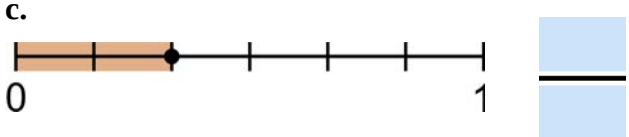
a.



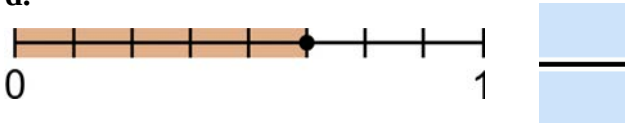
b.



c.



d.



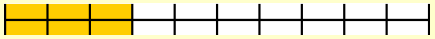
One Whole and Its Fractional Parts

A fraction always relates to some kind of *one whole*. Study the examples below:

Let's say the one whole is this square. It is divided into 12 parts.

Each part is $\frac{1}{12}$ of the whole. Also, we can write $1 = \frac{12}{12}$.

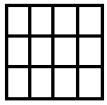
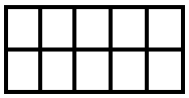
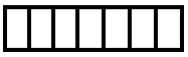
Maybe the one whole is this line, and $\frac{3}{10}$ of it is colored.



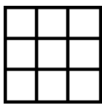
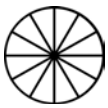
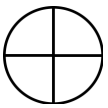
Maybe the one whole is Daddy's salary. To find 5/6 of it, imagine dividing the salary into 6 parts, and taking five of those parts. All six parts form the one whole, or $\frac{6}{6} = 1$

$\frac{7}{12}$ The top number is the **numerator**. It *numerates* or counts *how many pieces* there are.
The bottom number is the **denominator**. It *denominates* or *names* what kind of parts they are.

1. Color parts. Write the colored part *and* the white (uncolored) part as a fraction.

a. Color 1 part.	**b.** Color 5 parts.	**c.** Color 8 parts.	**d.** Color 3 parts.
			
$\frac{1}{12}$ and ___	and	and	and

2. Color and write one whole as a fraction.

				
a. 1 = ___	**b.** 1 = ___	**c.** 1 = ___	**d.** 1 = ___	**e.** 1 = ___

3. Solve.

a. The Jacksons ate $\frac{3}{4}$ of the pie. How much is left?	**b.** Jerry ate $\frac{1}{6}$ of the pizza. How much is left?
c. Five boys shared a chocolate bar equally. Each one got ___ of the bar.	

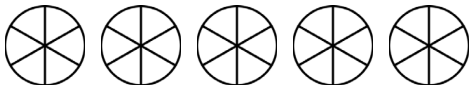
Mixed Numbers and Fractions

How to convert fractions to mixed numbers

Example 1. The fraction $\frac{23}{6}$ is more than one whole... so we can write it as a mixed number.

(How do we know it is more than 1? Because 1 = 6/6, and 23 sixths is more than six sixths.)

One way to write $\frac{23}{6}$ as a mixed number is to shade 23 sixths in the pie pictures:



If you do, you will get 3 whole pies, and 5 sixths shaded in one more pie. So the total is 3 5/6.

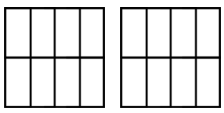
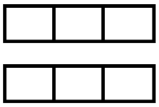
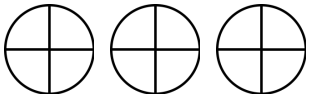
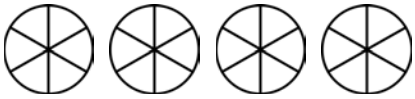
A quicker way is to use *division*. To find the number of whole pies we get from all those sixths, we need to figure how many 6's are in 23 (since each six slices makes a whole pie). The answer to that is found **by dividing <u>23 ÷ 6</u>.**

So, we think of the fraction $\frac{23}{6}$ as the division problem 23 ÷ 6. Since 23 ÷ 6 = 3 R5, we get three full pies, and five slices left over. This means that the mixed number we get is $3\frac{5}{6}$.

Example 2. To change $\frac{34}{5}$ into a mixed number, think of the fraction as a division, and <u>divide</u>.

We get 34 ÷ 5 = 6 R4. This means there are six whole pies, and four slices left over, or 6 4/5.

1. Write these fractions as mixed numbers. You can color parts to help.

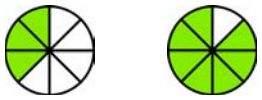
 **a.** $\frac{15}{8}$ =	 **b.** $\frac{5}{3}$ =	 **c.** $\frac{11}{5}$ =
 **d.** $\frac{11}{4}$ =	 **e.** $\frac{22}{6}$ =	 **f.** $\frac{7}{2}$ =

2. Write the fractions as mixed numbers.

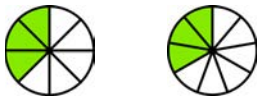
a. $\frac{13}{5}$ =	**b.** $\frac{11}{3}$ =	**c.** $\frac{23}{4}$ =	**d.** $\frac{17}{2}$ =
e. $\frac{26}{7}$ =	**f.** $\frac{55}{9}$ =	**g.** $\frac{22}{10}$ =	**h.** $\frac{58}{8}$ =

Comparing Fractions 5

1. Compare the fractions by writing < or > in the box between them.

If the fractions have the same *kind* of pieces, you can simply compare how many pieces they have.	**a.**  $\frac{3}{8}$ ☐ $\frac{7}{8}$	**b.**  $\frac{7}{12}$ ☐ $\frac{6}{12}$	**c.** $\frac{5}{9}$ ☐ $\frac{7}{9}$
			d. $\frac{6}{6}$ ☐ $\frac{2}{6}$

2. Compare the fractions by writing < or > in the box between them.

If the fractions have the same *amount* of pieces, you can simply compare the <u>size</u> of the pieces.	**a.**  $\frac{3}{8}$ ☐ $\frac{3}{9}$	**b.**  $\frac{1}{10}$ ☐ $\frac{1}{12}$	**c.** $\frac{5}{6}$ ☐ $\frac{5}{8}$
			d. $\frac{2}{6}$ ☐ $\frac{2}{5}$

Sometimes one fraction is more than 1/2 and the other is less.

Example 1. Compare $\frac{5}{6}$ and $\frac{3}{8}$.

Now, 3/8 is less than 1/2. How can you know? Because 4/8 would be exactly 1/2, so 3/8 is less than that. And, 5/6 is more than 1/2. (How do you know?) So, 5/6 > 3/8.

3. Write <, >, or = in the box. <u>Note:</u> Sometimes one of the fractions is actually *equal* to 1/2!

a. $\frac{1}{6}$ ☐ $\frac{3}{5}$ **b.** $\frac{4}{5}$ ☐ $\frac{2}{8}$ **c.** $\frac{3}{4}$ ☐ $\frac{2}{5}$ **d.** $\frac{5}{10}$ ☐ $\frac{4}{12}$

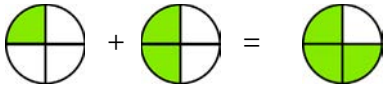
e. $\frac{4}{5}$ ☐ $\frac{3}{6}$ **f.** $\frac{1}{9}$ ☐ $\frac{2}{3}$ **g.** $\frac{3}{6}$ ☐ $\frac{5}{10}$ **h.** $\frac{4}{10}$ ☐ $\frac{7}{12}$

4. Write these fractions in order from the smallest to the greatest.

a. $\frac{6}{8}, \frac{3}{8}, \frac{3}{6}$	**b.** $\frac{6}{5}, \frac{2}{5}, \frac{5}{6}$	**c.** $\frac{1}{4}, \frac{1}{7}, \frac{5}{8}$

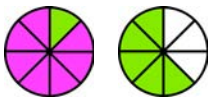
Adding Fractions

It is easy to add fractions that have the same kinds of parts. Study the examples.



$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$

Example 1. Here, think of the pie pieces or slices. One fourth means one piece, and two fourths means two pieces. In total we have three pieces, and they all are fourths. So, the answer is 3/4.

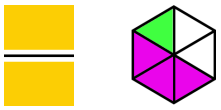


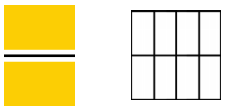
$\frac{7}{8} + \frac{6}{8} = \frac{13}{8} = 1\frac{5}{8}$

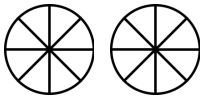
Example 2. In this picture we have shaded (added) seven slices and then another six slices. All the slices are eighth parts so we can simply count how many eighths we get: 13 eighths or 13/8.

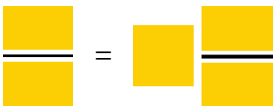
That makes more than one whole pie, so the answer is given as a mixed number: 1 5/8.

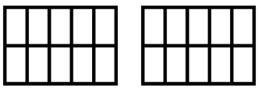
1. Solve. You can shade parts to help you. Give your answer as a mixed number when possible.

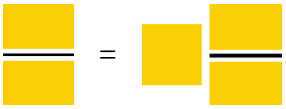
a. $\frac{1}{6} + \frac{3}{6} =$ 

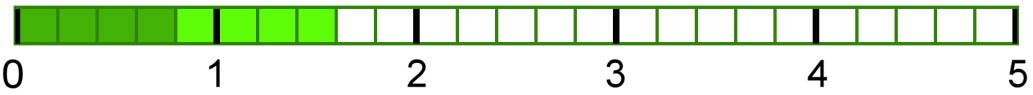
b. $\frac{2}{8} + \frac{5}{8} =$ 



c. $\frac{7}{8} + \frac{7}{8} =$ 



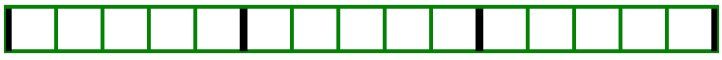
d. $\frac{7}{10} + \frac{5}{10} =$ 

This is a fraction strip: 

The shaded parts illustrate the addition $\frac{4}{5} + \frac{4}{5} = \frac{8}{5} = 1\frac{3}{5}$.

2. Add. Shade parts with different colors. Give your answer as a mixed number.

a. $\frac{3}{5} + \frac{4}{5} =$



b. $1\frac{2}{5} + \frac{4}{5} =$

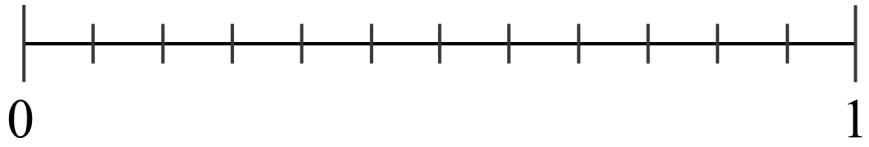
c. $\frac{13}{10} + \frac{6}{10} =$

d. $1\frac{3}{8} + \frac{6}{8} =$

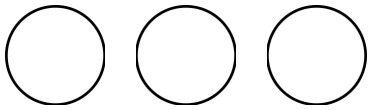
Practicing with Fractions

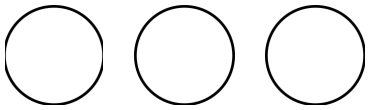
1. Mark these fractions on the number line. *Hint: Think of equivalent fractions.*

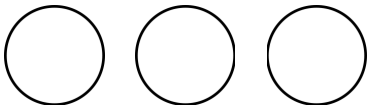
$\frac{5}{12}$, $\frac{11}{12}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{2}{3}$, $\frac{5}{6}$

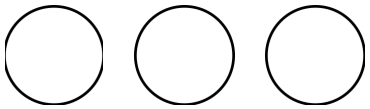


2. List the fractions in order from the smallest to the greatest. You can draw pie pictures to help.

a. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{2}{6}$ 

b. $\frac{1}{4}$, $\frac{1}{8}$, $\frac{3}{8}$ 

c. $\frac{2}{5}$, $\frac{3}{5}$, $\frac{1}{2}$ 

d. $\frac{4}{5}$, $\frac{3}{8}$, $\frac{3}{4}$ 

3. Each day, Robert runs 3/5 of a mile.
How far does he run in three days?
Give your answer as a mixed number.

4. Make chains of equivalent fractions. You can choose the other two fractions but they need to be equivalent to the first.

a. ◯ = ◯ = ◯

$\frac{1}{3}$ =

b. ◯ = ◯ = ◯

$\frac{1}{4}$ =

c. ◯ = ◯ = ◯

$\frac{3}{4}$ =

d. ◯ = ◯ = ◯

$\frac{2}{5}$ =

5. Multiply, and find the missing numbers.

a. $2 \times \frac{2}{7} =$	**b.** $5 \times \frac{2}{3} =$	**c.** $3 \times \frac{3}{4} =$
d. $\_\_\_\_ \times \frac{4}{7} = \frac{8}{7}$	**e.** $\_\_\_\_ \times \frac{1}{2} = 3\frac{1}{2}$	**f.** $\_\_\_\_ \times \frac{3}{4} = 1\frac{2}{4}$

6. Add or subtract. Give your final answer as a whole number or as a mixed number.

a. $\frac{3}{9} + \frac{6}{9} =$	**b.** $\frac{5}{8} + \frac{6}{8} =$
c. $2\frac{2}{5} + 4\frac{4}{5} =$	**d.** $\frac{11}{12} - \frac{3}{12} =$
e. $1\frac{1}{10} - \frac{3}{10} =$	**f.** $5\frac{1}{4} - \frac{3}{4} =$

7. Multiplying by one-half is actually the same as _____ by 2. Fill in.

a.

$2 \times \frac{1}{2} =$

$3 \times \frac{1}{2} =$

$4 \times \frac{1}{2} =$

$5 \times \frac{1}{2} =$

$6 \times \frac{1}{2} =$

b.

$7 \times \frac{1}{2} =$

$8 \times \frac{1}{2} =$

$9 \times \frac{1}{2} =$

$10 \times \frac{1}{2} =$

$11 \times \frac{1}{2} =$

c.

$15 \times \frac{1}{2} =$

$20 \times \frac{1}{2} =$

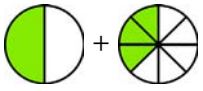
$17 \times \frac{1}{2} =$

$21 \times \frac{1}{2} =$

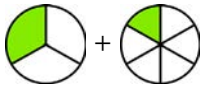
$32 \times \frac{1}{2} =$

Puzzle Corner

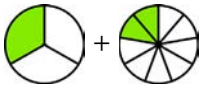
Add *un*like fractions. First change one fraction by splitting its pieces further, then add.



a. $\frac{1}{2} + \frac{3}{8} =$



b.



c.