Contents

Chapter 7: Systems of Linear Equations

Chapter 8: Bivariate Data

Foreword

Math Mammoth Grade 8, International Version, comprises a complete maths curriculum for the eighth grade mathematics studies. This curriculum is essentially the same as the *Math Mammoth Grade 8* sold in the United States, only customised for international use in a few aspects (listed below). The curriculum meets the Common Core Standards in the United States, but it may not properly align to the eighth grade standards in your country. However, you can probably find material for any missing topics in the neighbouring grades of Math Mammoth.

The International version of Math Mammoth differs from the US version in these aspects:

- The curriculum uses metric measurement units, not both metric and customary (imperial) units.
- The spelling conforms to British international standards.
- The pages are formatted for A4 paper size.
- Large numbers are formatted with a space as the thousands separator (such as 12 394).
  (The decimals are formatted with a decimal point, as in the US version.)

In 8th grade, students spend the majority of the time with algebraic topics, such as linear equations, functions, and systems of equations. The other major topics are geometry and statistics. The main areas of study are:

- Exponents laws and scientific notation
- Square roots, cube roots, and irrational numbers
- Geometry: congruent transformations, dilations, angle relationships, volume of certain solids, and the Pythagorean Theorem
- Solving and graphing linear equations;
- Introduction to functions;
- Systems of linear equations;
- Bivariate data.

This book, 8-B, covers the topic of graphing linear equations. The focus is on the concept of slope.

In chapter 6, our focus is on square roots, cube roots, the concept of irrational numbers, and the Pythagorean Theorem and its applications.

Next, in chapter 7, students solve systems of linear equations, using both graphing and algebraic techniques. There are also lots of word problems that are solved using a pair of linear equations.

The last chapter then delves into bivariate data. First, we study scatter plots, which are based on numerical data of two variables. Then we look at two-way tables, which are built from categorical bivariate data.

Part 8-A covers exponent laws, scientific notation, geometry, linear equations, and an introduction to functions.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching maths!

Maria Miller, the author

User Guide

Note: You can also find the information that follows online, at **https://www.mathmammoth.com/userguides/** .

Basic principles in using Math Mammoth Complete Curriculum

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. The two books (parts A and B) are like a "framework", but you still have some liberty in planning your student's studies. In eighth grade, chapters 2 (geometry), 3 (linear equations) and chapter 4 (functions) should be studied before chapter 5 (graphing linear equations). Also, chapters 3, 4, and 5 should be studied before chapter 7 (systems of linear equations) and before chapter 8 (statistics). However, you still have some flexibility in scheduling the various chapters.

Math Mammoth is not a scripted curriculum. In other words, it is not spelling out in exact detail what the teacher is to do or say. Instead, Math Mammoth gives you, the teacher, various tools for teaching:

- **The two student worktexts** (parts A and B) contain all the lesson material and exercises. They include the explanations of the concepts (the teaching part) in blue boxes. The worktexts also contain some advice for the teacher in the "Introduction" of each chapter.

  The teacher can read the teaching part of each lesson before the lesson, or read and study it together with the student in the lesson, or let the student read and study on his own. If you are a classroom teacher, you can copy the examples from the "blue teaching boxes" to the board and go through them on the board.

- There are **videos** matched to the curriculum available at https://www.mathmammoth.com/videos/ . There isn't a video for every lesson, but there are dozens of videos for each grade level. You can simply have the author teach your child or student!

- Don't automatically assign all the exercises. Use your judgement, trying to assign just enough for your student's needs. You can use the skipped exercises later for revision. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.

- For each chapter, there is a **link list to various free online games** and activities. These games can be used to supplement the maths lessons, for learning maths facts, or just for some fun. Each chapter introduction (in the student worktext) contains a link to the list corresponding to that chapter.

- The student books contain some **mixed revision lessons**, and the curriculum also provides you with additional **cumulative revision lessons**.

- There is a **chapter test** for each chapter of the curriculum, and a comprehensive end-of-year test.

- You can use the free online exercises at https://www.mathmammoth.com/practice/
  This is an expanding section of the site, so check often to see what new topics we are adding to it!

- There are answer keys for everything.

How to get started

Have ready the first lesson from the student worktext. Go over the first teaching part (within the blue boxes) together with your student. Go through a few of the first exercises together, and then assign some problems for the student to do on their own.

Repeat this if the lesson has other blue teaching boxes.

Many students can eventually study the lessons completely on their own — the curriculum becomes self-teaching. However, students definitely vary in how much they need someone to be there to actually teach them.

Pacing the curriculum

Worktext 8-A	
Chapter 1	13 days
Chapter 2	27 days
Chapter 3	21 days
Chapter 4	14 days
TOTAL	**75 days**

Worktext 8-B	
Chapter 5	15 days
Chapter 6	16 days
Chapter 7	17 days
Chapter 8	11 days
TOTAL	**59 days**

Each chapter introduction contains a suggested pacing guide for that chapter. You will see a summary on the right. (This summary does not include time for optional tests.)

Most lessons are 3 or 4 pages long, intended for one day. Some lessons are 5 pages and can be covered in two days.

It can also be helpful to calculate a general guideline as to how many pages per week the student should cover in order to go through the curriculum in one school year.

The table below lists how many pages there are for the student to finish in this particular grade level, and gives you a guideline for how many pages per day to finish, assuming a 160-day (32-week) school year. The page count in the table below *includes* the optional lessons.

Example:

Grade level	School days	Days for tests and revisions	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
8-A	84	8	214	76	2.8	14.1
8-B	76	8	189	68	2.8	13.9
Grade 8 total	160	16	403	144	2.8	14

The table below is for you to fill in. Allow several days for tests and additional revision before tests — I suggest at least twice the number of chapters in the curriculum. Then, to get a count of "pages to study per day", **divide the number of lesson pages by the number of days for the student book**. Lastly, multiply this number by 5 to get the approximate page count to cover in a week.

Grade level	Number of school days	Days for tests and revisions	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
8-A			214			
8-B			189			
Grade 8 total			403			

Now, something important. Whenever the curriculum has lots of similar practice problems (a large set of problems), feel free to **only assign 1/2 or 2/3 of those problems**. If your student gets it with less amount of exercises, then that is perfect! If not, you can always assign the rest of the problems for some other day. In fact, you could even use these unassigned problems the next week or next month for some additional revision.

In general, 8th graders might spend 45-75 minutes a day on maths. If your student finds maths enjoyable, they can of course spend more time with it! However, it is not good to drag out the lessons on a regular basis, because that can then affect the student's attitude towards maths.

Using tests

For each chapter, there is a **chapter test**, which can be administered right after studying the chapter. **The tests are optional.** The main reason for the tests is for diagnostic purposes, and for record keeping. These tests are not aligned or matched to any standards.

In the digital version of the curriculum, the tests are provided both as PDF files and as html files. Normally, you would use the PDF files. The html files are included so you can edit them (in a word processor such as Word or LibreOffice), in case you want your student to take the test a second time. Remember to save the edited file under a different file name, or you will lose the original.

The end-of-year test is best administered as a diagnostic or assessment test, which will tell you how well the student remembers and has mastered the mathematics content of the entire grade level.

Using cumulative revisions and the worksheet maker

The student books contain mixed revision lessons which revise concepts from earlier chapters. The curriculum also comes with additional cumulative revision lessons, which are just like the mixed revision lessons in the student books, with a mix of problems covering various topics. These are found in their own folder in the digital version, and in the Tests & Cumulative Revisions book in the printed version.

The cumulative revisions are optional; use them as needed. They are named indicating which chapters of the main curriculum the problems in the review come from. For example, “Cumulative Revision, Chapter 4” includes problems that cover topics from chapters 1-4.

Both the mixed and cumulative revisions allow you to spot areas that the student has not grasped well or has forgotten. When you find such a topic or concept, you have several options:

1. Check if the worksheet maker lets you make worksheets for that topic.
2. Check for any online games and resources in the Introduction part of the particular chapter in which this topic or concept was taught.
3. If you have the digital version, you could reprint the lesson from the student worktext, and have the student restudy that.
4. Perhaps you only assigned 1/2 or 2/3 of the exercise sets in the student book at first, and can now use the remaining exercises.
5. Check if our online practice area at https://www.mathmammoth.com/practice/ has something for the topic.
6. Khan Academy has free online exercises, articles, and videos for most any maths topic imaginable.

Concerning challenging word problems and puzzles

While this is not absolutely necessary, I heartily recommend supplementing Math Mammoth with challenging word problems and puzzles. You could do that once a month, for example, or more often if the student enjoys it.

The goal of challenging story problems and puzzles is to **develop the student’s logical and abstract thinking and mental discipline**. Math Mammoth curriculum contains lots of word problems, and they are usually multi-step problems. Several of the lessons utilise a bar model for solving problems. Even so, the problems I have created are usually tied to a specific concept or concepts. I feel students can benefit from solving problems and puzzles that require them to think “out of the box” or are just different from the ones I have written.

I recommend you use the free Math Stars problem-solving newsletters as one of the main resources for puzzles and challenging problems:

Math Stars Problem Solving Newsletter (grades 1-8)
https://www.homeschoolmath.net/teaching/math-stars.php

I have also compiled a list of other resources for problem solving practice, which you can access at this link:
https://l.mathmammoth.com/challengingproblems

Another idea: you can find puzzles online by searching for “brain puzzles for kids,” “logic puzzles for kids” or “brain teasers for kids.”

Frequently asked questions and contacting us

If you have more questions, please first check the FAQ at https://www.mathmammoth.com/faq-lightblue

If the FAQ does not cover your question, you can then contact us using the contact form at the Math Mammoth.com website.

Chapter 5: Graphing Linear Equations

Introduction

This chapter focuses on how to graph linear equations, and in particular, on the concept of slope in that context.

We start by graphing and comparing proportional relationships, which have the equation of the form $y = mx$. Students are already familiar with these, and know that m is the constant of proportionality. In this chapter, they learn that m is also the slope of the line, which is a measure of its steepness.

Then we go on to study slope in detail, its definition as the ratio of the change in y-values and the change in x-values. Students learn that it doesn't matter which two points on a line you use to calculate the slope, and study a geometric proof of this fact. They practise drawing a line with a given slope and that goes through a given point, and determine if three given points fall on the same line.

Then it is time to study the slope-intercept equation of a line, and connect the idea of an initial value of a function (chapter 4) with the concept of y-intercept in the context of graphing. Students graph lines given in the slope-intercept form, and write equations of lines from their graphs.

Next, we study horizontal and vertical lines and their simple equations. The standard form of a linear equation follows next. The last major topic is how the slope reveals to us whether two lines are parallel or perpendicular to each other.

Pacing Suggestion for Chapter 5

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing if you use the test.

The Lessons in Chapter 5	page	span	suggested pacing	your pacing
Graphing Proportional Relationships 1	13	*3 pages*	1 day	
Graphing Proportional Relationships 2	16	*3 pages*	1 day	
Comparing Proportional Relationships	19	*4 pages*	1 day	
Slope, Part 1	23	*4 pages*	1 day	
Slope, Part 2	27	*3 pages*	1 day	
Slope, Part 3	30	*5 pages*	2 days	
Slope-Intercept Equation 1	35	*4 pages*	1 day	
Slope-Intercept Equation 2	39	*3 pages*	1 day	
Write the Slope-Intercept Equation	42	*3 pages*	1 day	
Horizontal and Vertical Lines	45	*3 pages*	1 day	
The Standard Form	48	*3 pages*	1 day	
More Practice (optional)	51	*(2 pages)*	(1 day)	
Parallel and Perpendicular Lines	53	*3 pages*	1 day	
Mixed Revision Chapter 5	56	*3 pages*	1 day	
Chapter 5 Revision	59	*4 pages*	1 day	
Chapter 5 Test (optional)				
TOTALS		*48 pages*	15 days	
with optional content		*(50 pages)*	(16 days)	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr8ch5

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Graphing Proportional Relationships 1

We will now revise what it means when two variables are **in direct variation** or **in proportion**. The basic idea is that whenever one variable changes, the other varies (changes) proportionally or at the same rate.

Example 1. The wholesaler posted the following table for the price of potatoes:

weight (kg)	5	10	15	20	25	30
cost	$5.50	$11.00	$16.50	$22.00	$27.50	$33.00

Each pair of cost and weight forms a rate — and so does each pair of weight and cost. However, it is more common to look at the rate "cost over weight", such as $27.50/(25 kg), than vice versa.

If all of the rates in the table are equivalent, then the weight and the cost *are* proportional.

To check for that, we have several means. One is to calculate **the unit rate** (the rate for 1 kg) from each of these rates, and check whether you get the same unit rate.

In this case, that is so. The unit rate is $1.10/kg, no matter which rate from the table we'd use to calculate it.

One other way to check is, if one quantity doubles (or triples), will the other double (or triple) also?
This is especially useful for noticing if the quantities are *not* in direct variation.

Example 2. Here, when the weight doubles from 5 kg to 10 kg, the price also doubles. But what happens with the price when the weight doubles from 10 kg to 20 kg?

weight (kg)	5	10	15	20	25	30
cost	$6	$12	$18	$22	$26	$30

The price does not double! So, the quantities are not in proportion.
The seller is giving you some discount if you purchase higher quantities.

Also, if you calculate the unit rate from $6/(5 kg) and from $22/(20 kg), they are not equal. (Verify this.)

1. Are the quantities in a proportional relationship? If yes, list the unit rate.

a.

time (hr)	0	1	2	3	4	5
distance (km)	0	50	90	140	190	240

b.

time (hr)	0	1	2	3	4	5
distance (km)	0	45	90	135	180	225

c.

age (days)	0	1	2	3	4	5	6	7
height (cm)	0	0	0	2	4	6	8	10

d.

length (m)	0	0.5	1	1.5	2	4	5	10
cost ($)	0	3	6	9	12	24	30	60

2. Now consider the tables of values in #1 as functions, where the variable listed on top is the independent variable. For the ones where the quantities were in proportion, calculate the rate of change.

   What is its relationship to the unit rate?

When two quantities are in a proportional relationship, or in direct variation (the two terms are synonymous):

(1) Each rate formed by the quantities is equivalent to any other rate of the quantities.

(2) The equation relating the two quantities is of the form $y = mx$, where y and x are the variables, and m is a constant. The constant m is called the **constant of proportionality** and is also the unit rate.

(3) When plotted, the graph is a straight line that goes through the origin.

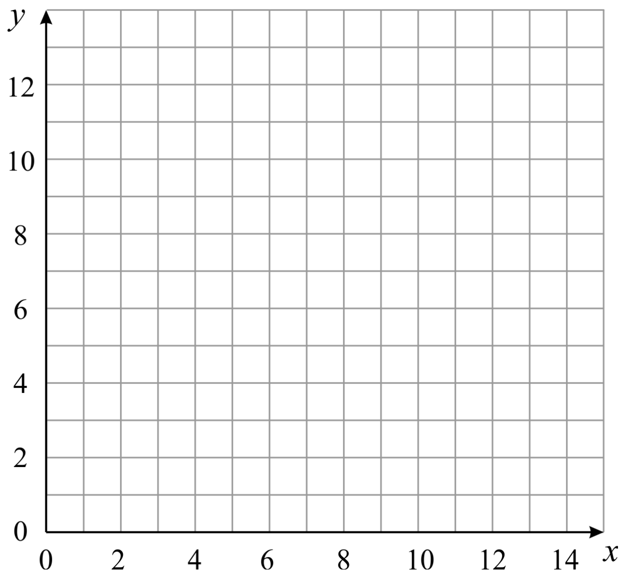
3. Choose an equation from below where the variables x and y are in direct variation (proportional):

$y = \frac{3}{x}$ $\qquad$ $y = 3x$

$xy = 3$ $\qquad$ $y = x^3$

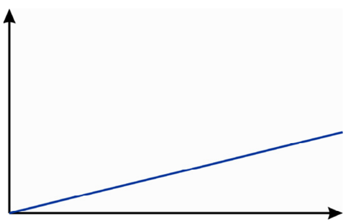
Then graph that equation in the grid.

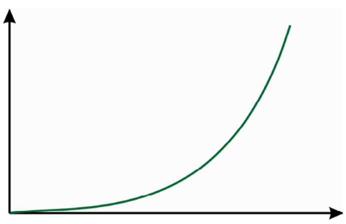
Hint: The point (0, 0) is always included in direct variation. All you need to do is plot one other point, and then draw a line through the origin and that point.



4. Choose the representations that show a proportional relationship.

a. 

b. 

c. 

d.

x	0	1	2	3	4	5
y	15	17	19	21	23	25

e. $y = 2x + 9$

f. $y = (3/4)x$

g.

x	0	4	8	12	16	20
y	0	3	6	9	12	15

5. Two of the above representations are the exact same relationship. Which ones?

Example 2. In a direct variation, $y = 9$ when $x = 12$. Write an equation for the relationship.

Since this is direct variation (proportional relationship), the equation is of the form $y = mx$, where m is the constant of proportionality.

The constant of proportionality is the ratio **(dependent variable)/(independent variable)**, so in this case it is $y/x = 9/12$, or 3/4. So, the equation is $y = (3/4)x$.

At this point, it is good to check that the point (12, 9) satisfies the equation, to check for errors:
Is it true that $9 = (3/4) \cdot 12$? Yes, it is.

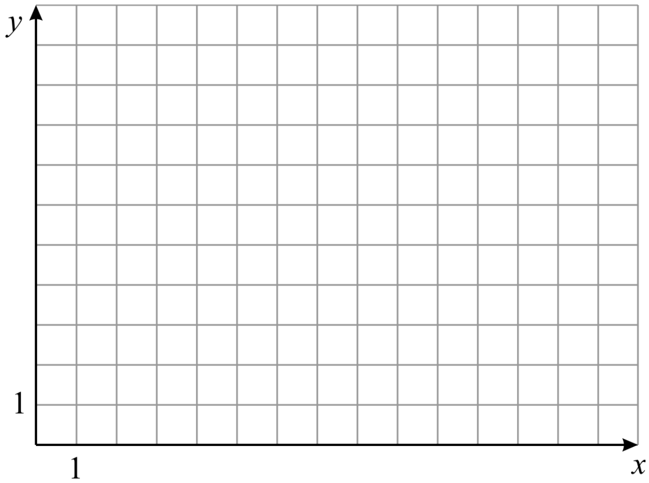
To graph the equation, we could simply plot the point (12, 9), and draw a line through it and the origin.

6. In a direct variation, when x is 14, y is 10.

a. Write an equation for this proportional relationship.

b. Graph a line for this relationship in the grid.

c. What is x when $y = 40$?

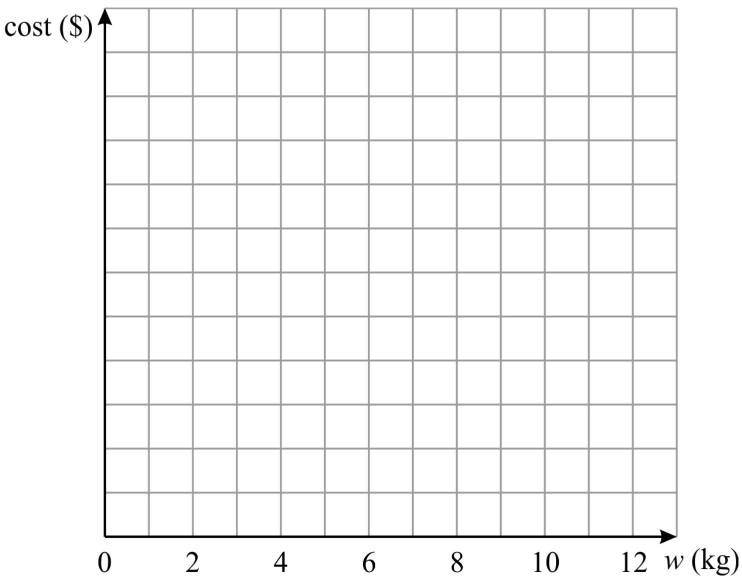


7. Organic rolled oats cost $34 for 4 kg.

a. Write an equation for this proportional relationship, using the variables C for cost and w for the amount (weight) of oats.

b. Graph the equation in the grid. Design the scaling on the cost-axis so that the point corresponding to 12 kg fits on the grid.

c. How much do 15 kg of the oats cost?



8. If y is 120 when x is 400 in a direct variation, then what is y when x is 80?

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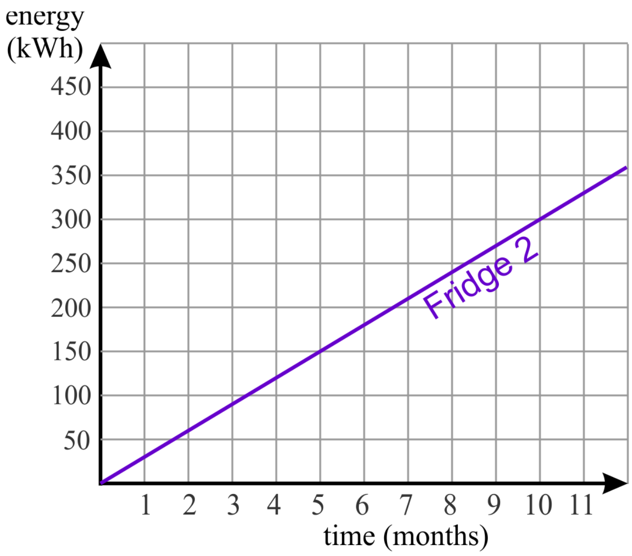
Chapter 5 Revision

1. Refrigerator companies make estimates of how much energy their fridges consume in typical usage. The table shows how many kilowatt-hours (kWh) of energy fridge 1 consumed over time, and the graph shows the same for fridge 2.

Fridge 1

time (mo)	energy (kWh)
2	75
4	150
6	225
8	300
10	375
12	450

Fridge 2



a. Which fridge consumes more electricity in a month?

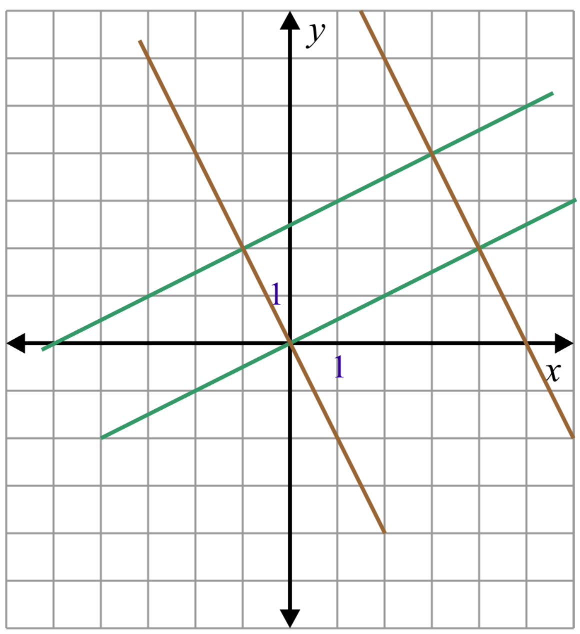
How much more?

b. Write an equation for each fridge, relating the energy (E, in kWh) and the time (t, in months).

c. Plot the equation for Fridge 1 in the grid.

d. Plot the point corresponding to the unit rate, for Fridge 1.

2. **a.** Find the equations of the four lines, in slope intercept form.



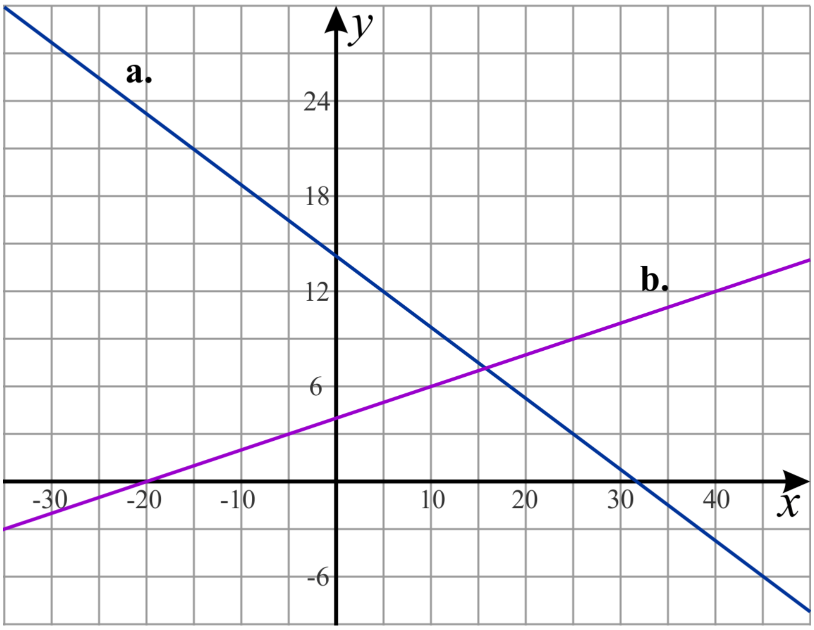
b. (optional) Find the area of the rectangle.

3. Find the equation of each line, in slope-intercept form:

a. has slope 3/4 and passes through (−2, 3)

b. is horizontal and passes through (9, −10)

4. Find the slope of the lines.
Notice the scaling.



a.

b.

Now find the equations for the lines.

a.

b.

5. Do the three points fall on one line? Explain your reasoning.

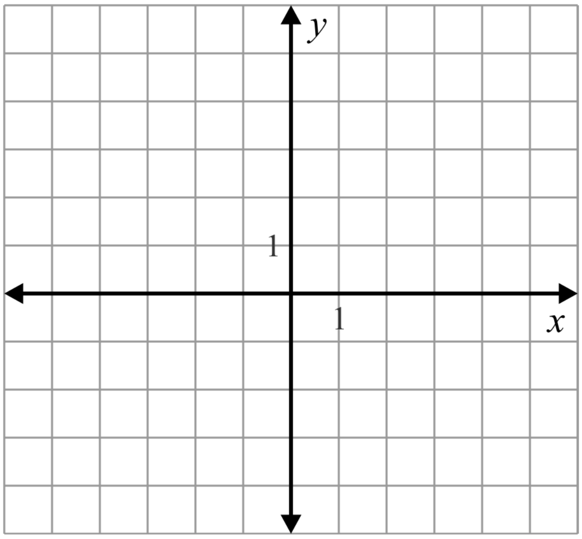
(−3, 1), (−1, −4), (1, −8)

6. Find *s* so that the point (*s*, 12) will fall on the same line as the points (3, 9) and (15, 18).

7. Line S passes through (−5, −2) and (0, 4). Line T is perpendicular to Line S, and passes through (1, 1).

a. Find the equation of line T, in slope-intercept form.

b. Write the equation also in the standard form.



8. Mr. Henson runs a garbage pick-up business, with 12 garbage trucks. To run one truck costs him $2100 per month in maintenance costs, plus $180 a day for fuel.

Consider the cost of running one truck as a function of time, in days (during one month only). Is this a linear relationship, a proportional relationship, or neither?

Write an equation for it.

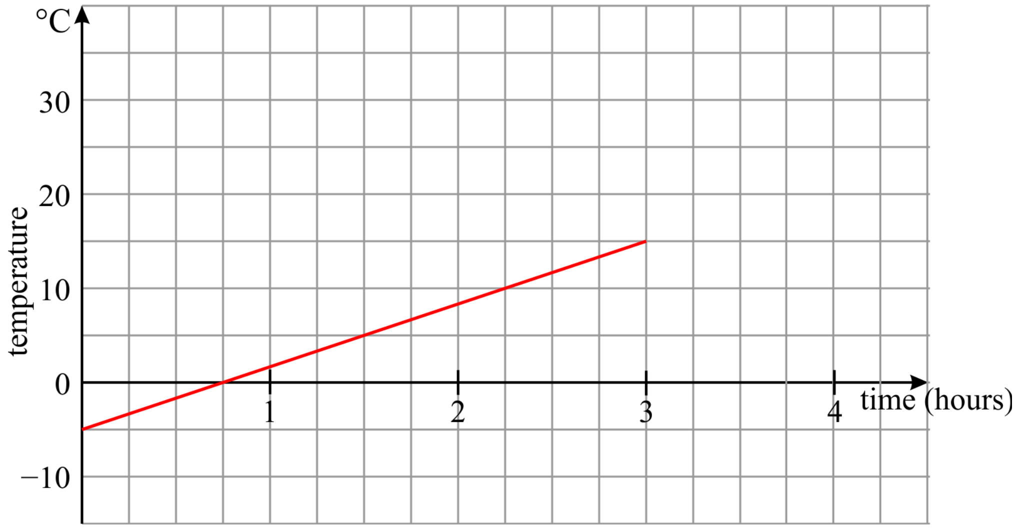
9. Match the descriptions and the equations.

$y = (-4/3)x - 7$	Is parallel to $x = 9$ and passes through (2, 7)
$3x - y = -21$	Has y-intercept −4 and is perpendicular to $y = -2x$.
$y = -4$	Passes through (−5, 6) and has slope 3.
$x - 2y = 8$	Passes through (−9, 5) and (−3, −3)
$x = 2$	Passes through (−3, 0) and (0, 9)
$y = 3x + 9$	Has y-intercept −4 and is parallel to $y = -2$.

10. Transform each equation of a line to the standard form, and then list its x and y-intercepts.

a. $y - 6 = 2(x + 2)$	**b.** $-\frac{1}{3}x - \frac{3}{2}y = 1$

11. A heater was turned on at 10 AM in a cold, uninhabited house, to prepare it for people later that day. The graph shows the temperature of the house. The count of hours starts at 10 AM.



a. Write an equation for the line.

b. If the temperature continues to rise in the same fashion, what will the temperature be at 2:30 PM?

c. When will the temperature reach 22°C?

d. Let's say the heater is turned off at 1:45. What is the temperature at that time?

e. If the house had started out at a temperature of −12°C instead, and the heating process worked in the same fashion (the temperature rose at the same rate), at what time would the house reach a temperature of 22°C?

Chapter 6: Irrational Numbers and the Pythagorean Theorem
Introduction

We start out this chapter by studying the concept of a square root, as the opposite operation to squaring a number. In the next lesson, on irrational numbers, students find values of square roots by hand. They make a guess and then square the guess, and based on how close the square of their guess is to the radicand, they refine their guess until desired accuracy is reached. This will help solidify the concept of a square root, while also showing how most square roots are non-ending decimal numbers, and how in real life, we need to use approximations of them to do calculations. Students also practise placing irrational numbers on the number line, using mental maths to find their approximate location.

Next, the chapter has a revision lesson on how to convert fractions to decimals. The following lesson has to do with writing decimals as fractions, and teaches a method for converting repeating decimals to fractions.

Then it is time to learn to solve simple equations that involve taking a square or cube root, over the course of two lessons. After learning to solve such equations, students are now fully ready to study the Pythagorean Theorem and apply it.

The Pythagorean Theorem is introduced in the lesson by that name. Students learn to verify that a triangle is a right triangle by checking whether it fulfils the Pythagorean Theorem. They apply their knowledge about square roots and solving equations to solve for an unknown side in a right triangle when two of the sides are given.

Next, students solve a variety of geometric and real-life problems that require the Pythagorean Theorem. This theorem is extremely important in many practical situations. Students should show their work for these word problems to include the equation that results from applying the Pythagorean Theorem to the problem and its solution.

There are literally hundreds of proofs for the Pythagorean Theorem. In this book, we present one easy proof based on geometry (not algebra). As an exercise, students are asked to supply the steps of reasoning to another geometric proof of the theorem. Students also study a proof for the converse of the theorem, which says that if the sides of a triangle fulfil the equation $a^2 + b^2 = c^2$ then the triangle is a right triangle.

Our last topic is distance between points in the coordinate grid, as this is another simple application of the Pythagorean Theorem.

Pacing Suggestion for Chapter 6

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing if you use the test.

A Proof of the Pythagorean Theorem and of Its Converse	96	*4 pages*	1-2 days
Applications of the Pythagorean Theorem 2	100	*4 pages*	1 day
Distance Between Points	104	*3 pages*	1 day
Mixed Revision Chapter 6	107	*3 pages*	1 day
Chapter 6 Revision	110	*6 pages*	2 days
Chapter 6 Test (optional)			
TOTALS		*49 pages*	15-16 days
with optional content		*(51 pages)*	(16-17 days)

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr8ch6

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Square Roots

The **square** of a number is that number multiplied by itself. For example, six squared = $6^2 = 6 \cdot 6 = 36$. (Recall that the square of 6 tells us the area of a square with sides 6 units long.)

Taking a **square root** is the opposite operation to squaring: the square root of 36 is the number that when squared, gives you 36.

There are actually two such numbers: 6 and −6. The positive one, 6, is **the principal square root** of 36. We use the "$\sqrt{\ }$" symbol (called the "radical sign" or "radix") to signify the principal square root of a number. For example, $\sqrt{25} = 5$ because $5^2 = 25$.

> The words "radish" and "radical" both come from the Latin word *radix*, meaning **root**.

Taking a square root allows us to find the side length of a square when its area is given.

Here is a way to remember what a square root is. In the picture on the right, the area of a square is written inside the square and the length of the side is written to the side:

[49] 7

Now, imagine the square is a radical sign that "houses" the number for the area:

To find the (principal) **square root of a number, think of a square with that area, and find the side length of that square.**

$$\sqrt{49} = 7$$

1. Find the (principal) square roots.

a. $\sqrt{100}$	**b.** $\sqrt{64}$	**c.** $\sqrt{4}$	**d.** $\sqrt{0}$
e. $\sqrt{81}$	**f.** $\sqrt{144}$	**g.** $\sqrt{1}$	**h.** $\sqrt{10\,000}$

2. It is especially easy to find square roots of numbers that are **perfect squares:** numbers we get by squaring whole numbers. For example, 49 is a perfect square because it is 7^2. Fill in the list of perfect squares from 1^2 to 20^2 at the right:

x	x^2
1	1
2	4
3	9
4	______
______	______
______	______
______	49
8	______
9	______
______	______

x	x^2
11	______
12	______
13	______
14	______
15	______
______	256
______	289
______	324
______	______
______	______

3. Find the square roots of these perfect squares.

a. $\sqrt{169}$ **b.** $\sqrt{900}$

c. $\sqrt{225}$ **d.** $\sqrt{121}$

e. $\sqrt{441}$ **f.** $\sqrt{8100}$

g. $\sqrt{324}$ **h.** $\sqrt{400}$

i. $\sqrt{6400}$ **j.** $\sqrt{25\,600}$

k. $\sqrt{16\,900}$ **l.** $\sqrt{1\,000\,000}$

Most whole numbers are *not* perfect squares, and their square roots are unending decimals. (In fact, their square roots are **irrational numbers**, which means they cannot be written as a fraction, and their decimal expansions are unending decimals without any repeating patterns in the digits.)

We can handle that situation in at least three ways:

1. We can find an approximate value of such square roots **with a calculator**, rounding the answer to a reasonable accuracy. This is necessary if we're dealing with a real-life application.
2. We can find an approximate value using **guess and check**, and decimal multiplication. For example, we know that the value of $\sqrt{17}$ will be between 4 and 5 (since $\sqrt{16} = 4$ and $\sqrt{25} = 5$). We can also tell that it will be closer to 4 than 5, since 17 is very close to 16. So, we could guess that it is 4.1, square that, and based on the result, refine our guess.
3. We can **indicate such values using the square root symbol**, and not find a decimal approximation. For example, the side of a square with an area of 2 square units is $\sqrt{2}$ units. This is the preferred way in pure mathematics, and any time you want to convey an accurate value.

4. Between which two consecutive whole numbers do the following square roots lie? Do not use a calculator. Tell also which of those whole numbers the root is closer to.

a. $\sqrt{5}$ **b.** $\sqrt{24}$ **c.** $\sqrt{47}$ **d.** $\sqrt{83}$

5. Tell the side of the square (exact value) when its area is given. Indicate the side length using the square root symbol, if the area is not a perfect square. Note: u^2 signifies square units, and u signifies a unit.

a. area = 25 u^2 side = __________	**b.** area = 1600 u^2 side = __________	**c.** area = 5 u^2 side = __________	**d.** area = 11 u^2 side = __________

6. **a.** What is the area of a square, if its side measures $\sqrt{8}$ units?

b. What is the value of $(\sqrt{7})^2$?

c. What is the side of a square with an area of 130 square metres? Give an exact value.

Example 1. Since $0.5 \cdot 0.5 = 0.25$, then $\sqrt{0.25} = 0.5$.

Example 2. Since $\frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}$, then $\sqrt{\frac{4}{9}} = \frac{2}{3}$.

7. Find the square roots.

a. $\sqrt{0.16}$ **b.** $\sqrt{0.01}$ **c.** $\sqrt{1.21}$

d. $\sqrt{\frac{16}{25}}$ **e.** $\sqrt{\frac{100}{9}}$ **f.** $\sqrt{\frac{49}{36}}$

Example 3. The area of a square is 42.5 m^2. What is the side of the square?

From a calculator, $\sqrt{42.5} \approx 6.5192024052026487145829715574292$. Even this long decimal is not giving us all the decimal digits! In reality, the number would continue in an unending manner, without any patterns in the decimal digits (it is an *irrational* number).

The area was given to three significant digits, so we will do the same here, and use three significant digits in our answer. The side measures 6.52 metres.

Note: On some calculators, you first push the square root button, then the number of which you are taking the square root. On others, you first enter the number and then push the square root button. Find out which way your calculator works.

8. Find the value of these square roots with a calculator, to three decimal digits.

a. $\sqrt{70}$	**b.** $\sqrt{3}$	**c.** $\sqrt{1450}$
d. $\sqrt{0.45}$	**e.** $\sqrt{\frac{5}{6}}$	**f.** $\sqrt{\frac{31}{7}}$

The radical sign acts as a grouping symbol: it is as if there were brackets around the expression under the square root. In other words, $\sqrt{15 + 10}$ means $\sqrt{(15 + 10)}$.

Example 4. Simplify $\sqrt{5 \cdot (70 + 10)}$.

We simplify the expression under the square root first, and take the square root last:

$$\sqrt{5 \cdot (70 + 10)} = \sqrt{5 \cdot 80} = \sqrt{400} = 20$$

9. Calculate. Do not use a calculator.

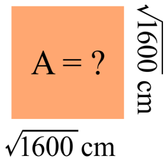
a. $\sqrt{9 + 16}$	**b.** $\sqrt{11 \cdot 11}$	**c.** $\sqrt{2 \cdot (41 - 9)}$
d. $\sqrt{225 - 9^2}$	**e.** $\sqrt{10^2 - 8^2}$	**f.** $\sqrt{13^2 - 12^2}$

10. Find the value of these expressions to three decimal digits with a calculator. Note: if your calculator doesn't automatically follow the order of operations, you need to use brackets when entering the expressions. Another option is to write the intermediate results down or load them into the calculator's memory.

a. $\sqrt{5.6^2 - 2.1^2}$	**b.** $\sqrt{45.7^2 + 38.12^2}$

Use these exercises for more practice.

11. **a.** What is the area of a square if its side measures $\sqrt{1600}$ cm?



b. What is the area of a square if its side measures $\sqrt{37}$ m?

12. **a.** Sketch a square with an area of 18 square centimetres.

b. What is its perimeter, to two decimal digits?

13. **a.** Sketch a square with a perimeter of 18 cm.

b. What is its area, to two decimal digits?

14. Place the letter of each expression in the box below its value, solving the riddle.

R $\sqrt{4.41}$	**O** $\sqrt{1.2 \cdot 0.3}$	**E** $\sqrt{0.16}$	**H** $\sqrt{169}$
G $\sqrt{100}$	**T** $\sqrt{\frac{225}{9}}$	**Q** $\sqrt{6^2 - 1}$	**R** $\sqrt{10^2 - 6^2}$
S $\sqrt{\frac{25}{4}}$	**E** $\sqrt{\frac{64}{25}}$	**U** $\sqrt{900}$	**S** $\sqrt{\frac{49}{100}}$
V $\sqrt{144}$	**E** $\sqrt{1.21}$	**A** $\sqrt{100(20 + 5)}$	**I** $\sqrt{41 \cdot 41}$
O $\sqrt{43 + 51}$	**M** $\sqrt{0.0016}$	**S** $\sqrt{10\ 000}$	**T** $\sqrt{4(20 - 8)}$

Why do plants hate maths?

Because it...

10	41	12	0.4	2.5		5	13	1.1	0.04		100	$\sqrt{35}$	30	50	2.1	1 3/5		8	$\sqrt{94}$	0.6	$\sqrt{48}$	0.7

Puzzle Corner

Make number 19 on a broken calculator that only has these buttons:

[5] [6] [+] [×] [√] [( ] [ )]

Irrational Numbers

The square roots of perfect squares are whole numbers. However, most numbers, such as 2, 5, and 17, are not perfect squares. We can find a **decimal approximation** to these types of square roots, either with a calculator, or manually, by using the techniques of squaring and guess-and-check.

Example 1. Find the value of $\sqrt{19}$ to two decimal digits, without using a calculator's square root function.

First we find two consecutive perfect squares so that 19 is between them: **16** < 19 < **25**. From that we know that $4 < \sqrt{19} < 5$. Also, since 19 is closer to 16 than to 25, we expect $\sqrt{19}$ to be closer to 4 than to 5.

So let's choose 4.3 and 4.4 as our initial guesses for the value of $\sqrt{19}$, square the guesses, and check how close to 19 we get.

Low Guess	$(LG)^2$	$(HG)^2$	High Guess
4.3	18.49	19.36	4.4

Note: $(LG)^2$ means low guess squared, and $(HG)^2$ means high guess squared.

From the table above, we can see that $\sqrt{19}$ is indeed between 4.3 and 4.4, and that it is probably closer to 4.4 than it is to 4.3 (because 19.36 is closer to 19 than 18.49 is). Let's try 4.36 and 4.37 next.

Low Guess	$(LG)^2$	$(HG)^2$	High Guess
4.3	18.49	19.36	4.4
4.36	19.0096	19.0969	4.37

Oops! $\sqrt{19}$ is not between 4.36 and 4.37. Both of those are too high. Let's try 4.35 and 4.36 next.

Low Guess	$(LG)^2$	$(HG)^2$	High Guess
4.3	18.49	19.36	4.4
4.35	18.9225	19.0096	4.36

Now we know that $\sqrt{19}$ is between 4.35 and 4.36 and closer to 4.36 than it is to 4.35 (because 19.0096 is much closer to 19 than 18.9225 is). This means that **to two decimal digits, $\sqrt{19}$ = 4.36**.

1. Continue refining the decimal approximation to $\sqrt{19}$, to three decimal digits. You may use a calculator to multiply, but do not use its square root function.

Low Guess	$(LG)^2$	$(HG)^2$	High Guess
4.35	18.9225	19.0096	4.36

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Fractions to Decimals

(This lesson is revision, and optional.)

Each fraction is a rational number (by definition!). Each fraction can be written as a decimal. It will either be a terminating decimal, or a non-terminating repeating decimal.

It is easy to rewrite a fraction as a decimal when the denominator is a power of ten. However, when it is not (which is most of the time), simply treat the fraction as a division and divide. You will get either a **terminating decimal** or a non-terminating **repeating decimal**. See the examples below.

1. The denominator is a power of ten or the fraction can be simplified so that it is. In this case, writing the fraction as a decimal is straightforward. Simply write out the numerator. Then add the decimal point based on the fact that the number of zeros in the power of ten tells you the number of decimal digits.

Examples 1. $\frac{7809}{100} = 78.09$ $\frac{1458}{1000} = 1.458$ $\frac{506}{100\,000} = 0.00506$ $\frac{33}{30} = \frac{11}{10} = 1.1$

2. The denominator is a factor of a power of ten. Convert the fraction into one with a denominator that is a power of ten. Then do as in case (1) above.

Examples 2. $\frac{9}{20} = \frac{45}{100} = 0.45$ $\frac{2}{125} = \frac{16}{1000} = 0.016$ $\frac{9}{8} = \frac{1125}{1000} = 1.125$

3. Use division (long division or with a calculator). This method works in all cases, even if the denominator happens to be a power of ten or a factor of a power of ten.

Example 3. Write $\frac{31}{40}$ as a decimal.

```
      0 0.7 7 5
    __________
40 ) 3 1.0 0 0 0
   - 2 8 0
       3 0 0
     - 2 8 0
         2 0 0
       - 2 0 0
             0
```

This division terminates (comes out even) after just three decimal digits.

We get $\frac{31}{40} = 0.775$. This is a **terminating decimal**.

(The fact the division was even means that the denominator 40 is a factor of some power of ten, and so we could have used method 2 from above. In this case, 1000 = 40 · 25.)

Example 4. Write $\frac{18}{11}$ as a decimal.

```
      0 1.6 3 6 3
    ____________
11 ) 1 8.0 0 0 0
   - 1 1
       7 0
     - 6 6
         4 0
       - 3 3
           7 0
         - 6 6
             4 0
           - 3 3
               7
```

We write 18 as 18.0000 in the long division "corner" and divide by 11. Notice how the digits "63" in the quotient, and the remainders 40 and 70, start repeating.

So $\frac{18}{11} = 1.\overline{63}$.

The fraction 18/11 equals $1.\overline{63}$, which is a **repeating decimal**.

1. Write the fractions as decimals.

a. $\frac{2}{100}$ =	**b.** $\frac{278}{10\,000}$ =	**c.** $\frac{55\,073}{1\,000\,000}$ =
d. $\frac{4508}{1000}$ =	**e.** $\frac{56\,330}{100}$ =	**f.** $\frac{40\,309}{10\,000}$ =

2. Write the fractions as decimals.

a. $\frac{2}{5}$ =	**b.** $\frac{24}{25}$ =	**c.** $\frac{54}{200}$ =
d. $\frac{7}{4}$ =	**e.** $\frac{330}{250}$ =	**f.** $\frac{7}{125}$ =

3. Write as decimals. Use long division, and calculate each answer to at least six decimal places. If you find a repeating pattern, give the repeating part. If you don’t, round your answer to five decimals.

a. $\frac{5}{9}$	**b.** $\frac{508}{27}$	**c.** $\frac{23}{61}$

Decimals to Fractions

When a decimal terminates, it is straightforward to write it as a fraction:

1. Copy all the digits (without a decimal point and the leading zeros) to be the numerator.
2. You will find the denominator by checking the number of decimal places: for two decimal places, the denominator is 100, for four decimal places, the denominator is 10 000, and for n decimal places, it is 10^n.

Example 1. $0.00507 = \dfrac{507}{100\,000}$ $\quad 5.256 = \dfrac{5256}{1000}$ $\quad 0.0818 = \dfrac{818}{10\,000}$

When a decimal does not terminate and has a repeating pattern in the decimal digits, it is a rational number, and we can write it as a fraction. The examples show you how.

Example 2. To write $0.2\overline{38}$ as a fraction, we multiply it by some power of ten in such a manner that when we subtract this multiple of $0.2\overline{38}$ and $0.2\overline{38}$, the repeating digits are eliminated in the subtraction.

Since there are *two* digits that repeat, we multiply $0.2\overline{38}$ by 10^2 or 100. The digits will repeat in $100 \cdot 0.2\overline{38}$ <u>in the same places</u> that they do in $0.2\overline{38}$. This means that when we subtract $100 \cdot 0.2\overline{38} - 0.2\overline{38}$, the repeating digits will be eliminated.

Let $x = 0.2\overline{38}$. We will calculate $100x$ and then subtract $100x$ and x. See the equations on the right.

Remember to line up the decimal points carefully, so that the digits that repeat will be in the same places.

We subtract the left sides of the equations, and also the right sides. This leaves $99x$ on the left side, and 23.6 on the right side. From this new equation $99x = 23.6$, we can solve x: it is 23.6/99.

Lastly, we multiply both the numerator and the denominator of that expression by 10, to get 236/990. This can still be simplified to <u>**118/495**</u>.

$100x = 23.8~~383838...~~$
$-\ \ x = 0.2~~38383838...~~$
$99x = 23.6$

$x = 23.6/99 = 236/990$

$= 118/495$

Example 3. Write $0.41\overline{509}$ as a fraction.

Let $y = 0.41\overline{509}$. Since there are three repeating digits, we will multiply y by 10^3 or 1000, and then subtract $1000y$ and y.

See the process on the right. We get $y = \dfrac{41\,468}{99\,900}$.

(Now, this *can* be simplified to 10 367/24 975 but since factorization takes time, you do not have to simplify the fractions in the exercises when the numerators and the denominators are large.)

$1000y = 415.09~~509509...~~$
$-\ y = 0.41~~509509509...~~$
$999y = 414.68$

$y = 414.68/999 = 41\,468/99\,900$

1. Write as a fraction. Simplify if you can, but it is not required.

a. 93.82	**b.** 0.333	**c.** 2.05056
d. 61.098	**e.** 0.0000045	**f.** 4.932048

2. Are the decimals $0.\overline{2}$ and 0.222 the same? If not, what is their difference?

3. Write each repeating decimal as a fraction.

a. $0.\overline{4}$	**b.** $0.2\overline{1}$
c. $0.\overline{954}$	**d.** $2.5\overline{32}$

4. Match the fractions and the decimals.

0.666	$0.\overline{51}$	$0.\overline{6}$	$0.0\overline{6}$	0.51	0.051	0.066	$0.0\overline{51}$
$\frac{51}{100}$	$\frac{2}{3}$	$\frac{66}{1000}$	$\frac{66}{990}$	$\frac{51}{99}$	$\frac{51}{990}$	$\frac{666}{1000}$	$\frac{51}{1000}$

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Chapter 7: Systems of Linear Equations
Introduction

This chapter covers how to analyse and solve pairs of simultaneous linear equations. (The equations studied contain only two variables.)

The first lesson, *Equations with Two Variables*, is optional. It reinforces the idea that a point on a line satisfies the equation of the line, and thus prepares the way for the main topics in the book.

First, students learn to solve systems of linear equations by graphing. Since each equation is an equation of a line, this is a simple technique, but it has its limitations, thus, algebraic solution methods will be taught also, in later lessons.

In the next lesson, we look at the number of solutions that a system of two linear equations can have. The three possible situations are easy to see based on the graphs of the equations: either one solution (the lines intersect in one point), no solutions (lines are parallel), or an infinite number of solutions (the lines are the same).

In the next lesson, students learn the algebraic method of solving systems of equations by substitution. This is a straightforward technique that many students will grasp easily. However, one has to be careful not to make simple mistakes. The lesson has some practice problems where students practise finding errors in solutions. Instruct the student(s) to check their solutions each time, as that is the best way to catch errors.

As for me (Maria, the author), as I wrote the answer key, I immediately checked my solution for each system of equations, and several times found an error that way. (The funniest errors were when I had switched from x to y in the middle of the solution!) So, checking the solution is important. To save space the answer key does not include the checks, but the student should always do that, whether with mental maths or with a calculator.

The following lesson, *Applications, Part 1*, has a variety of word problems that students can now solve using a system of equations.

After that, students learn another algebraic method for solving systems of equations: the addition or elimination method. This is useful when the coefficients of the variables are such that you can easily find their least common multiple. Students also practise solving more complex systems, where the equations first have to be transformed and simplified, or include fractions.

Then it is time for more word problems, in the lesson *Applications, Part 2*. One lesson is devoted to problems about speed, time, and distance, and another for mixtures and comparisons. Making a chart is very helpful in these situations.

Pacing Suggestion for Chapter 7

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing if you use the test.

The Lessons in Chapter 7	page	span	suggested pacing	your pacing
Equations with Two Variables	119	*4 pages*	1 day	
Solving Systems of Equations by Graphing	123	*5 pages*	1-2 days	
Number of Solutions	128	*4 pages*	1 day	
Solving Systems of Equations by Substitution	132	*7 pages*	2 days	
Applications, Part 1	139	*4 pages*	1 day	
The Addition Method, Part 1	143	*5 pages*	1 day	
The Addition Method, Part 2	148	*5 pages*	1 day	
More Practice	153	*4 pages*	1 day	
Applications, Part 2	157	*3 pages*	1 day	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr8ch7

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Equations with Two Variables

(This lesson is optional.)

The equation $2x + 3y = 16$ has **two variables**, x and y. One solution to the equation is $x = 2$ and $y = 4$, because when we substitute those values to the equation, it checks, or is a true equation:

$$2(2) + 3(4) = 16$$

But it also has the solution $x = 0.5$ and $y = 5$:

$$2(0.5) + 3(5) = 16$$

In fact, we can choose any number we like for the value of x, and then *calculate* the value of y, and thus find another solution to the equation.

For example, if we choose $x = -1$, then we get

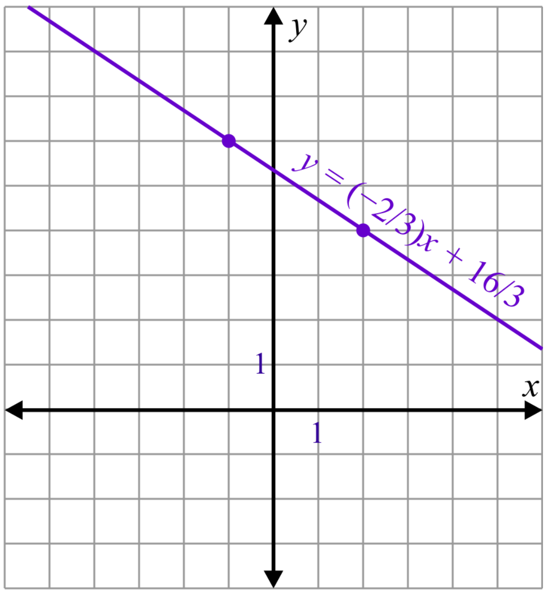
$$2(-1) + 3y = 16$$

from which $y = (16 + 2)/3 = 6$. So, $x = -1$, $y = 6$ is yet another solution.

All of these solutions, having both x and y values, are **number pairs**, and can be considered as **points on the coordinate plane.**

We can make a table of some of the possible (x, y) values (solutions):

x	y
−1	6
0	16/3
0.5	5
2	4



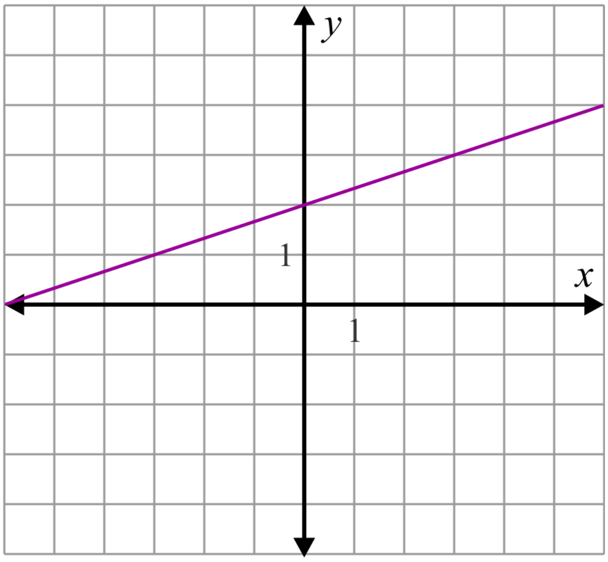
...and there are many more. When plotted, **these points fall on a line** — and you can probably guess, the equation of that line is $2x + 3y = 16$!
(Or, in slope-intercept form, $y = (-2/3)x + 16/3$.)

A line in the coordinate plane represents all the solutions to the equation that is the equation of the line. In other words, **each point on the line is a solution to the equation.**

1. Find three solutions to the equation $5x + 2y = 32$.

2. Find three solutions to the equation $-4x + y = -6$.

3. **a.** What is the equation if its solution set is represented by this line?



b. List two distinct integer number pairs that are solutions to the equation.

4. A certain linear equation with two variables has as solutions (0, −5), (2, 3) and (4, 11). Find the equation.

5. A certain linear equation with two variables has as solutions (−1, −5) and (2, 8). Find the equation.

6. Party hats cost $2 apiece and party whistles cost $3 apiece. Randy bought *x* hats and *y* whistles.

a. Write an expression depicting the total cost (C).

b. Now write an equation stating that the total cost is $48.

How many hats and how many whistles could Randy have bought?

c. Find two other solutions to your equation.

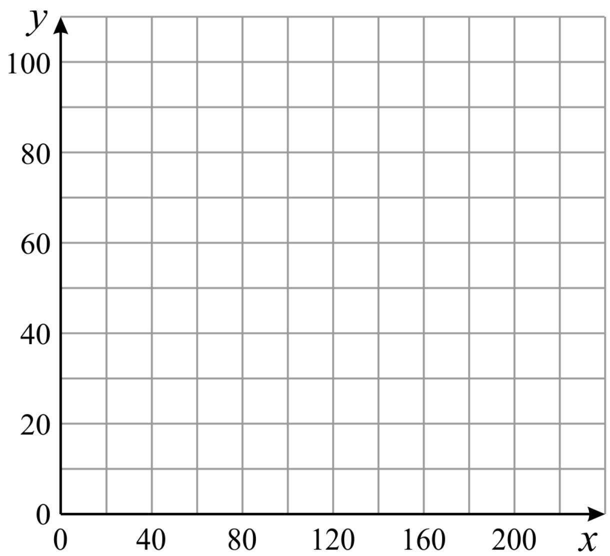
7. Recall the formula tying together distance (d), constant speed (v), and time (t): $\boldsymbol{d = vt}$.
Sarah jogs at the speed of 9 km per hour, and she rides her bicycle at the speed of 18 km per hour.

a. Convert these speeds to kilometres per minute.

b. Write an expression for the total distance (d) Sarah covers in x minutes of jogging plus y minutes of bicycling.

c. What distance does Sarah cover if she jogs for 20 minutes and bicycles for 10 minutes?

d. Let's say the distance Sarah covers, jogging and bicycling, is 30 km. Write an equation stating this. How many minutes could she have jogged/bicycled? Find three possible solutions.

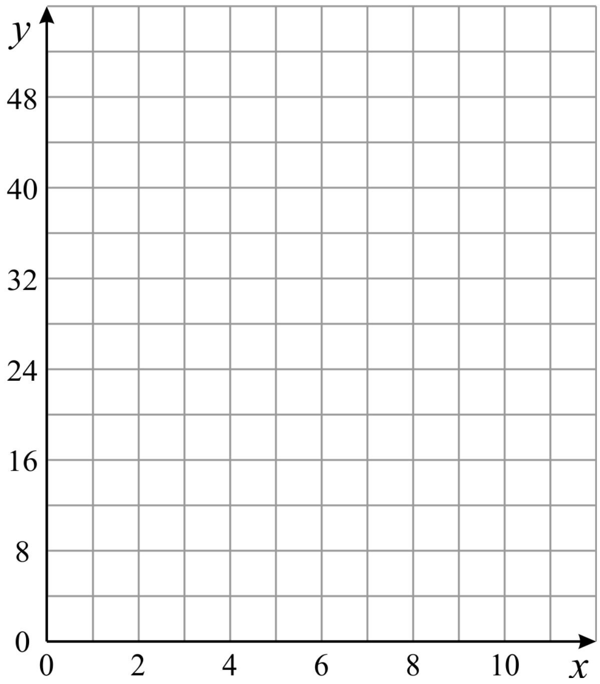


e. Write the equation in slope-intercept form and plot it.

8. General admission to a gardening seminar was \$15 but seniors paid only \$10. If the total of the admission fees was \$900, give three possibilities as to how many non-seniors and how many seniors could have attended.

9. A mystery basket contains a mixture of adult cats and kittens (it could even contain zero adults or zero kittens).
Each cat weighs 4 kg and each kitten weighs 0.5 kg.
The total weight of the cats and kittens is 20 kg.

a. If there are x cats and y kittens, write an equation to match the situation.



b. How many adult cats and how many kittens could there be? Find at least three different solutions.

c. Plot your equation from (a).

d. If $x = 1.5$, what is y?
Why is this not a valid solution?

Plot the individual points on the graph that *are* valid solutions.

10. Ava and her family went to stay in a resort for a few nights. Each night cost $120 (for the whole family). The resort offered horse rides for $20 per person.

a. If the family stayed for x nights and did y horse rides in total, write an expression for the total cost of these two things.

b. In total, Ava's family spent $760 on the horse rides plus the nights they stayed.
How many nights and how many horse rides could they have paid for?

11. The equation $2x^2 - 6x - y = 5$ is a quadratic equation because the variable x is squared. If $x = 0$, then $y = -5$, so $(0, -5)$ is one solution to the equation. Find two other solutions to it.

Solving Systems of Equations by Graphing

A system of equations consists of several equations that have the same variables.

A **solution** to a system of equations is a list of values of the variables that satisfy *all* the equations in the system. For two equations, this is an ordered pair.

Example 1. This system of equations consists of two equations. We signify the system with a bracket.

$$\begin{cases} 5x + 4y = 12 \\ y = -x + 2 \end{cases}$$

The solution to the above system is the ordered pair (4, −2), because those values make both equations true: 5(4) + 4(−2) does equal 12, and −2 does equal −4 + 2.

Example 2. The equation $y = (3/2)x - 4$ has an infinite number of solutions, and we can represent those solutions with a line drawn in the coordinate plane.

Similarly, the equation $y = -2x + 3$ has infinitely many solutions.

Here is a system of equations consisting of both:

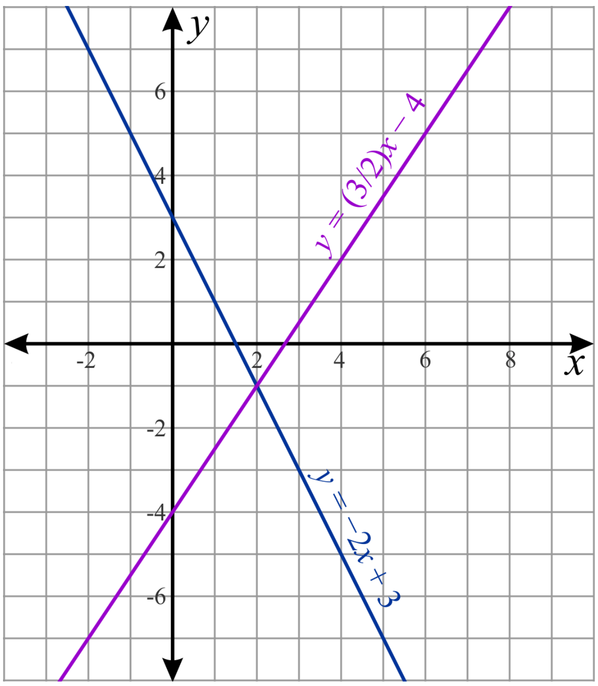
$$\begin{cases} y = (3/2)x - 4 \\ y = -2x + 3 \end{cases}$$

Since the solutions to the first equation form a line, and the solutions to the second also form a line, what would the point of intersection (2, −1) signify?

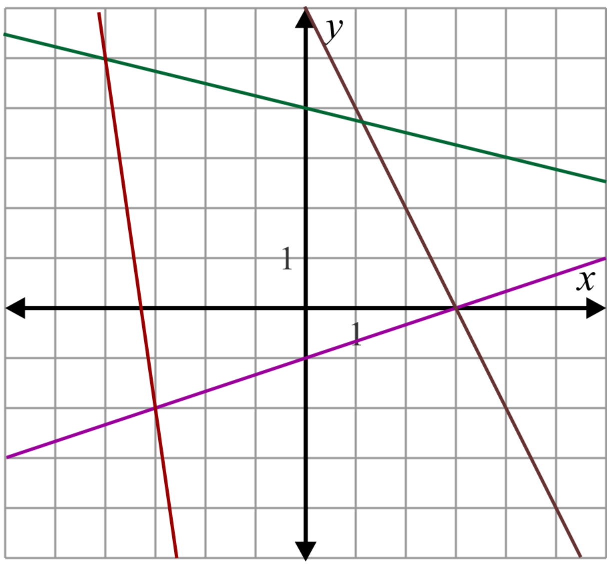
____

____

(The answer is found at the end of the lesson.)



1. Solve each system of equations using the image. The lines are already plotted in it.



a. $\begin{cases} y = -7x - 23 \\ y = (1/3)x - 1 \end{cases}$

Solution: (_____, _____)

b. $\begin{cases} y = -(1/4)x + 4 \\ y = -2x + 6 \end{cases}$

Solution: (_____, _____)

c. $\begin{cases} -(1/3)x + y = -1 \\ 2x + y = 6 \end{cases}$

Solution: (_____, _____)

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Speed, Time, and Distance Problems

There are many jokes about algebra word problems where a train leaves a station at a certain hour. You can now solve these types of problems with your knowledge of systems of equations. One of the most effective ways to do so is to first build a chart.

Example 1. A train leaves a station at 9:00 AM and travels with a constant speed of 90 km/h. Another train leaves the same station 10 minutes later, travelling to the same direction at the speed of 100 km/h. At what time will the second train reach the first?

We will be using the formula $d = vt$ extensively in these problems. Let's build a chart. The goal is to have TWO, not three or more, variables present in the chart. The formula $d = vt$ has three variables, and since the speed, distance, and time can be different for each train, theoretically we could have six variables. However, invariably, the problem gives information for one or some of these variables, and something about the situation means that the distance or the time or the speed is the same for both trains.

To get started, we gather some information in the chart. The distance that train 1 and train 2 travel until they meet is the same, so that is why we use the same variable, d, for it.

	distance	***velocity***	***time***
Train 1	d	90 km/h	t_1
Train 2	d	100 km/h	t_2

The times (t_1 and t_2) are different, but we do know that they differ by 10 minutes, so, actually we will get by using only *one* variable for time, like this:

	distance	***velocity***	***time***
Train 1	d	90 km/h	t
Train 2	d	100 km/h	$t - 10$

The chart now contains only two variables. However, we have one more thing to change. The speed is in km/h, whereas the 10 has to do with minutes. For our equation to work, the time units need to be the same, so we will change the 10 to 1/6 (in hours).

	distance	***velocity***	***time***
Train 1	d	90 km/h	t
Train 2	d	100 km/h	$t - 1/6$

The equations always follow the same formula: $d = vt$, and we use that same formula for both Train 1 and Train 2. So, the two equations we get are:

$$\begin{cases} d = 90t \\ d = 100(t - 1/6) \end{cases}$$

The quickest way to solve this system is to set $90t$ equal to $100(t - 1/6)$ and solve for t.

1. Solve the system of equations from example 1 and answer the question: At what time will the second train reach the first? Is the answer surprising?

$$\begin{cases} d = 90t \\ d = 100(t - 1/6) \end{cases}$$

2. Your friend starts walking at a speed of 6 km/h from your home to his. Exactly 15 minutes later, you decide you want to join him so you take your bicycle and start after him, with a speed of 18 km/h. How far have you ridden by the time you reach your friend?

	distance	***velocity***	***time***
Your friend			
You			

3. A tortoise and hare race a distance of 100 m. The hare gives the tortoise a 10-minute lead time. Then he quickly runs the 100 metres and wins the race. After the hare has finished, the tortoise takes an *additional* 6 minutes to reach the finish line. If the speed of the hare is 15 m/s, find the time the tortoise takes to finish the race and the tortoise's speed.

Hint: since the speed is in metres per second, and the distance is in metres, the time unit will be <u>seconds</u>.

	distance	***velocity***	***time***
Tortoise			
Hare			

Example 2. A train leaves Turin (Italy), heading for Milan (Italy), a distance of 125 km, at 1 PM and travels with a constant speed of 90 km/h. Another train leaves Milan, heading for Turin and travelling at a constant speed, at the same time. They meet 45 minutes later. (We hope they don't crash!) What is the speed of the second train? What distance has the second train travelled by that time?

We fill our chart again. The time, 45 minutes, is 3/4 hour. The speed of the second train, v_2, is unknown.

There is a relationship between the two distances, because $d_1 + d_2 = 125$ km. So, we can get by with just one variable for the distance:

	distance	*velocity*	*time*
Train 1	d	90 km/h	3/4
Train 2	$125 - d$	v_2	3/4

Our equations are:

$$\begin{cases} d = 90(3/4) \\ 125 - d = (3/4)v_2 \end{cases}$$

Here, it is handy to use the substitution method, since we have an expression for d. So, we substitute 90(3/4), which equals 67.5, in place of d in the second equation:

$$\begin{aligned} \textbf{(2)} \quad 125 - \mathbf{67.5} &= (3/4)v_2 \\ 57.5 &= (3/4)v_2 \quad | \cdot 4 \\ 230 &= 3v_2 \\ v_2 &= 76.\overline{6} \end{aligned}$$

So, the speed of the second train is $76.\overline{6}$ km/h. The problem also asked what distance the second train has travelled by that time. To find that, we use the formula $d = vt$: $d = 76.\overline{6}$ km/h · (3/4 h) = 57.5 km.

4. Two trains leave the same station at the same time, one travelling due east and the other travelling due west. Train 1 travels at a speed of 120 km/h and Train 2 at the speed of 100 km/h. When are the trains 50 km apart from each other?

	distance	*velocity*	*time*
Train 1			
Train 2			

5. Train 1 leaves the station heading due south and Train 2 leaves the same station at the same time, heading due north. Train 1 travels at the speed of 110 km/h. After 30 minutes, the trains are 120 km apart. How fast is Train 2 travelling?

	distance	***velocity***	***time***
Train 1			
Train 2			

6. **a.** Two horses, Ranger and Chip, start racing at the same time. Ranger runs at a steady speed of 16 m/s. After 100 seconds, they are 600 m apart from each other, Ranger leading. How fast is Chip running?

	distance	***velocity***	***time***
Ranger			
Chip			

b. What would Chip's speed need to be, so that after 100 seconds, he would only be 50 m behind Ranger?

Example 3. A motorboat travels downstream on the river a distance of 8 km, in 20 minutes. Doing the same trip upstream takes it 4 minutes longer. How fast is the river flowing? What is the speed of the boat in still water?

Our chart method will still work. We are dealing with two speeds: that of the boat (v_b) (in still water), and that of the water (v_w) in the river. Going downstream, the boat's actual speed is its own speed PLUS the speed of the water. Going upstream, it has to fight the current and its actual speed is $v_b - v_w$.

This is what the chart looks like. Using these quantities, the speeds will end up being in km per minute.

	distance	***velocity***	***time***
Downstream	8 km	$v_b + v_w$	20 min
Upstream	8 km	$v_b - v_w$	24 min

Our equations are: $\begin{cases} 8 = 20(v_b + v_w) \\ 8 = 24(v_b - v_w) \end{cases}$

7. Solve the problem in example 3.

8. An aeroplane travels from City A to City B in 2 hours 30 minutes, flying with the wind, and in 2 hours 45 minutes flying against the wind. If the speed of the aeroplane in still air is 900 km/h, find the distance between the cities to the nearest 10 km.

9. With a tailwind, an aeroplane can fly from City 1 to City 2, a distance of 650 km, in 40 minutes. If the speed of the wind is 30 km/h, find the time the aeroplane takes to fly the same distance against the wind.

10. You swim downstream, from a dock to a certain rock in the middle of the river, in 43 seconds. Swimming back (upstream) takes you 10 seconds longer. If your swimming speed in still water is 1.6 km/h, what is the speed of the water in the river?

Puzzle Corner

The following are “trick” problems. Have fun!

(1) Train 1 leaves Jackson at 1:30 PM, travelling at 95 km/h towards Atlanta, a distance of 560 km, and Train 2 leaves Atlanta, heading towards Jackson at the same time, travelling at 105 km/h. When they meet, which train is closer to Atlanta?

(2) At 6:30 PM, you board a train in Dallas, heading south, and 10 minutes later, your friend boards a train at Kansas City, heading north. If both trains travel at 100 km/h, when do you pass each other?

(3) Two trains leave a station at the same time, one heading east, the other heading west. After 15 minutes, they are 64 km apart. Which train is travelling faster?

Mixtures and Comparisons

The chart method lends itself well also for solving problems about mixtures.

Example 1. Raisins cost $8/kg and almonds cost $13.70/kg. Ashley made a mixture of both so that the cost of 1 kilogram is $10.00. What amount of the mixture is raisins?

Let r be the weight of the raisins and a be the weight of the almonds in the mixture (in kilograms). We can fill in a chart:

	weight (kg)	***cost per kg***	***cost* ($)**
Raisins	r	$8/kg	$8r$
Almonds	a	$13.70/kg	$13.7a$
Mixture	1	$10/kg	$10.00

There are *two* unknowns, r and a. Now all we need is *two* equations. What allows us to build those equations? What is equal to what? Think about it first. Then check the end of the lesson for the two equations. The rest of the solution is left for an exercise.

1. Finish solving the problem in Example 1.

2. A mixture of grains meant for a bird feed is 16% protein by weight. It consists of corn (12% of protein by weight) and chickpeas (22% protein by weight). Find out how much corn and how much chickpeas is in 2 kg of this mixture.

   The given information is already put in the chart below. Choose two variables and fill in the chart. Then write two equations using the information in the chart. Check your equations before going to exercise #3.

	weight (kg)	***protein (%)***	**protein (weight)**
Corn		12	
Chickpeas		22	
Mixture	2	16	0.16(2)

   Equations:

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Chapter 8: Bivariate Data
Introduction

The last chapter of grade 8 covers statistical topics that have to do with bivariate data, or data involving two variables.

The first lesson introduces scatter plots. Students analyse the data in a variety of scatter plots, and determine visually whether there is an association between the variables. Next, they learn about basic patterns we often see in scatter plots, such as positive and negative association, linear association, clusters, and outliers. They also make scatter plots from given data, describe any special features in the plot, and answer a variety of questions related to the data.

In the following lesson, students fit a line (informally) to the data points displayed in a scatter plot. Mathematicians have developed several algorithms for finding a line of best fit, such as linear regression, but we are not using those here. Students use the basic idea of trying to leave close to an equal number of points on each side of the line, and also judging the fit by the closeness of the points to the line. This resembles the thought behind the linear regression algorithm, which finds the line of best fit by minimising the squares of the distances of the data points to the line.

The last topic relating to scatter plots is the equation of the trend line. Students use the equation of the trend line to solve problems in the context of the data, interpreting the slope and intercept of the equation.

Then we turn our attention to categorical bivariate data, that is, data involving two variables that may or may not be numerical, but is divided into categories. Students learn that bivariate categorical data can be summarised in a two-way table, and if there is a pattern of association between the variables, it can be seen in the table.

Students construct and interpret two-way tables summarising data on two categorical variables. In the last lesson, they calculate relative frequencies for rows or columns, and use those to describe the possible association between the two variables.

Pacing Suggestion for Chapter 8

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing if you use the test.

The Lessons in Chapter 8	page	span	suggested pacing	your pacing
Scatter Plots	183	*3 pages*	1 day	
Scatter Plot Features and Patterns	186	*4 pages*	1 day	
Fitting a Line	190	*4 pages*	1 day	
Equation of the Trend Line	194	*5 pages*	1-2 days	
Two-Way Tables	199	*3 pages*	1 day	
Relative Frequencies	202	*5 pages*	2 days	
Mixed Revision Chapter 8	207	*5 pages*	2 days	
Chapter 8 Revision	212	*3 pages*	1 day	
Chapter 8 Test (optional)				
TOTALS		*32 pages*	10-11 days	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

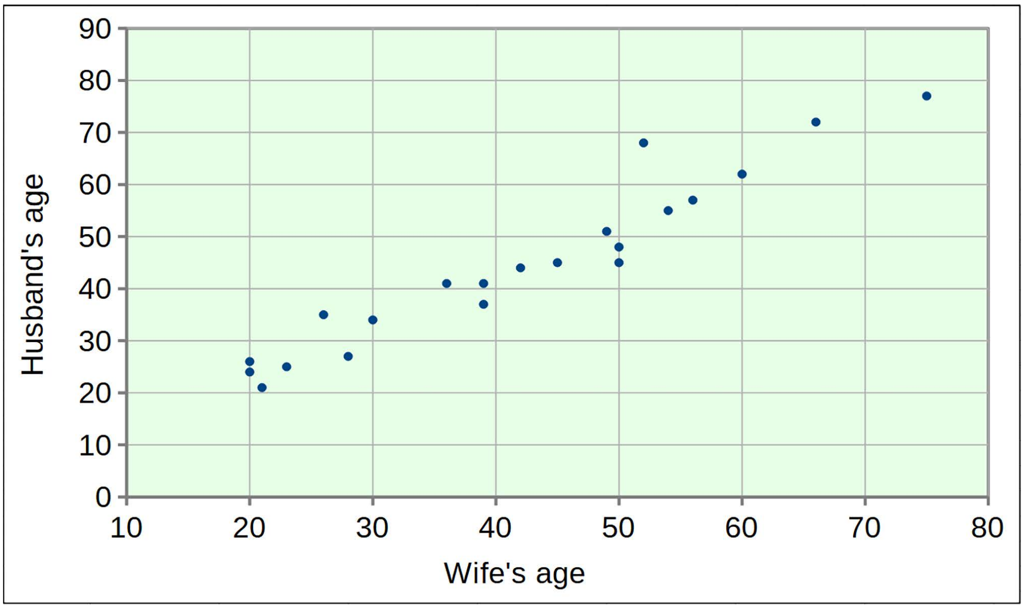
We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr8ch8

Scan me

Scatter Plots

A **scatter plot** depicts **bivariate data**, meaning that the data involves **two variables**. In the scatter plot below, the variables are the husband's age and the wife's age. Each dot in this scatter plot represents a husband-wife couple. In other words, the coordinates of the dot give us the ages of the husband and the wife.



1. Refer to the scatter plot above.

a. Locate the dot with coordinates (36, 41). What does it signify?

b. Find two couples where the wife is the same age in both cases. Estimate the ages of their husbands.

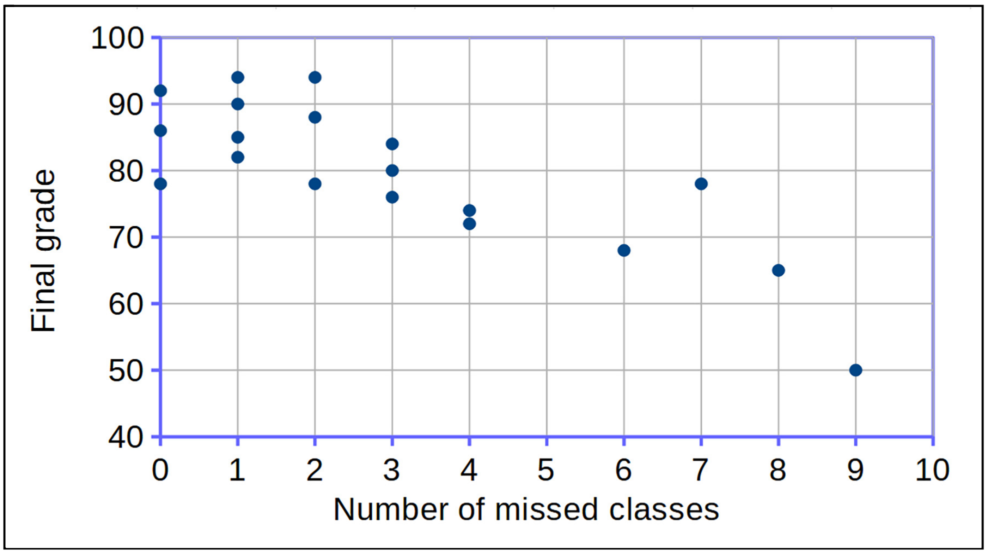
c. Find the couple with the third oldest husband in this data set. How old is his wife?

d. Is it true that the youngest wife is married to the youngest husband? Explain.

e. Is it true that the oldest wife is married to the oldest husband? Explain.

f. Do you notice a relationship between the two variables? Explain what you see.

2. The scatter plot below shows the final grade and the number of missed classes for a group of students who took a 15-week course on nursing care.



a. What are the two variables depicted?

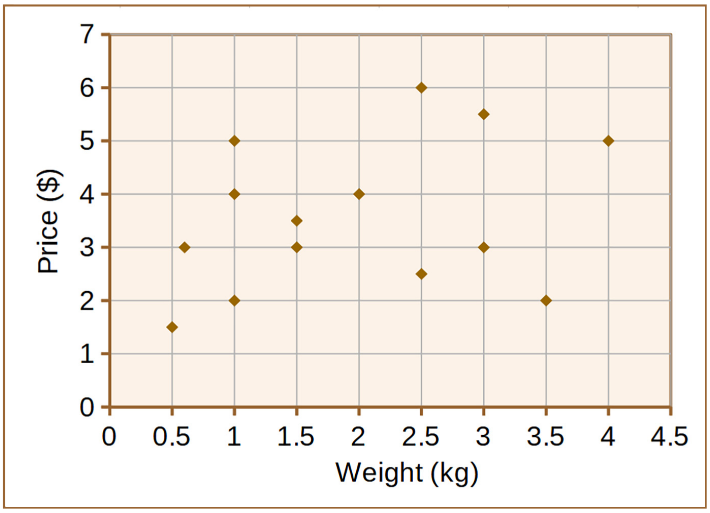
b. How many times did the two students with the highest grade miss class?

c. Add to the plot a point depicting a student with nearly perfect attendance but with a mediocre grade (from 60 to 75).

d. Do you notice a relationship between the two variables? Explain what you see.

e. Is the relationship ***causal***? In other words, is the number of missed classes directly causing the student's final grade?

3. Each dot in this plot depicts a bag of rice.



a. What does the point (2.5, 6) signify?

b. Find two bags of rice that cost $2 per kilogram.

c. Add a point for a 4-kg bag of rice with a very low price.

d. Which bag gives you the best value for your money?

e. Could we say that as the weight increases, the price also increases?

4. The data below shows the shots and the goals for a mini-tournament of four games for each player on a soccer team. (A shot is when the player kicks the ball towards the goal.) Each row has the data for one player in a team.

a. Make a scatter plot of the data. Choose the scaling for both axes wisely, so that all your data fits in the graph. Note that the scaling does not have to start from 0. See question 2 for an example where the vertical axis started at 40.

Shots	Goals
20	2
21	1
21	2
22	3
22	2

Shots	Goals
22	3
24	4
24	2
25	4
25	5

Shots	Goals
26	3
26	2
27	4
28	6
30	5

b. Do you notice a relationship between the two variables? Explain what you see.

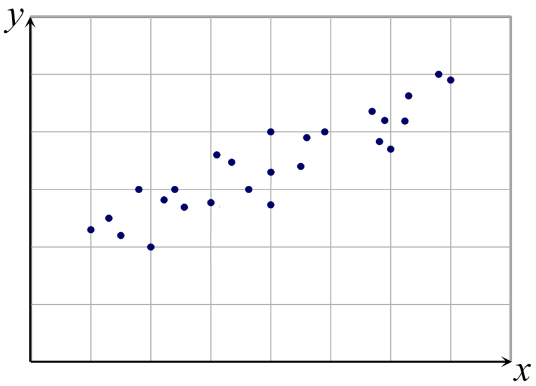
c. Which players do you feel are the best? Explain why.

Scatter Plot Features and Patterns

When studying data in two variables, we are usually interested in knowing whether there is any association between them; in other words, whether the two things we are studying are connected in any way.

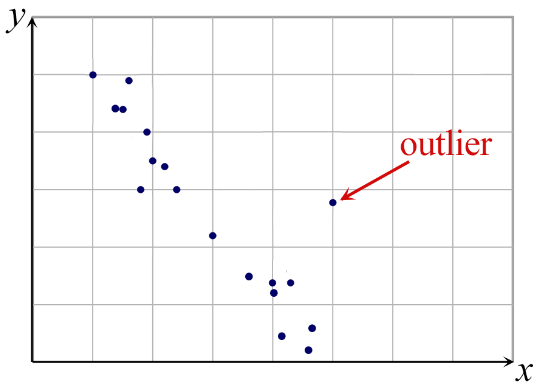
An association will be seen visually in a scatter plot when the points lie in a visible pattern. The data can also have outliers and clusters.

A positive linear association



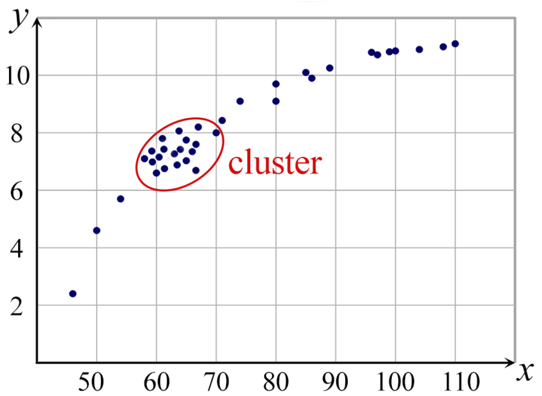
The pattern of the points is as if on a line (linear). As the *x* values increase, the *y*-values increase also, so the association is **positive**.

A negative linear association



The points lie in a linear pattern. As the *x* values increase, the *y*-values *decrease*, so the association is **negative**. One point lies far from the others; it is an **outlier**.

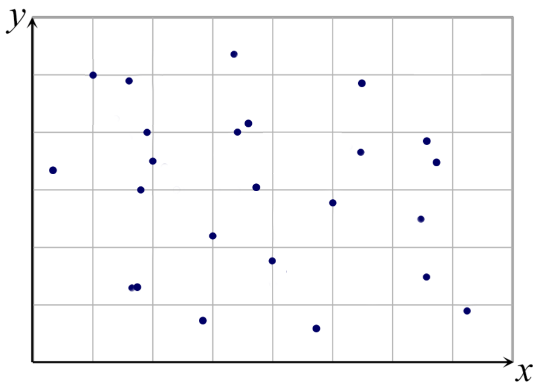
A positive nonlinear association



Here as the *x*-values increase, the *y*-values increase also, so the association is positive. But the pattern of dots follows a curve, not a straight line, so we say that the association is **nonlinear**.

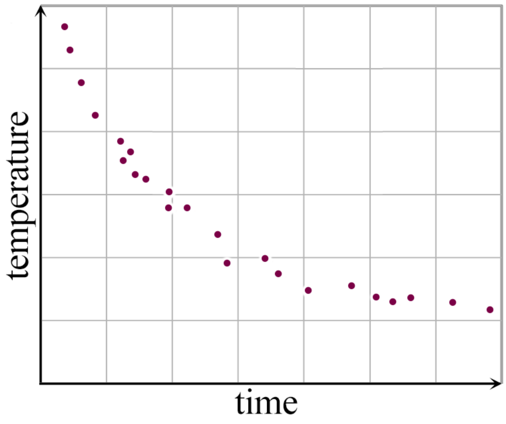
Additionally, we see a **cluster** — many points close together in a small area, around the *x* values of 58 to 70 and *y*-values of 7.5 to 8.3.

No association

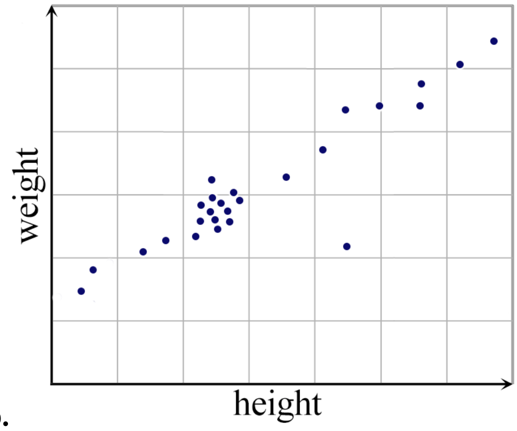


Here, there is no association between the variables. The dots are scattered and do not lie in any visible pattern.

1. Describe the patterns and features you see in each scatter plot.

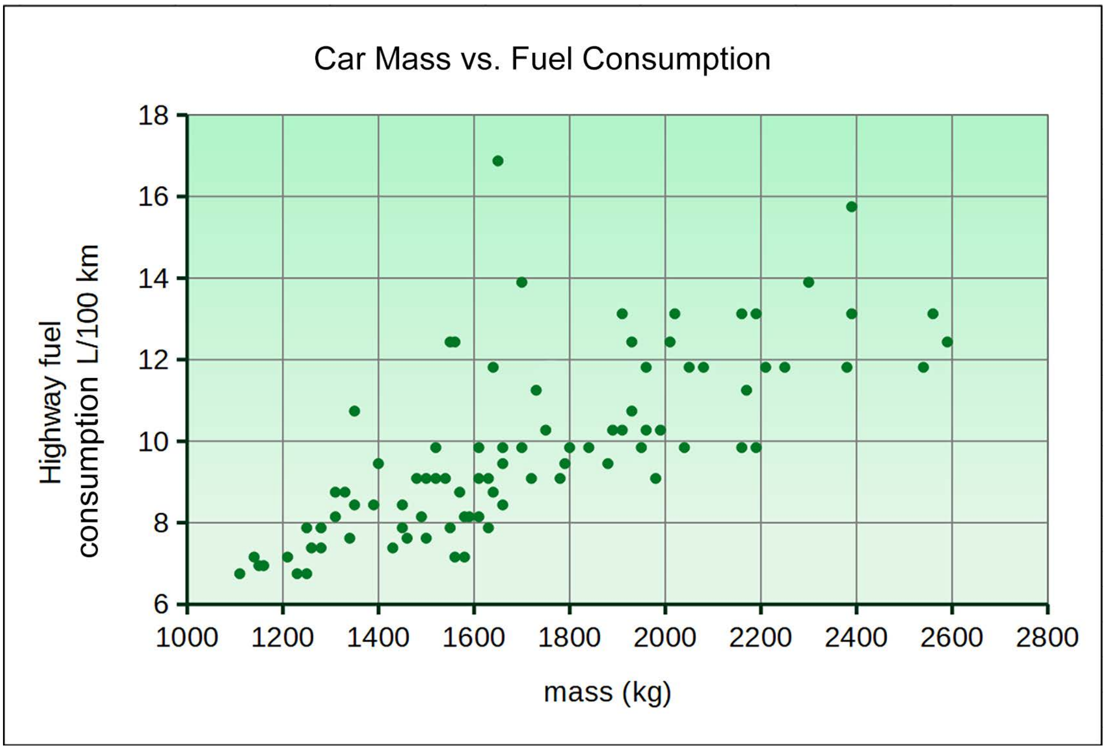


a.



b.

2. This graph shows the mass and fuel consumption (in litres per 100 km, for highway driving) of various cars.



a. Describe the general pattern and any special features of the plot.
Do heavier cars use more or less fuel, in general, than lighter cars?

b. Find the heaviest car that uses less than 10 L per 100 km. How much does it weigh?

c. List the mass and the fuel consumption of the car that is so different from the others that it may be even an error in the data, such as a typo.

d. Among cars weighing from 1800 to 2200 kg, how much fuel does the car with the least fuel consumption use?