Contents

Chapter 3: Linear Equations

Chapter 4: Introduction to Functions

Foreword

Math Mammoth Grade 8, International Version, comprises a complete maths curriculum for the eighth grade mathematics studies. This curriculum is essentially the same as the *Math Mammoth Grade 8* sold in the United States, only customised for international use in a few aspects (listed below). The curriculum meets the Common Core Standards in the United States, but it may not properly align to the eighth grade standards in your country. However, you can probably find material for any missing topics in the neighbouring grades of Math Mammoth.

The International version of Math Mammoth differs from the US version in these aspects:

- The curriculum uses metric measurement units, not both metric and customary (imperial) units.
- The spelling conforms to British international standards.
- The pages are formatted for A4 paper size.
- Large numbers are formatted with a space as the thousands separator (such as 12 394).
  (The decimals are formatted with a decimal point, as in the US version.)

In 8th grade, students spend the majority of the time with algebraic topics, such as linear equations, functions, and systems of equations. The other major topics are geometry and statistics. The main areas of study are:

- Exponents laws and scientific notation
- Square roots, cube roots, and irrational numbers
- Geometry: congruent transformations, dilations, angle relationships, volume of certain solids, and the Pythagorean Theorem
- Solving and graphing linear equations;
- Introduction to functions;
- Systems of linear equations;
- Scatter plots/bivariate data.

We start with a study of exponent laws, using both numerical and algebraic expressions. The first chapter also covers scientific notation (both with large and small numbers), significant digits, and calculations with numbers given in scientific notations.

In chapter 2, students learn about geometric transformations (translations, reflections, rotations, dilations), common angle relationships, and volume of prisms, cylinders, spheres, and cones.

Next, in chapter 3, our focus is on linear equations. Students both revise and learn more about solving linear equations, including equations whose solutions require the usage of the distributive property and equations where the variable is on both sides.

Chapter 4 presents an introduction to functions. Students construct functions to model linear relationships, learn to use the rate of change and initial value of the function, and they describe functions qualitatively based on their graphs.

In part 8-B, students graph linear equations, learn about irrational numbers and the Pythagorean Theorem, solve systems of linear equations, and investigate patterns of association in bivariate data (scatter plots).

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching maths!

Maria Miller, the author

User Guide

Note: You can also find the information that follows online, at **https://www.mathmammoth.com/userguides/** .

Basic principles in using Math Mammoth Complete Curriculum

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. The two books (parts A and B) are like a “framework”, but you still have some liberty in planning your student’s studies. In eighth grade, chapters 2 (geometry), 3 (linear equations) and chapter 4 (functions) should be studied before chapter 5 (graphing linear equations). Also, chapters 3, 4, and 5 should be studied before chapter 7 (systems of linear equations) and before chapter 8 (statistics). However, you still have some flexibility in scheduling the various chapters.

Math Mammoth is not a scripted curriculum. In other words, it is not spelling out in exact detail what the teacher is to do or say. Instead, Math Mammoth gives you, the teacher, various tools for teaching:

- **The two student worktexts** (parts A and B) contain all the lesson material and exercises. They include the explanations of the concepts (the teaching part) in blue boxes. The worktexts also contain some advice for the teacher in the “Introduction” of each chapter.

  The teacher can read the teaching part of each lesson before the lesson, or read and study it together with the student in the lesson, or let the student read and study on his own. If you are a classroom teacher, you can copy the examples from the “blue teaching boxes” to the board and go through them on the board.

- There are **videos** matched to the curriculum available at https://www.mathmammoth.com/videos/ . There isn’t a video for every lesson, but there are dozens of videos for each grade level. You can simply have the author teach your child or student!
- Don’t automatically assign all the exercises. Use your judgement, trying to assign just enough for your student’s needs. You can use the skipped exercises later for revision. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.
- For each chapter, there is a **link list to various free online games** and activities. These games can be used to supplement the maths lessons, for learning maths facts, or just for some fun. Each chapter introduction (in the student worktext) contains a link to the list corresponding to that chapter.
- The student books contain some **mixed revision lessons**, and the curriculum also provides you with additional **cumulative revision lessons**.
- There is a **chapter test** for each chapter of the curriculum, and a comprehensive end-of-year test.
- You can use the free online exercises at https://www.mathmammoth.com/practice/
  This is an expanding section of the site, so check often to see what new topics we are adding to it!
- And there are answer keys to everything.

How to get started

Have ready the first lesson from the student worktext. Go over the first teaching part (within the blue boxes) together with your student. Go through a few of the first exercises together, and then assign some problems for the student to do on their own.

Repeat this if the lesson has other blue teaching boxes.

Many students can eventually study the lessons completely on their own — the curriculum becomes self-teaching. However, students definitely vary in how much they need someone to be there to actually teach them.

Pacing the curriculum

Worktext 8-A	
Chapter 1	13 days
Chapter 2	27 days
Chapter 3	21 days
Chapter 4	14 days
TOTAL	**75 days**

Each chapter introduction contains a suggested pacing guide for that chapter. You will see a summary on the right. (This summary does not include time for optional tests.)

Most lessons are 3 or 4 pages long, intended for one day. Some lessons are 5 pages and can be covered in two days.

It can also be helpful to calculate a general guideline as to how many pages per week the student should cover in order to go through the curriculum in one school year.

The table below lists how many pages there are for the student to finish in this particular grade level, and gives you a guideline for how many pages per day to finish, assuming a 180-day (36-week) school year. The page count in the table below *includes* the optional lessons.

Example:

Grade level	School days	Days for tests and revisions	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
8-A	90	8	204	82	2.5	12.5
8-B	90	8	182	82	2.2	11
Grade 8 total	180	16	386	164	2.4	12

The table below is for you to fill in. Allow several days for tests and additional revision before tests — I suggest at least twice the number of chapters in the curriculum. Then, to get a count of "pages to study per day", **divide the number of lesson pages by the number of days for the student book**. Lastly, multiply this number by 5 to get the approximate page count to cover in a week.

Grade level	Number of school days	Days for tests and revisions	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
8-A			204			
8-B			182			
Grade 8 total			386			

Now, something important. Whenever the curriculum has lots of similar practice problems (a large set of problems), feel free to **only assign 1/2 or 2/3 of those problems**. If your student gets it with less amount of exercises, then that is perfect! If not, you can always assign the rest of the problems for some other day. In fact, you could even use these unassigned problems the next week or next month for some additional revision.

In general, 8th graders might spend 45-75 minutes a day on maths. If your student finds maths enjoyable, they can of course spend more time with it! However, it is not good to drag out the lessons on a regular basis, because that could affect the student's attitude towards maths.

Using tests

For each chapter, there is a **chapter test**, which can be administered right after studying the chapter. **The tests are optional.** Some families might prefer not to give tests at all. The main reason for the tests is for diagnostic purposes, and for record keeping. These tests are not aligned or matched to any standards.

In the digital version of the curriculum, the tests are provided as PDF files. You can edit them (such as to change the numbers in them) to provide a different test using PDF apps that have editing capabilities. You can even use the annotation tools (such as text boxes) available in most PDF apps. Remember to save the edited file under a different file name, or you will lose the original.

The end-of-year test is best administered as a diagnostic or assessment test, which will tell you how well the student remembers and has mastered the mathematics content of the entire grade level.

Using cumulative revisions and the worksheet maker

The student books contain mixed revision lessons which revise concepts from earlier chapters. The curriculum also comes with additional cumulative revision lessons, which are just like the mixed revision lessons in the student books, with a mix of problems covering various topics. These are found in their own folder in the digital version, and in the Tests & Cumulative Revisions book in the print version.

The cumulative revisions are optional; use them as needed. They are named indicating which chapters of the main curriculum the problems in the revision come from. For example, "Cumulative Revision, Chapter 4" includes problems that cover topics from chapters 1-4.

Both the mixed and cumulative revisions allow you to spot areas that the student has not grasped well or has forgotten. When you find such a topic or concept, you have several options:

1. Check if the worksheet maker lets you make worksheets for that topic.
2. Check for any online games and resources in the Introduction part of the particular chapter in which this topic or concept was taught.
3. If you have the digital version, you could reprint the lesson from the student worktext, and have the student restudy that.
4. Perhaps you only assigned 1/2 or 2/3 of the exercise sets in the student book at first, and can now use the remaining exercises.
5. Check if our online practice area at https://www.mathmammoth.com/practice/ has something for the topic.
6. Khan Academy has free online exercises, articles, and videos for most any maths topic imaginable.

Concerning challenging word problems and puzzles

While this is not absolutely necessary, I heartily recommend supplementing Math Mammoth with challenging word problems and puzzles. You could do that once a month, for example, or more often if the student enjoys it.

The goal of challenging story problems and puzzles is to **develop the student's logical and abstract thinking and mental discipline**. Math Mammoth curriculum contains lots of word problems, and they are usually multi-step problems. Several of the lessons utilise a bar model for solving problems. Even so, the problems I have created are usually tied to a specific concept or concepts. I feel students can benefit from solving problems and puzzles that require them to think "out of the box" or are just different from the ones I have written.

I recommend you use the free Math Stars problem-solving newsletters as one of the main resources for puzzles and challenging problems:

Math Stars Problem Solving Newsletter (grades 1-8)
https://www.homeschoolmath.net/teaching/math-stars.php

I have also compiled a list of other resources for problem solving practice, which you can access at this link:
https://l.mathmammoth.com/challengingproblems

Another idea: you can find puzzles online by searching for "brain puzzles for kids," "logic puzzles for kids" or "brain teasers for kids."

Frequently asked questions and contacting us

If you have more questions, please first check the FAQ at https://www.mathmammoth.com/faq-lightblue

If the FAQ does not cover your question, you can contact us using the contact form at the MathMammoth.com website.

Chapter 1: Exponents and Scientific Notation

Introduction

The first chapter of Math Mammoth Grade 8 starts out with a study of basic exponent laws and scientific notation.

We begin with a revision of the concept of an exponent and of the order of operations. The next lesson first revises multiplication of integers, and then focuses on powers with negative bases, such as $(-5)^3$.

Then we get to the "meat" of the chapter: the various laws of exponents. The first lesson on that topic allows students to explore and to find for themselves the product law and the quotient law of exponents. After that, students find out the logical way to define negative and zero exponent by looking at patterns. They practise simplifying various expressions with exponents, both with numerical values and with variables.

The lesson "More on Negative Exponents" focuses on expressions with a negative exponent in the numerator, such as $7/(a^{-4})$. This is to prepare students for calculations that ask them to find how many times bigger one number is than another, when the numbers are written in scientific notation.

Next, in the lesson "Laws of Exponents, Part 2", students practise applying the power of a power law: $(a^n)^m = a^{nm}$.

Then the chapter has one more lesson on the laws of exponents ("Laws of Exponents, Part 3"), which summarises the laws and gives more practice. This lesson is not absolutely essential if you're following Common Core Standards. It is presented here to give a summary, to give practice on all exponent laws, including the power of a quotient law which was not dealt with a lot in the previous lessons. This lesson also allows the book to meet the Florida B.E.S.T. standards for 8th grade.

Then we turn our attention to scientific notation, first learning how it is used with large numbers and then with small numbers. The lesson on significant digits follows, helping students to know how to round final answers in calculations with measurements.

The last topic of the chapter is calculations with numbers given in scientific notations. These calculations, naturally, involve many scientific topics such as the atomic world or astronomy.

Pacing Suggestion for Chapter 1

This table does not include the chapter test as it is found in a different book (or file). Please add one day to the pacing if you use the test.

The Lessons in Chapter 1	page	span	suggested pacing	your pacing
Powers and the Order of Operations	13	*3 pages*	1 day	
Powers with Negative Bases	16	*3 pages*	1 day	
Laws of Exponents, Part 1	19	*3 pages*	1 day	
Zero and Negative Exponents	22	*3 pages*	1 day	
More on Negative Exponents	25	*2 pages*	1 day	
Laws of Exponents, Part 2	27	*3 pages*	1 day	
Laws of Exponents, Part 3	30	*2 pages*	1 day	
Scientific Notation: Large Numbers	32	*3 pages*	1 day	
Scientific Notation: Small Numbers	35	*2 pages*	1 day	
Significant Digits	37	*3 pages*	1 day	
Using Scientific Notation in Calculations, Part 1	40	*3 pages*	1 day	
Using Scientific Notation in Calculations, Part 2	43	*3 pages*	1 day	
Chapter 1 Revision	46	*2 pages*	1 day	
Chapter 1 Test (optional)				
TOTALS		*35 pages*	13 days	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr8ch1

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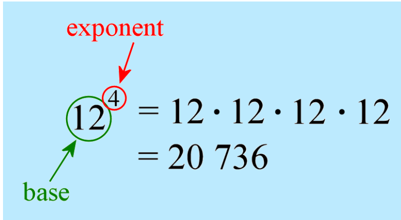
Powers and the Order of Operations

You will recall that we use **exponents** as a shorthand for writing repeated multiplications by the same number. For example, $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$ is written 7^5.



The tiny raised number is called the **exponent**. It tells us how many times the **base** number is multiplied by itself.

The entire expression, 7^5, is a **power.** We read it as "seven to the fifth power," "seven to the fifth," or "seven raised to the fifth power." Similarly, 0.5^8 is read as "five tenths to the eighth power" or "zero point five to the eighth."

The "powers of 8" are the various expressions where 8 is raised to some power: for example, 8^3, 8^4, 8^{45}, and 8^{99} are powers of 8.

The expression 9^1 equals simply 9. In general, $a^1 = a$.

Powers of 2 are usually read as something "**squared.**" For example, 11^2 is read as "eleven squared." That is because it gives us the area of a square with sides 11 units long.

Similarly, if the exponent is 3, the expression is usually read using the word "**cubed.**" For example, 1.5^3 is read as "one point five cubed" because it is the volume of a cube with edges 1.5 units long.

A calculator is not needed for the exercises of this lesson.

1. Evaluate.

a. four cubed

b. 2^4

c. 5^3

d. 0.2^3

e. 1^{60}

f. 100 squared

2. **a.** Which is more, 4^2 or 2^4?

b. Which is more, 2^5 or 5^2?

3. Complete the patterns.

a.	**b.**	**c.**
$10^1 =$	$2^1 =$	$0.1^1 =$
$10^2 =$	$2^2 =$	$0.1^2 =$
$10^3 =$	$2^3 =$	$0.1^3 =$
$10^4 =$	$2^4 =$	$0.1^4 =$
$10^5 =$	$2^5 =$	$0.1^5 =$
$10^6 =$	$2^6 =$	$0.1^6 =$
$10^7 =$	$2^7 =$	$0.1^7 =$

The order of operations dictates that powers (expressions with exponents) are solved before multiplication, division, addition, and subtraction. **Example 1.** Find the value of $5 \cdot 0.1^3 + 0.2^2$. First the powers: $0.1^3 = 0.1 \cdot 0.1 \cdot 0.1 = 0.001$, and $0.2^2 = 0.2 \cdot 0.2 = 0.04$. The expression becomes $5 \cdot 0.001 + 0.04 = 0.005 + 0.04 = \underline{0.045}$.	**The Order of Operations (BEMDAS)** (“*Best Excuse My Dear Aunt Sally*”) 1) Solve what is within brackets **(B)**. 2) Solve exponents **(E)**. 3) Solve multiplication **(M)** and division **(D)** from left to right. 4) Solve addition **(A)** and subtraction **(S)** from left to right.

4. Find the value of the expressions.

a. $4 \cdot 10^3 - 5 \cdot 10^2$	**b.** $4(5^2 - 2^3)$	**c.** $\dfrac{3}{1^8} + \dfrac{5}{3^2}$
d. $7 \cdot 10^3 - 5(800 - 10^2)$	**e.** $500 - \dfrac{3 \cdot 8}{2^3} + 2 \cdot 8^2$	**f.** $\dfrac{2 \cdot 17 + 2^4}{7 \cdot 7 - 3^2} + 20$

5. Find the value of the expressions.

a. $0.5^2 - 0.2^2 - 0.1^2$	**b.** $3(0.1^2 - 0.2^3)$	**c.** $0.6^2 + 2(1 - 0.3^2)$

6. The table on the right shows a list of powers of 4.

$4^1 = 4$
$4^2 = 16$
$4^3 = 64$
$4^4 = 256$
$4^5 = 1024$
$4^6 = 4096$

a. Find the value of 4^7 using the value for 4^6. (Do not use a calculator.)

b. Which power of 4 is equal to 65 536? Use estimation and the table, not a calculator.

c. Use the table to check whether $4^2 + 4^3 = 4^5$.

d. Use the table to check whether $4^2 \cdot 4^3 = 4^5$.

7. **a.** Find a power of 3 that is greater than seven squared.

b. Find a power of 5 that is greater than ten cubed.

c. Find a power of 1 that is greater than three squared.

8. **a.** If $3^6 = 729$, find the value of 3^8.

b. If $2^8 = 256$, find the value of 2^{11}.

9. Find the missing exponents.

a. $10^4 = 100^{\square}$	**b.** $2^6 = 4^{\square}$	**c.** $9^2 = 3^{\square}$	**d.** $0 = 0^{\square}$
e. $0.1^{\square} = 0.0001$	**f.** $0.2^{\square} = 0.00032$	**g.** $625 = 5^{\square}$	**h.** $128 = 2^{\square}$

10. Find the value of these powers.

a. $\left(\frac{1}{6}\right)^2 =$	**b.** $\left(\frac{3}{10}\right)^3 =$	**c.** $\left(\frac{2}{3}\right)^4 =$	**d.** $\left(\frac{3}{4}\right)^3 =$

Example 2. Simplify $3 \cdot s \cdot s \cdot s \cdot 3 \cdot t \cdot s \cdot t \cdot t$.

We can multiply in any order, so let's reorganise the expression as $3 \cdot 3 \cdot s \cdot s \cdot s \cdot s \cdot t \cdot t \cdot t$.

The variable s is multiplied by itself four times, and t three times. Naturally, $3 \cdot 3$ is 9.

So, we get $3 \cdot 3 \cdot s \cdot s \cdot s \cdot s \cdot t \cdot t \cdot t = \mathbf{9s^4t^3}$.

11. Simplify.

a. $2 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot 7$	**b.** $4 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot 9 \cdot x \cdot y \cdot x$
c. $5 \cdot a \cdot b \cdot b \cdot a \cdot a \cdot 2 \cdot b \cdot 6$	**d.** $0.3 \cdot p \cdot r \cdot p \cdot r \cdot r \cdot 0.2 \cdot r \cdot 10$

12. **a.** Find the value of the expression $10a^4b^2$ when $a = 2$ and $b = 3$.

b. Find the value of the expression $14x^3y^5$ when $x = 2$ and $y = 0$.

13. When you fold a sheet of paper in half, its area is now only 1/2 of the area of the original paper. Let's say you repeat this process, and fold that paper again in half, and again, and again. How many times do you need to fold a sheet of paper in order for the area of the folded piece to be 1/64 of the area of the original?

Puzzle Corner What is the simple value of $\frac{9^6}{9^5}$? There is no need for actual calculations!

Powers with Negative Bases

Revision. To multiply and divide integers, simply multiply and divide the absolute values of the numbers, and then determine the sign of the final product. For two integers, the sign is given by this chart (applies also to division): (negative) · (positive) = (negative) (positive) · (negative) = (negative) (positive) · (positive) = (positive) (negative) · (negative) = (positive) We will look at the situation of three or more integers in the exercises.	**Example 1.** $-8 \cdot 3 = -24$ $7 \cdot (-2) = -14$ $-6 \cdot (-6) = 36$ $5 \div (-5) = -1$ $\frac{-40}{-2} = 20$

1. Multiply and divide.

a. $-2 \cdot (-7) =$	**b.** $-72 \div 8 =$	**c.** $7 \cdot (-8) =$	**d.** $54 \div (-9) =$
e. $\frac{12}{-3} =$	**f.** $9 \cdot (-9) =$	**g.** $(-7) \cdot (-4) =$	**h.** $\frac{-36}{-4} =$

2. Use repeated addition to explain why a product of a positive and a negative integer must be negative.

To multiply three or more integers, you can multiply any two of them first. Then multiply their product by the third integer, and so on.	**Example 2.** Solve $-5 \cdot 3 \cdot (-6) \cdot 2$. We can first multiply -5 and -6 to get 30. The problem now becomes $30 \cdot 3 \cdot 2 = 180$

3. Multiply. Note that you can multiply in any order.

a. $(-2) \cdot (-7) \cdot (-1) =$	**b.** $5 \cdot (-2) \cdot (-2) =$	**c.** $(-1) \cdot 6 \cdot 5 =$
d. $(-1) \cdot (-2) \cdot (-3) \cdot (-2) =$	**e.** $(-2) \cdot 2 \cdot (-5) \cdot (-1) =$	**f.** $(-2) \cdot (-1) \cdot (-3) \cdot 5 \cdot (-2) =$
g. $6 \cdot \frac{48}{-6} \cdot (-5) =$	**h.** $\frac{-12}{16} \cdot \frac{-15}{25} \cdot (-3) =$	**i.** $-3 \cdot \frac{-8}{-16} \cdot \frac{6}{5} \cdot (-5) =$

4. Find the missing factors.

a. $(-8) \cdot$ ______ $\cdot (-2) = -40$	**b.** $2 \cdot$ ______ $\cdot (-4) = 64$	**c.** ______ $\cdot (-3) \cdot 4 = -36$

Chapter 2: Geometry
Introduction

The second chapter of Math Mammoth Grade 8 covers geometric transformations, angle relationships, and the volume of prisms, cylinders, pyramids, cones, and spheres.

The chapter starts out with the basics of congruent transformations: translations, reflections, rotations. Students use transparent paper to perform several of these transformations hands-on, so as to gain an understanding of the attributes that are preserved in these transformations.

Next we practise these same transformations in the coordinate grid. Students learn how the coordinates of the points change when a figure is translated or reflected in the *x* or *y*-axis. They also explore rotating figures in the coordinate grid; here we limit the rotations to 90°, 180°, or 270° degrees.

Then it is time to study sequences of transformations, which enable us to describe more complex transformations. The key idea here is to understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of transformations.

All of this work has related to congruent transformations, which means the size of the figure has not changed. Now we turn our attention to dilations. In a dilation, the figure is transformed so that its size changes but its shape does not. Such figures are called similar figures. Yet another term describing the same process is scaling a figure.

Next, we study angle relationships. The first lesson in this section reviews certain angle relationships from 7th grade (complementary, supplementary, and vertical angles). Then students learn about angles formed when a transversal crosses two parallel lines: corresponding angles, alternate interior angles, and alternate exterior angles. They also investigate angle relationships related to triangles and learn how these relationships allow us to deduce angle measurements of other angles.

In all of this work, students are guided to reason using mathematical facts they have learned, and to justify their reasoning, thus becoming familiar with the process of mathematical proof.

The last major topic of the chapter is volume of various three-dimensional figures. Students solve a variety of real-world and mathematical problems involving multiple three-dimensional shapes.

Pacing Suggestion for Chapter 2

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing if you use the test.

The Lessons in Chapter 2	page	span	suggested pacing	your pacing
Geometric Transformations and Congruence, Part 1	51	*4 pages*	1 day	
Geometric Transformations and Congruence, Part 2	55	*3 pages*	1 day	
Translations in the Coordinate Grid	58	*3 pages*	1 day	
Reflections in the Coordinate Grid	61	*3 pages*	1 day	
Translations and Reflections	64	*3 pages*	1 day	
Rotations in the Coordinate Grid	67	*4 pages*	1 day	
Sequences of Transformations	71	*3 pages*	1 day	
Sequences of Transformations, Part 2	74	*2 pages*	1 day	
Dilations	76	*3 pages*	1 day	
Dilations in the Coordinate Grid	79	*3 pages*	1 day	
Similar Figures, Part 1	82	*3 pages*	1 day	
Similar Figures, Part 2	85	*2 pages*	1 day	

The Lessons in Chapter 2	page	span	suggested pacing	your pacing
Similar Figures: More Practice	87	*3 pages*	1 day	
Revision: Angle Relationships	90	*3 pages*	1 day	
Corresponding Angles	93	*2 pages*	1 day	
More Angle Relationships with Parallel Lines	95	*2 pages*	1 day	
The Angle Sum of a Triangle	97	*3 pages*	1 day	
Exterior Angles of a Triangle	100	*3 pages*	1 day	
Angles in Similar Triangles, Part 1	103	*2 pages*	1 day	
Angles in Similar Triangles, Part 2	105	*2 pages*	1 day	
Volume of Prisms and Cylinders	107	*2 pages*	1 day	
Volume of Pyramids and Cones	109	*3 pages*	1 day	
Volume of Spheres	112	*2 pages*	1 day	
Volume Problems	114	*2 pages*	1 day	
Chapter 2 Mixed Revision	116	*2 pages*	1 day	
Chapter 2 Revision	118	*5 pages*	2 days	
Chapter 2 Test (optional)				
TOTALS		*72 pages*	27 days	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr8ch2

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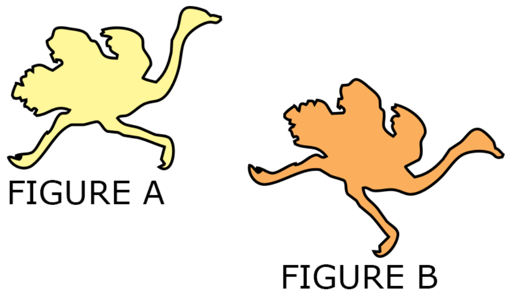
Geometric Transformations and Congruence, Part 1

Two figures are congruent when they are, you might say, identical in the sense that they have the same shape and size (but may be of different colour). We can define congruency as follows:

Two figures are **congruent** if they perfectly match, when one is placed on top of the other.

The figures don't have to be in the same position or orientation. For example, these two figures are congruent — if you rotate and move figure A, you can place it exactly on top of figure B.

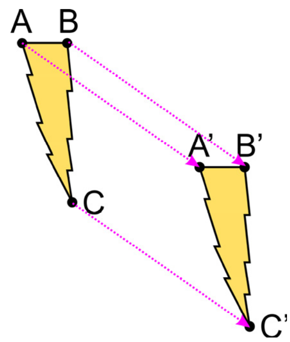


We will now study three geometric **transformations**, or basic ways to move a point, or by extension, a figure, since a figure can be considered to consist of many points.

1. A **translation** of a figure means sliding or moving it a certain distance in a certain direction, without turning or rotating it. The arrows show how three individual points of the figure were moved.

   We say the translation maps point A onto point A' (read "A prime"), point B onto point B', and point C onto point C'.

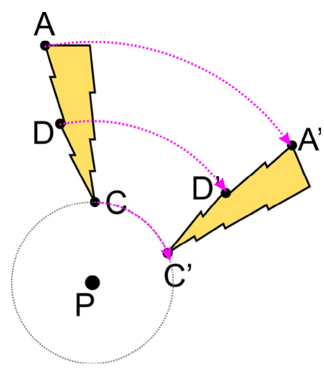
   We also say that point A' is the image of point A under the translation.



2. A **rotation** means turning a figure around a certain point.

   Here, the lightning figure is rotated around point P. Each point of the figure moves in a **circular arc around point P**.

   A rotation is measured in degrees, just like angles are. In this example, the lightning figure was rotated 67 degrees clockwise around point P.

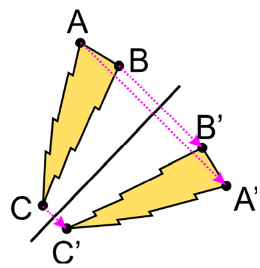


3. A **reflection** across a line means mirroring the figure in that line. You could also say the figure was "flipped".

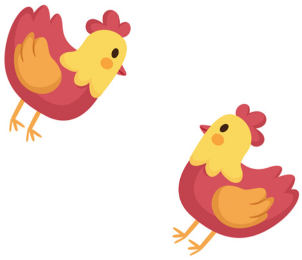
   In a reflection, the distance from each point to the reflection line and the distance of its image to the line are equal (measured along a line segment that is perpendicular to the line).

   For example, the distance from point C to the line equals the distance from point C' to the line.

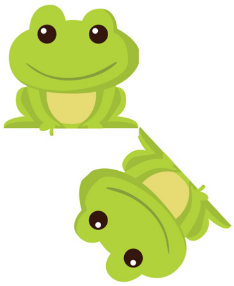
   A reflected figure is congruent to the original.



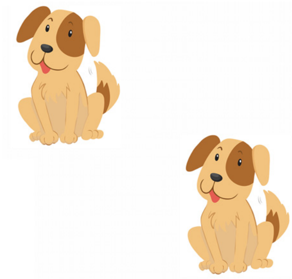
1. Name the transformation that was used to transform the figure on the left to the figure on the right.



a.



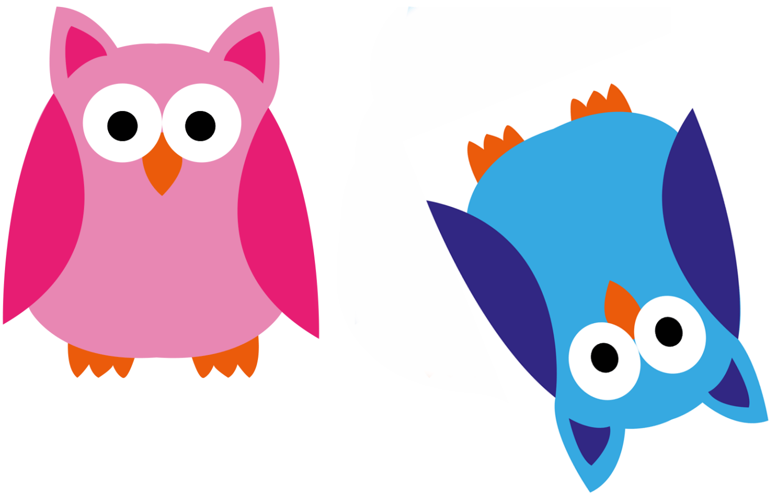
b.



c.

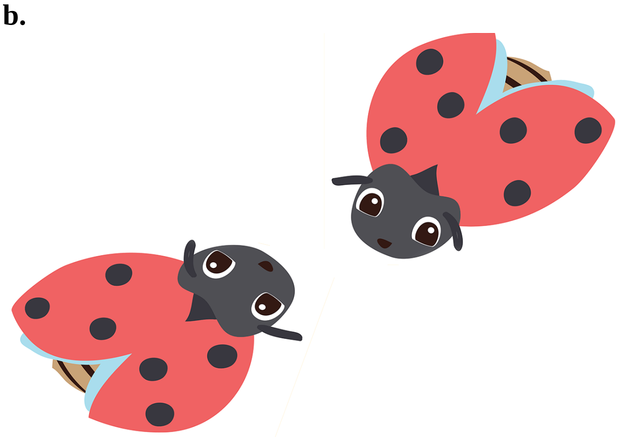
In continuation, we will explore geometric transformations and how they relate to congruence with the help of tracing paper (patty paper) or a transparency.

2. Use tracing paper to determine whether the two figures are congruent. You may move, turn, and/or flip the tracing paper. First, copy the outline of **one** figure to the tracing paper. (Note: when checking for congruency, we ignore the colours.)

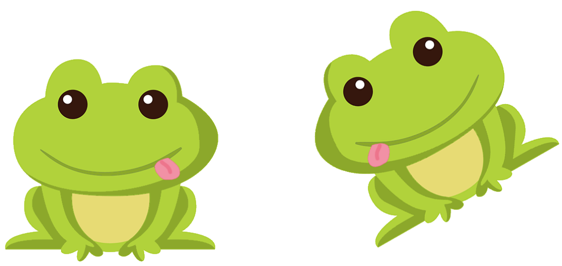


a.

b.

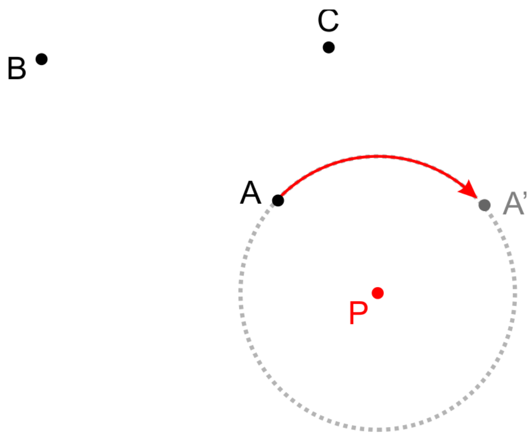


c.



3. The image below shows how point A was mapped to point A' in a rotation. We will now do the same rotation to points B and C using tracing paper. This is how:

i. Put a thumbtack or a pin through the tracing paper at P so that you can turn the paper around P.
ii. Copy points A, B, and C to the paper.
iii. Then rotate the paper around point P so that **point A is mapped to point A'**.
iv. Now, draw the points B' and C'. You can use a pin to mark where these points are (through the tracing paper). Drawing the points with a pencil on the tracing paper may also make a faint mark in the underlying paper. Then remove the tracing paper and draw the points.



a. Connect A, B, and C with line segments, and also A', B', and C', so that you get two triangles.

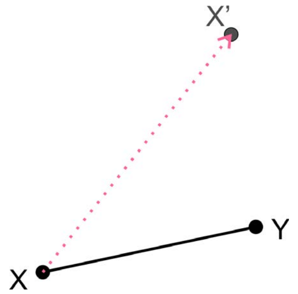
b. Measure the side lengths of both triangles. What do you notice?

c. Measure the angles BAC and B'A'C' and also the angles ACB and A'C'B'. What do you notice?

4. Point X' is the image of point X under a translation along the dashed arrow.

a. Sketch the image of point Y in the same translation. Mark it as point Y'.

You may optionally do this translation with tracing paper. However, it is difficult to do this accurately.



b. What can we know about the length of the segment $\overline{X'Y'}$? Choose one answer:

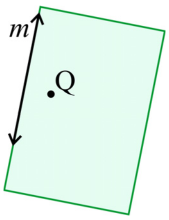
(i) $\overline{XY}$ and $\overline{X'Y'}$ are congruent (have the same length).

(ii) $\overline{XY}$ and $\overline{X'Y}$ are not congruent.

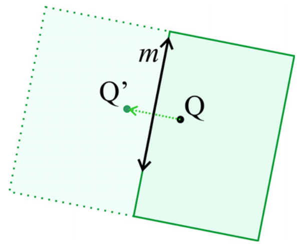
(iii) We cannot know for sure whether $\overline{XY}$ and $\overline{X'Y}$ are congruent or not.

How to reflect a point across a line using tracing paper or a transparency

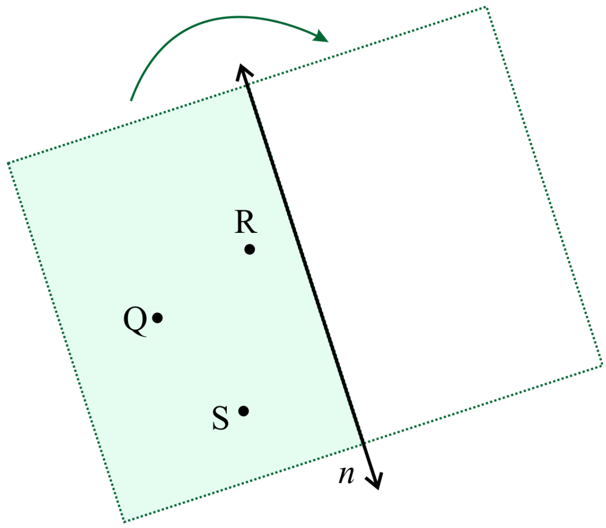
Step 1. Align the paper so that one of its edges is along the reflection line *m*.



Step 2. Flip the paper. You can use a pin to mark the image of the point in question.



5. **a.** Cut out a piece of transparent paper that fits inside the light-coloured rectangle in the image on the right (approximately 3.2 cm by 4.8 cm). Use tracing paper to reflect the points Q, R, and S across line *n*. Label the reflected points as Q', R', and S'.

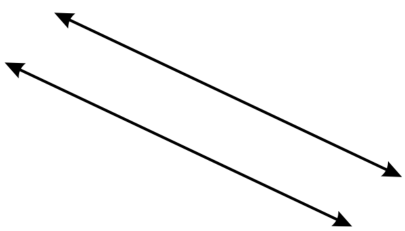


b. Connect the points Q and R, R and S, Q' and R', and R' and S' with line segments.

c. Measure the length of the line segments $\overline{QR}$ and $\overline{Q'R'}$, and also $\overline{RS}$ and $\overline{R'S'}$. What do you notice?

d. Measure also the angles ∠QRS and ∠Q'R'S'. What do you notice?

6. Predict what will happen to parallel lines under translation, rotation, and reflection. You may want to use tracing paper (as needed) to confirm your prediction.



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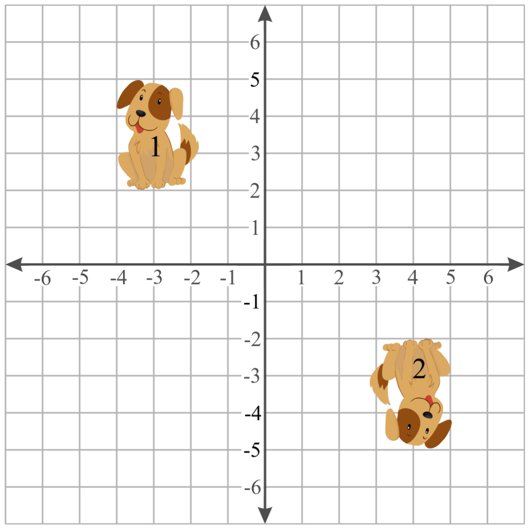
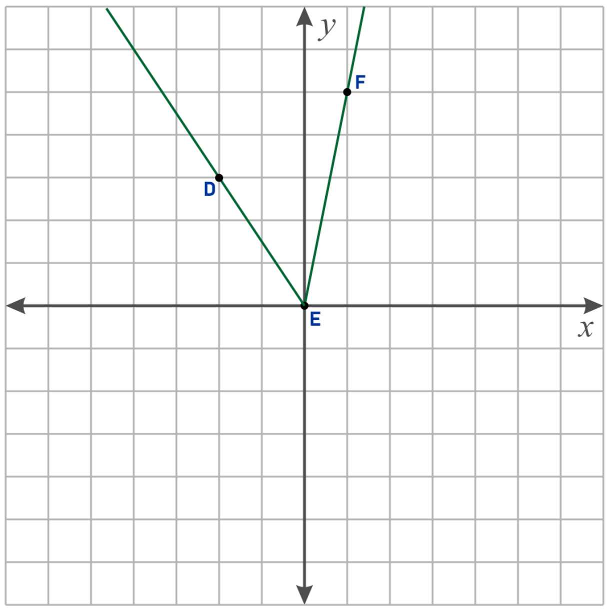
Chapter 2 Revision

1. Describe a sequence of transformations that can map figure 1 to figure 2.

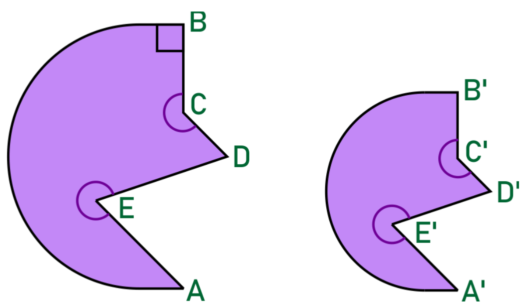


2. Rotate angle DEF both 90 degrees and also 180 degrees clockwise around the origin.



3. A quadrilateral was first reflected in the *y*-axis, and then rotated around the origin clockwise 90 degrees. Its vertices are now at points (3, −5), (5, −2), (4, −1), and (1, −4). What were the coordinates of its vertices before these transformations?

4. Figure A'B'C'D'E' is a dilation of figure ABCDE with scale factor 3/4. Angle B is a right angle. Check all the statements that are true.



a. Angle B' is a right angle.

b. The measure of ∠CDE is 3/4 of the measure of ∠C'D'E'.

c. ∠E is equal to ∠E'.

d. If CD = 1 cm, then C'D' = 3/4 cm.

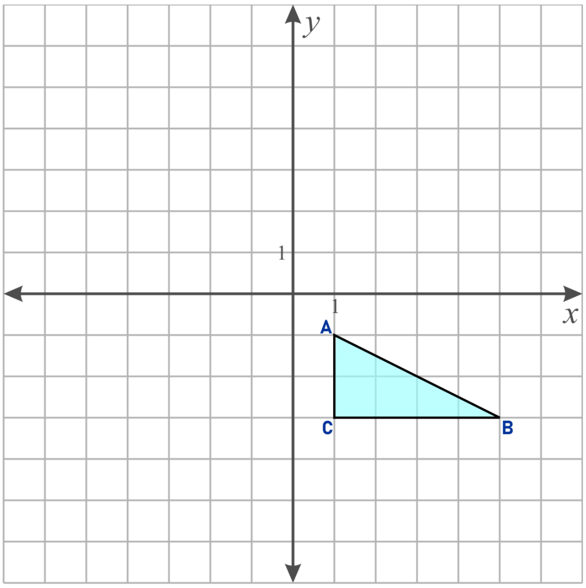
e. ∠D is equal to ∠E'.

f. If the perimeter of figure ABCDE is 20 cm, then the perimeter of A'B'C'D'E is 12 cm.

5. **a.** Perform the following sequence of transformations to triangle ABC:

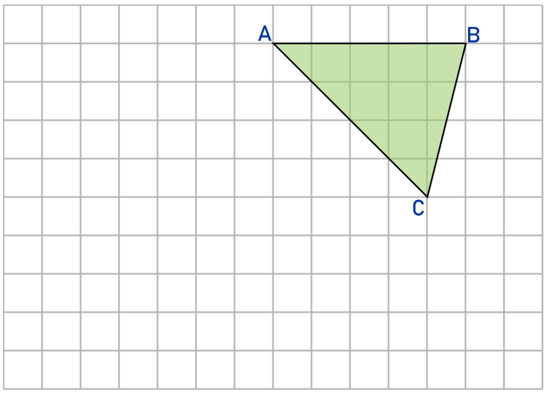
First rotate it counterclockwise around point C 90 degrees.
Then reflect it in the vertical line at $x = -1$.
Lastly, translate it two units to the right and three down.

b. Find another, different sequence of transformations that does the same as the sequence in (a), and starts with a reflection.

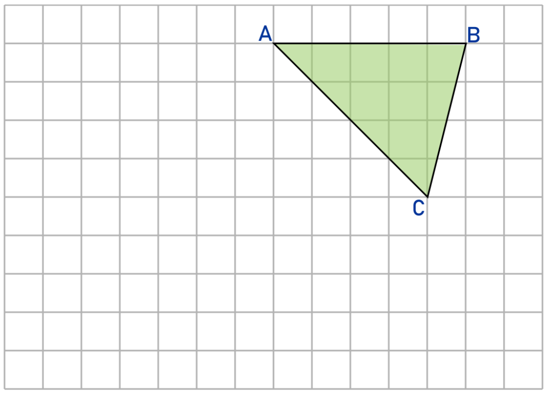


6. Draw a dilation of triangle ABC...

a. from point C and scale factor 1/2.



b. from point B and scale factor 2.



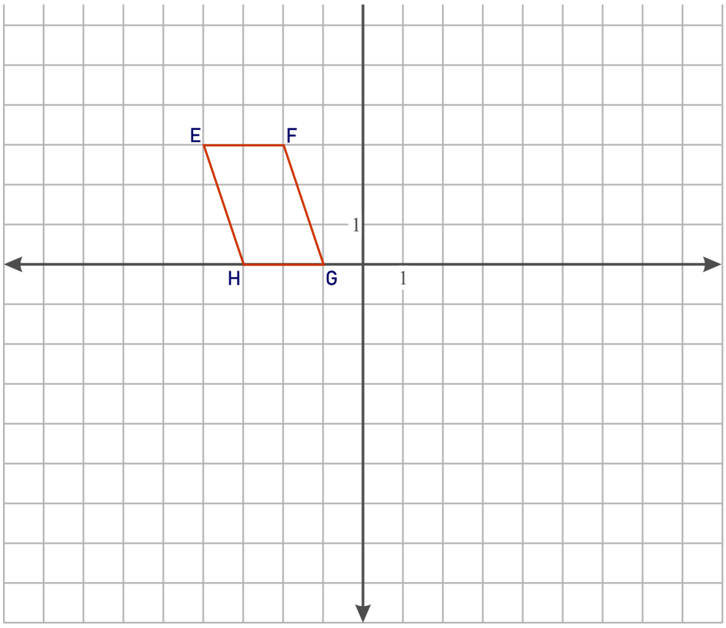
7. Parallelogram EFGH underwent the following transformations :

1. Reflection in the vertical line at $x = -0.5$.

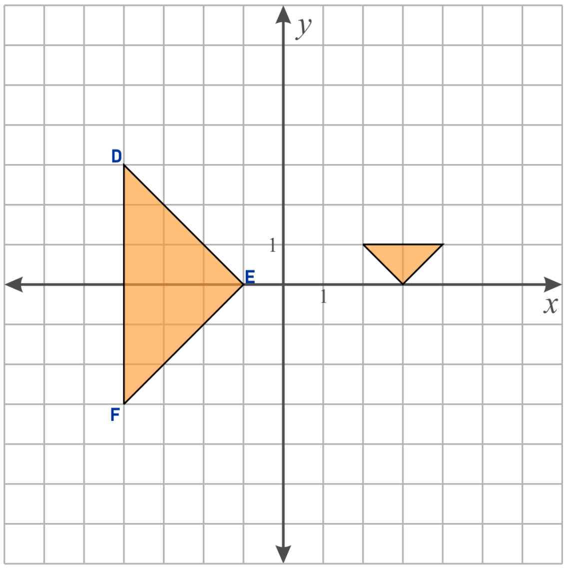
2. Translation 3 units to the left and 4 units down.

3. Dilation from point E" with scale factor 2.

What are the coordinates of the image of point F after all these transformations?

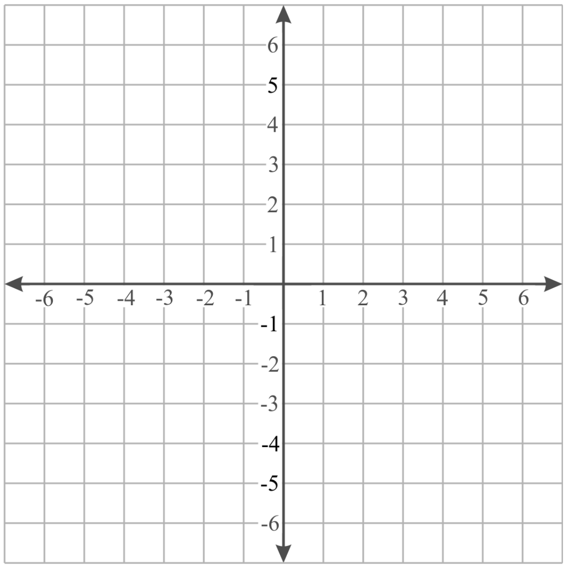


8. Show that the two triangles are similar by describing a sequence of transformations that could map △DEF to the smaller triangle



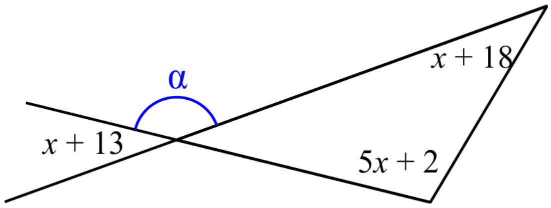
9. Figure PQRS underwent a dilation, then a rotation. Study the coordinates to find out the details about each transformation, then fill in the missing coordinates.

Original figure	**Dilation**	**Rotation**
P(−5, 3)	P'(−6, 5)	P"(___ , ___)
Q(0, 3)	Q'(4, 5)	Q"(___ , ___)
R(−1, 1)	R'(___ , ___)	R"(−4 , −5)
S(−4, 1)	S'(−4, 1)	S"(−4, 1)

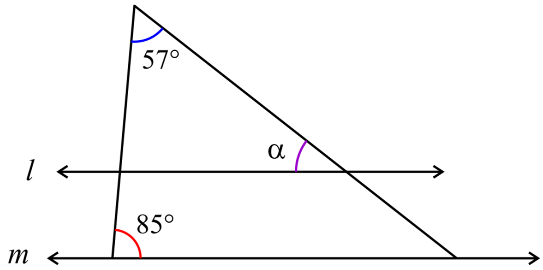


10. **a.** Find the value of *x*.

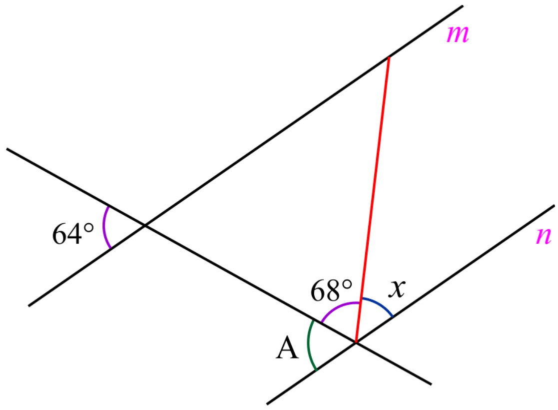
b. Find the value of α.



11. Lines *l* and *m* are parallel. Figure out the measure of the angle α. (You may need to mark more angles in the diagram.)



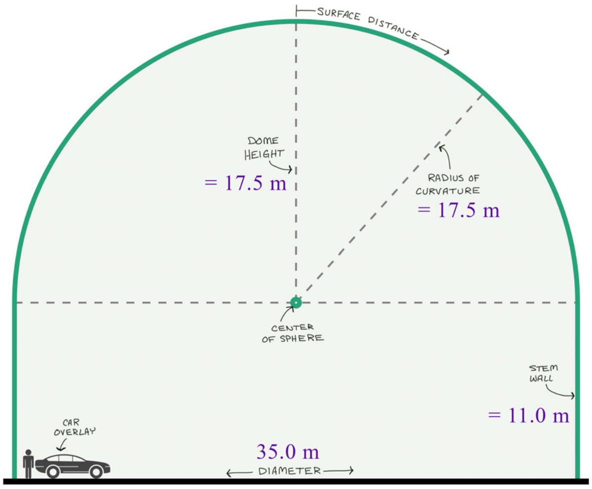
12. Lines *m* and *n* are parallel. Find the measure of angle *x*, and prove why it is what you find it to be. In other words, explain and justify your reasoning. You may need to mark more angles in the diagram.

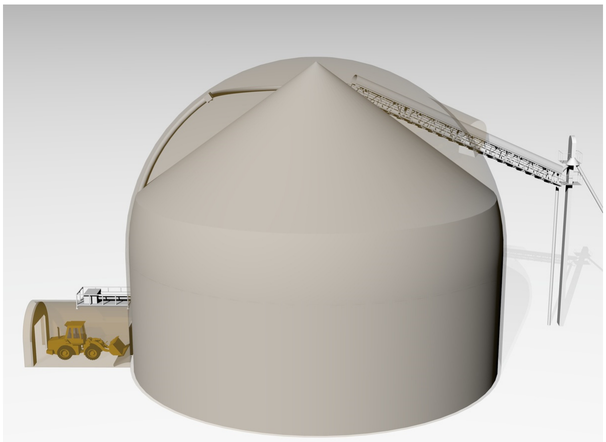


13. A shampoo bottle is in the shape of a circular cylinder. It says it contains 473 ml of shampoo. Its inner diameter is 6.0 cm and its height is 17 cm. What percent of the bottle does the shampoo take up?

14. Compare a sphere with radius 5 cm with a cone with the same radius. What is the height of the cone, given the two have the same volume?

15. Elkhart Ammonium Nitrate Storage in Elkhart, Texas, is a large building consisting of a half sphere on top of a circular cylinder. The stem wall is 11.0 m high, and the diameter of the cylinder (which is also the diameter of the sphere) is 35.1 m. When the storage is filled with ammonium nitrate, the top part of it (the part inside the half-sphere) forms a cone.





Images courtesy of Monolithic Dome Institute, www.monolithic.com

Find the volume of the ammonium nitrate mound when the cone reaches the top of the structure, to the nearest hundred cubic metres.

Chapter 3: Linear Equations
Introduction

The third chapter of Math Mammoth Grade 8 focuses both on the mechanics of solving linear equations and on problem solving.

The chapter starts with a vocabulary reference sheet. The first actual lesson is a revision of integer addition and subtraction, which you can omit at your discretion. The next several lessons after that revise simple equations of the form $px + q = r$ and $p(x + q) = r$ and the distributive property from 7th grade.

The next step towards solving more complex equations is the lesson *Combining Like Terms*. Students add and subtract like terms, including with decimal or fractional coefficients, and solve equations where like terms need combined first.

Having learned this, students then tackle some typical algebraic word problems in the following lesson.

Then it is time to learn to solve equations where the variable is on both sides. There are often several possible solution pathways. Students also learn about the common error of adding or subtracting “across the sides.”

The lesson *Simplifying Linear Expressions* focuses on how to remove brackets after a minus sign, such as in the expression $2(3 + 2y) - 7(3 - 5y)$. After that, it is time for more practice and word problems, including age and coin word problems.

Then we turn our attention to equations with fractions, and the student learns to multiply both sides of the equation by a common multiple of the denominators. In the lessons on formulas, the student both solves various formulas for a variable in it, and uses formulas to solve a variety of word problems.

The lesson *More on Equations* deals with equations that have an infinite number of solutions (identities) or no solutions.

The chapter ends with two more lessons on word problems (percent word problems and miscellaneous problems).

Pacing Suggestion for Chapter 3

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing if you use the test.

The Lessons in Chapter 3	**page**	**span**	**suggested pacing**	**your pacing**
Algebra Terms (For Reference)	125	*(1 page)*		
Revision: Integer Addition and Subtraction	126	*3 pages*	1 day	
Equations Revision, Part 1	129	*4 pages*	1 day	
The Distributive Property	133	*3 pages*	1 day	
Equations Revision, Part 2	136	*4 pages*	1 day	
Equations Revision, Part 3	140	*4 pages*	1 day	
Combining Like Terms	144	*3 pages*	1 day	
Word Problems	147	*4 pages*	1 day	
A Variable on Both Sides	151	*4 pages*	1 day	
Word Problems and More Practice	155	*3 pages*	1 day	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr8ch3

Scan me

Algebra Terms
For Reference

Expressions in mathematics consist of:

- numbers;
- mathematical operations (+ , − , · , ÷ , exponents);
- and letter variables, such as *x*, *y*, *a*, T, and so on.

Note: Expressions do *not* have an equals sign!

Examples of expressions:

$\frac{3}{5}x^2 - 3x + 5$ $\quad$ 5 $\quad$ $\left(\frac{3x}{y^2}\right)^2$

$T - 29$ $\quad$ $2^x - 5^y$

An **equation** has two expressions separated by an equals sign:

(expression 1) = **(expression 2)**

Examples of equations:

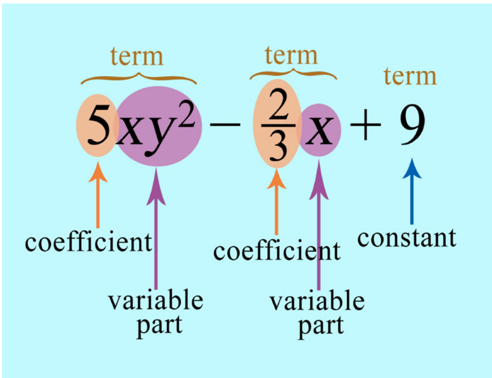
$0 = 0$ $\quad$ $2(z - 9) = -z^2$

$9 = -8$ (a false equation) $\quad$ $\frac{x + 3}{2} = -1.5$

A **term** is an expression that consists of numbers and/or variables that are *multiplied*. For example, $7x$ is a term and so is $0.6mn^2$.

A single number or a single variable is also a term. If the term is a single number, such as 4.5 or ¾, we call it a **constant**.

In the expression on the right, we have three terms: $5xy^2$, $\frac{2}{3}x$, and 9, that are separated by subtraction and addition.



If a term is not a single number, then it has a **variable part** and a **coefficient**.

- The coefficient is the single number by which the variable or variables are multiplied.
- The variable part consists of the variables and their exponents.

For example, in $4.3ab$, 4.3 is the coefficient, and ab is the variable part.

Note: a term that consists of variables only still has a coefficient: it is one. For example, the coefficient of the term x^3 is one, because you can write x^3 as $1 \cdot x^3$.

Example. Is $s - 5$ a term? No, it is not since it contains subtraction. Instead, $s - 5$ is an expression consisting of two terms, s and 5, separated by subtraction.

1. Write the expression based on the clues.

- It has four terms.
- The constant term is the square of the third smallest prime.
- The variable parts of the variable terms are ab, a^2, and a, respectively.
- The coefficients of the variable terms are the three consecutive integers with a sum of 21.
- The first two terms are separated by subtraction, the rest by addition.

Revision: Integer Addition and Subtraction

Integers consist of the counting numbers (1, 2, 3, 4, ...), zero, and the negative counterparts of the counting numbers (−1, −2, −3, −4, ...). So, the set of integers is {..., −4, −3, −2, −1, 0, 1, 2, 3, 4, ...}.

An **absolute value** of an integer is its distance from zero, and is marked with two vertical lines.
For example, $|2| = 2$ and $|-18| = 18$.

We obtain the **opposite** or **negation** of an integer by changing its sign from positive to negative, or vice versa. For example, the opposite or negation of 17 is −17. The opposite of −4 is 4.
We can use the negative sign "−" to signify this: −(−5) means the opposite of −5, which is 5.

To **add several negative integers**, simply add their absolute values and write the answer as negative.

Example 1. To find the sum −8 + (−3) + (−7) + (−11), add 8 + 3 + 7 + 11 = 29.
The value of the original sum is −29.

To **add a negative and a positive integer**, find the difference in their absolute values. The integer with the bigger absolute value determines the sign of the final answer.

Example 2. In the sum −9 + 11, the absolute values of the two integers are 9 and 11. Their difference is 11 − 9 = 2. This means the answer is either 2 or −2. To determine which, check the sign of the integer with the larger absolute value. In our case it is 11 (which is positive), so the answer is 2 (and not −2).

Example 3. In the sum 7 + (−12), the absolute values of the two integers are 7 and 12. Their difference is 12 − 7 = 5. This means the answer is either 5 or −5. To determine which, check the sign of the integer with the larger absolute value. Here it is −12 which is negative, so the answer is −5 (and not 5).

So, this is the mechanical rule, but you don't have to use it if you have learned other methods, such as visualising a number line.

To **add several integers** where some are negative, some positive, first calculate the partial sums of all the negative integers and of all the positive ones. Lastly add those sums.

Example 4. −8 + 12 + (−9) + (−1) + 5 + (−6) = ?

Positives: 12 + 5 = 17

Negatives: −8 + (−9) + (−1) + (−6) = −24

Total: 17 + (−24) = **−7**

1. Add.

a. (−4) + 8 =	**b.** 15 + (−25) =	**c.** −12 + 6 =	**d.** −11 + (−32) =

e. −12 + (−2) + (−5) =	**f.** 6 + (−1) + (−5) + 2 =	**g.** −7 + 10 + (−6) + 1 =

h. −11 + (−2) + 7 + (−5) + 4 + (−3) =	**i.** −6 + (−5) + 8 + (−12) + 24 + 1 =

To **subtract two integers**, you can often think with the help of the number line model. For example, you can visualise 2 − 6 as starting at 2, and moving 6 steps to the left on the number line.

Mathematicians actually define the subtraction of two numbers, $a - b$, as the sum of a and the opposite of b.

In symbols: $a - b = a + (-b)$

In other words, to subtract an integer, change the subtraction to an addition of the opposite number.
From this definition it also follows that $a - (-b)$ simplifies to $a + b$.

(Why? In $a - (-b)$ we subtract $-b$, and the opposite of $-b$ is b. Instead of subtracting $(-b)$, you add *its opposite*, or b.)

Example 5.

5 − 7	−6 − 8	2 − (−4)	−3 − (−9)
↓	↓	↓	↓
5 + (−7) = −2	−6 + (−8) = −14	2 + 4 = 6	−3 + 9 = 6

2. Write each subtraction as an addition, and solve.

a. 8 − (−6) ↓ 8 + 6 = ______	**b.** −9 − (−14) ↓ ____ + ____ = ______	**c.** −21 − 8 ↓ ____ + ____ = ______	**d.** 3 − 15 ↓ ____ + ____ = ______

3. Subtract.

a. 2 − 9 =	**b.** −2 − 9 =	**c.** −2 − (−9) =	**d.** 2 − (−9) =
e. −7 − 4 =	**f.** −7 − (−4) =	**g.** 7 − (−4) =	**h.** 4 − 7 =

4. Can any addition be changed to a subtraction? See if you can find matching subtractions for these additions.

a. 7 + (−10) ____ − ____ = _____	**b.** −2 + (−1) ____ − ____ = ______	**c.** 14 + 3 ____ − ____ = ______	**d.** −8 + 5 ____ − ____ = ______

So, any subtraction can be written as an addition. The converse is also true: any addition can be written as a subtraction. For example, the sum 5 + 4 can be written as 5 − (−4), and the sum 2 + (−13) can be written as the subtraction 2 − 13.

In symbols, $c + d = c - (-d)$. Instead of adding d, you subtract the opposite (or negation) of d.

However, since this usually does not simplify the calculation, it does not get used often.

5. Solve, working in order from left to right.

a. −2 − 5 + 6 =	**b.** 7 + (−12) − 5 − 8 =	**c.** −1 + 9 − 14 + 7 =
d. −8 − 12 − 5 + 9 =	**e.** 2 − (−12) − 10 − (−3) =	**f.** −21 − 13 + 8 − (−5) =

Adding negative fractions **Example 6.** Add $\frac{2}{5} + \left(-\frac{3}{7}\right)$. We could use the regular process for integer addition: figure out which fraction has a larger absolute value, then subtract the smaller absolute value from the larger one, and so on. But the easier way is this: Simply add the fractions normally and treat the negative fraction $-\frac{3}{7}$ as $\frac{-3}{7}$. You will end up with an *integer addition* in the numerator. See the full solution on the right.	$\frac{2}{5} + \left(-\frac{3}{7}\right)$ $\frac{2}{5} + \frac{-3}{7}$ $\frac{14}{35} + \frac{-15}{35}$ $\frac{14 + (-15)}{35} = \frac{-1}{35} = -\frac{1}{35}$

6. Add and subtract.

a. $\frac{2}{7} + \left(-\frac{3}{4}\right)$	**b.** $\frac{1}{8} + \left(-\frac{1}{2}\right) + \left(-\frac{3}{4}\right)$
c. $-\frac{5}{6} + \frac{2}{9}$	**d.** $-\frac{2}{3} + \frac{2}{9} - \frac{1}{6}$
e. $-\frac{7}{8} + \left(-\frac{1}{10}\right)$	**f.** $\frac{1}{6} - \left(-\frac{7}{8}\right)$

Equations Revision, Part 1

An **equation** consists of two expressions, separated by an equals sign:

expression 1 = **expression 2**

For example, $40 = w + 32$ is an equation, and so is $2 = 5$, the latter being a *false* equation.

A **solution** or a **root** to an equation is a value of the unknown that makes the equation *true*; in other words, makes the two expressions on both sides to have the same value.

Example 1. Is 20 a root to the equation $11 = \frac{1}{2}x + 3$?

To check that, we substitute 20 in place of x and check whether the two sides of the equation have the same value:

$$11 \stackrel{?}{=} \frac{1}{2}(20) + 3$$

$$11 \neq 10 + 3$$

No, 20 does not fulfil this equation, so it is not a root.

1. **a.** Is 2 a root to the equation $\frac{3x^2 - 7}{5} = x$? Explain.

   b. Is −90 a root to the equation $\frac{2}{3}y + 11 = -49$? Explain.

2. Without solving the equation, check whether $x = -3$ is a solution to the equation $x + 4x + 6x - 8 = -5(x + 8)$. Before you start, think: would you be allowed to simplify the left side of the equation?

3. Write three different equations with the solution of $x = -5$.

4. If $2w + 6 = 50$ and $3w - 15 = 51$, then does $2w + 6$ equal $3w - 15$? Justify your reasoning.

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Chapter 4: Introduction to Functions
Introduction

The fourth chapter of Math Mammoth Grade 8 covers various introductory topics from the theory of functions. These topics prepare students for studying functions in great detail in high school maths, and even include preparatory ideas for calculus (rate of change).

The first lesson focuses on the basic definition of a function, as a relationship between two sets that assigns exactly one output for each input. It also briefly explains the range and domain of a function, even though those terms are not required in the CCS.

Next, we study the rate of change in the context of linear functions. Students calculate the rate of change from functions given as a table of values or from their graphs. They also encounter nonlinear functions and calculate the rate of change for those in specific intervals.

Then, students learn about the initial value of a function (its value when the input is zero), and learn that the equation $y = mx + b$ defines a linear function. They write and plot equations of that form to model linear relationships. We also spend one lesson looking at linear versus nonlinear relationships.

The following major topic is describing functions. Students analyse a graph and tell whether a function is increasing, decreasing, or constant; linear or nonlinear. They sketch a graph matching a given verbal description, and interpret given graphs of nonlinear functions in a variety of the real-life contexts.

Lastly, students compare properties of two functions represented in different ways (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, distance as a function of time is given as an equation for one aeroplane, and as a graph for another, and students answer questions concerning the speed and distance of the two aeroplanes.

Pacing Suggestion for Chapter 4

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing if you use the test.

The Lessons in Chapter 4	**page**	**span**	**suggested pacing**	**your pacing**
Functions	191	*4 pages*	1 day	
Linear Functions and the Rate of Change 1	195	*4 pages*	1 day	
Linear Functions and the Rate of Change 2	199	*3 pages*	1 day	
Linear Functions as Equations	202	*3 pages*	1 day	
Linear versus Nonlinear Functions	205	*3 pages*	1 day	
Modelling Linear Relationships	208	*4 pages*	1 day	
Describing Functions 1	212	*3 pages*	1 day	
Describing Functions 2	215	*3 pages*	1 day	
Describing Functions 3	218	*4 pages*	1 day	
Comparing Functions 1	222	*3 pages*	1 day	
Comparing Functions 2	225	*2 pages*	1 day	
Chapter 4 Mixed Revision	227	*3 pages*	1 day	
Chapter 4 Revision	230	*4 pages*	2 days	
Chapter 4 Test (optional)				
TOTALS		*43 pages*	14 days	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr8ch4

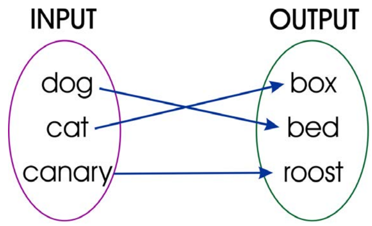
Scan me

Functions

A **function** is a rule or a relationship between two sets that assigns **exactly one output for each input.** We also use the word **mapping** for a function.

Example 1. The illustration below shows a simple function that maps each animal to its favourite sleeping place.

Each animal has a sleeping place, and only one, so this is a function.

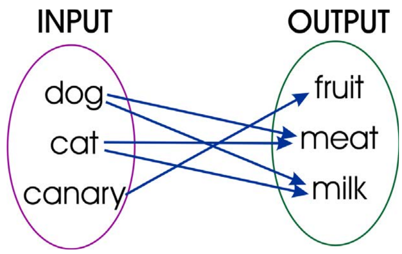


Example 2. The table lists the name of seven children, and the month when each child has their birthday. Notice that several of them have their birthday in December. Is this a function?

Input	Allie	Julie	Danny	Juan	Pete	Bob	Samantha
Output	September	December	December	June	August	December	February

Yes. The definition only requires that there has to be exactly one output for each input; **the outputs don't have to be unique.**

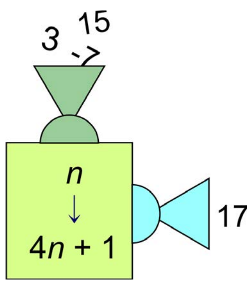
1. The relationship shown on the right is *not* a function. Why?



2. A function machine "ingests" a number (the input) and "spits out" another (the output) based on some rule. This function machine turns any number n into $4n + 1$.

   a. Number −7 is just going in. What will be the output?

   b. Number 17 just came out. What was the input?



3. Potatoes cost $3 per kilogram. Fill in the tables #1 and #2.

   Does each table represent a function? Explain.

#1

(Input) **Weight**	(Output) **Cost**
1 kg	$3
2 kg	
3 kg	
5 kg	
12 kg	

#2

(Input) **Cost**	(Output) **Weight**
$12	
$30	
$48	
$72	
$90	

4. The table lists seven children, and each child's favourite colour.

Input	Allie	Julie	Danny	Juan	Pete	Bob	Samantha
Output	pink and blue	blue	grey	yellow	blue and red	?	purple

Is this a function? If not, change it in some manner(s) so it *is* a function.

5. T is a function that maps the name of a month to the number of days in it.

a. Create a depiction of T using a diagram like in example 1.

b. If you reverse the inputs and outputs, is the resulting relationship a function? Explain.

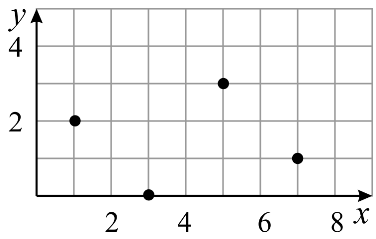
If the inputs and outputs are numbers, we can plot **a graph of the function** in the coordinate grid. Each input-output pair is viewed as an ordered pair (a single point).

We also use the terms "independent variable" for the input, and "dependent variable" for the output.

Example 3. Let F be the function (1, 2), (3, 0), (5, 3), (7, 1).

Note: A function *can* be given as a list of ordered pairs.

The image on the right is the plot of F; yet the plot is *not* F. The function F is the specific list of inputs and outputs, or the relationship itself.



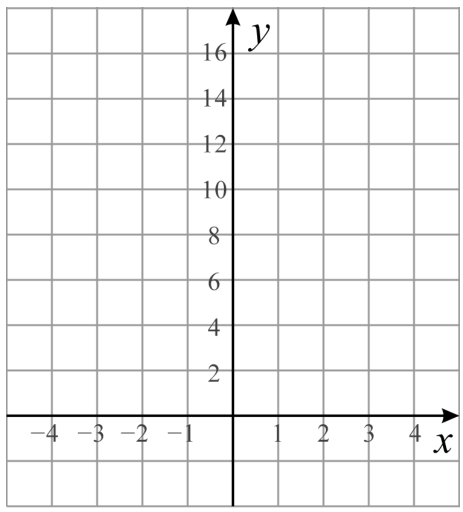
6. Let G be the function that maps each integer from −4 to 4 to its square minus one.

a. Fill in the table, listing the ordered pairs of G.

Input (x)	−4	−3	−2						
Output (y)	15								

b. Make a plot of G.

c. If you reversed the inputs and the outputs, would the relationship still be a function? Explain.



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Modelling Linear Relationships

Reminder: The equation $y = mx + b$ defines a linear function where:

- x is the independent variable (input)
- y is the dependent variable (output)
- b is the initial value, and
- m is the rate of change.

If a function is linear, we can always write an equation for it in this format.

1. The table shows the cost of renting a specialty car for a day. The base cost is $120, with a 300-km allowance. Any distance you drive over that 300 km, you will pay an additional fee.

Km over allowance	0	20	40	60	80	100	120	140	160	180
Cost	$120	$124	$128	$132	$136	$140	$144	$148	$152	$156

a. What is the rate of change?

What does it signify in this situation?

b. What is the initial value?

What does it signify in this situation?

c. Write an equation to model this relationship.

d. How much is the rental cost if you drive a total of 481 km?

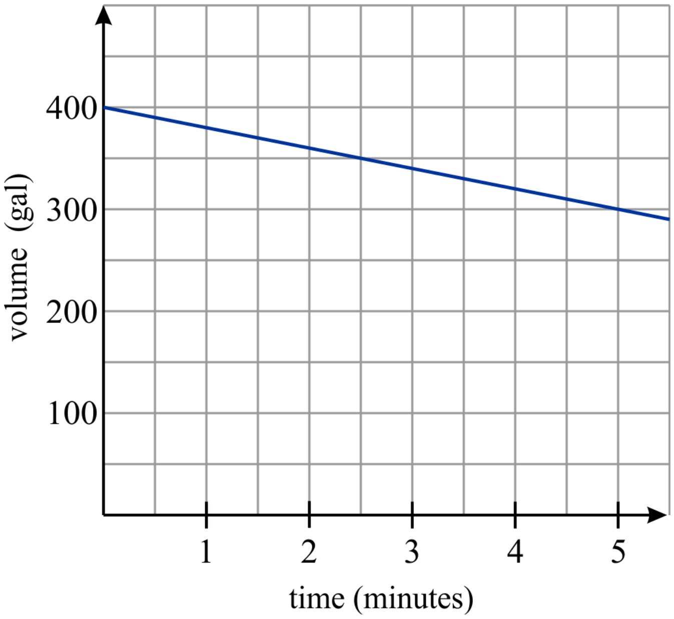
e. How many kilometres over the allowance can you drive with $170?

2. Andrew has already saved $140, and from now on, he will be putting $25 into his savings every week.

a. Write an equation for the total savings he has, as a function of time.

b. When will Andrew have saved $490?

3. The plot shows the amount of water in a water tank as a function of time.



a. What is the rate of change?

What does it signify in this situation?

b. What is the initial value?

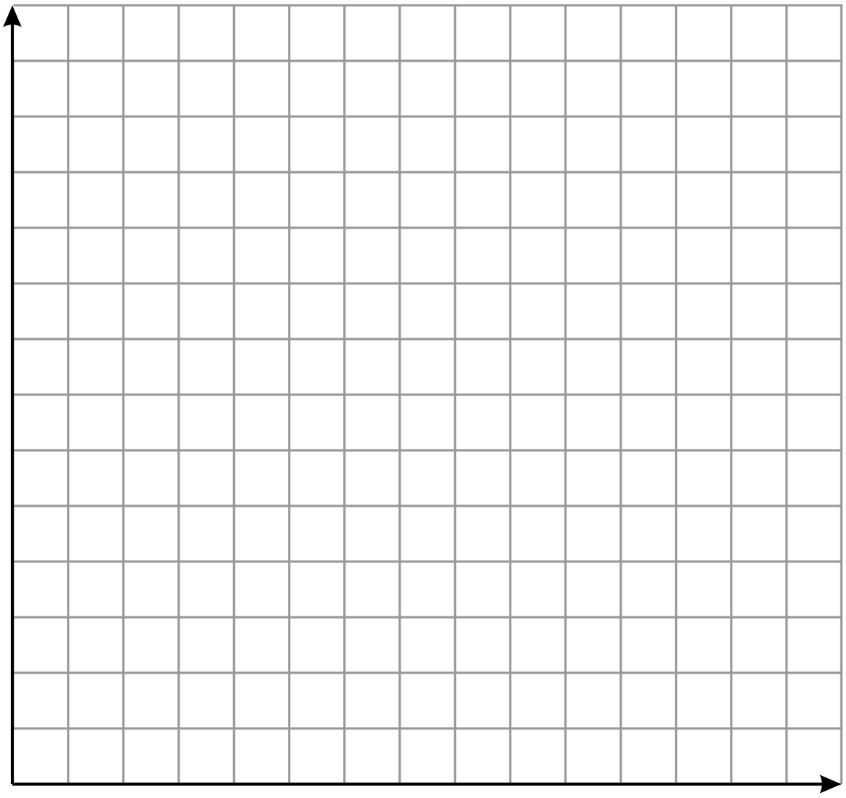
What does it signify in this situation?

c. Write an equation to model this relationship.

4. Sarah took a $6500 loan, and she is paying it back at the rate of $350 each month.

a. Write an equation to represent the amount of debt (D) she has, as a function of time (*t*, in months).

b. Plot your equation. Make sure that the point corresponding to *time* = 12 months fits in it.

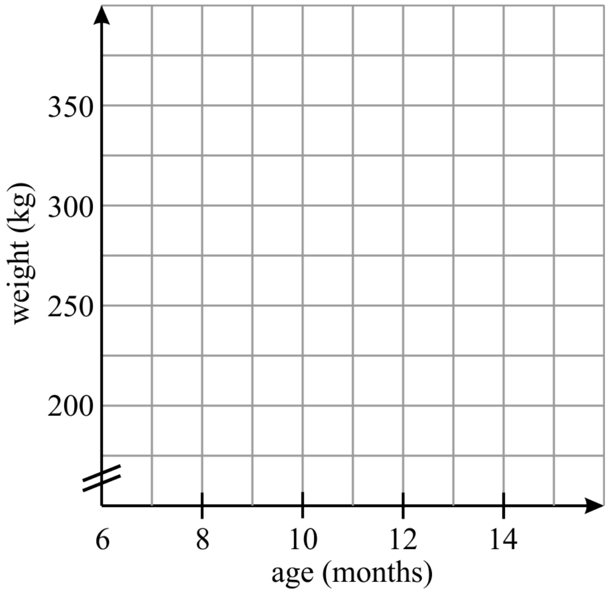


5. The weight gain of a heifer is approximately linear between the ages of 6 and 14 months.

Age (months)	6	7	8	9	10	11	12	13	14
W (kg)	180	205	230	255	280	305	330	355	380

a. What is the rate of change of this function? What does it signify in this situation?

b. Make a plot of this function.



c. (Challenge) Write an equation for the relationship between the weight (W) and the age (A).

6. The table below shows how Celsius and Fahrenheit degrees correspond at different temperatures. Consider the linear function that inputs the temperature in Celsius (C), and outputs the temperature in Fahrenheit (F).

Celsius (C)	0	5	10	15	20	25	30
Fahrenheit (F)	32	41	50	59	68	77	86

a. What is the initial value?

What does it signify in this situation?

b. What is the rate of change?

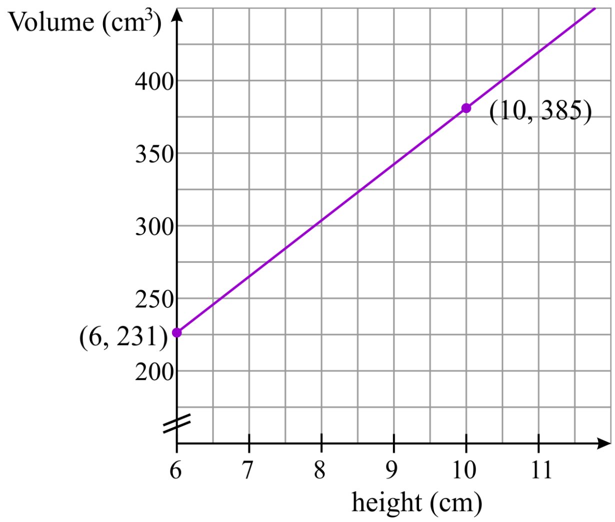
What does it signify in this situation?

c. Write an equation to model this relationship.

d. What temperature in F° corresponds to 18C°?

e. What temperature in C° corresponds to 100F°?

7. The graph shows the volume of a cylindrical drinking glass with a 7-cm diameter, as a function of its height. In other words, the diameter of the glass is fixed or decided, but the height is not, and we're looking at how the volume changes as the height changes.



a. What is the initial value? (It is not shown on the graph, but you can figure it out with common sense.)

b. Write an equation to model this relationship, using the given points.

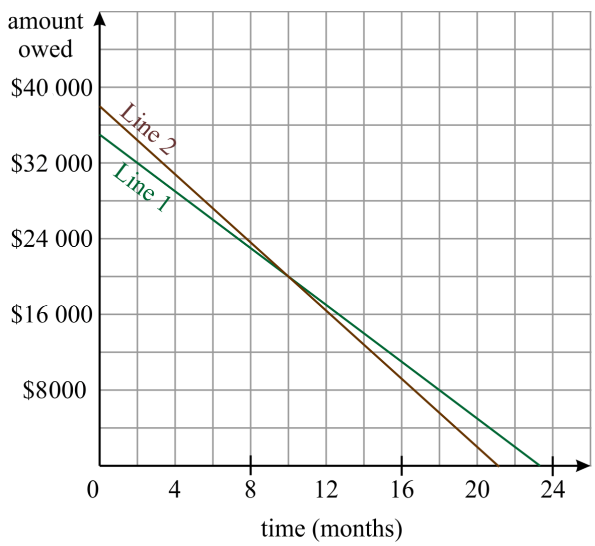
c. How tall would the glass need to be in order to have a volume of 500 cm^3?

d. What is the volume of the glass if its height is 7.4 cm?

e. Now use your knowledge of geometry. Write a formula for the volume of this glass. Compare it to the equation you wrote in (c). (Note: the reason they are slightly different is because the numbers in the graph above are rounded.)

Puzzle Corner

Natalie is deciding between two different cars. The first one, Car 1, costs $35 000, and she would pay for it in monthly payments of $1500. Car 2 costs Y dollars, and she would pay for it in monthly payments of $1800.



a. Match each line in the picture with the correct car/payment plan.

b. The two plans meet at (10 months, $20 000). Write an equation for the line for Car 2.

Describing Functions 1

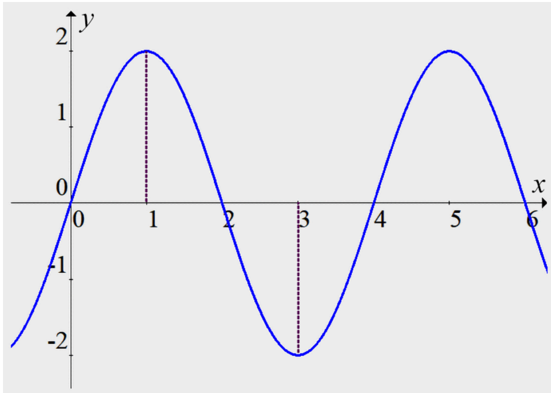
A function is said to be **increasing** if its graph is continually going upwards. If its graph is continually going downwards, it is **decreasing**. A function can also be **constant** — its graph is a horizontal line.

We typically use these terms to describe a function in a certain interval. We use the notation [*a*, *b*] to denote an interval from *a* to *b*.

Example 1. From the graph we can see that this function is decreasing in the interval [1, 3] (in other words, from $x = 1$ to $x = 3$).

It is increasing in the interval [3, 5].

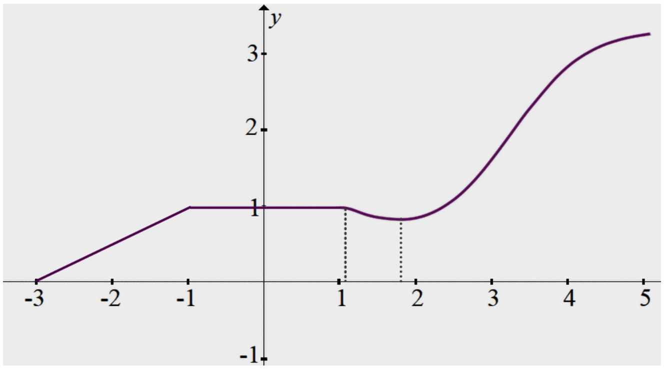
In the interval [0, 5] it is neither increasing nor decreasing.



Example 2. This function is linear and increasing in the interval [−3, −1].

Next, it is constant in the interval [−1, 1.1]. (Its output value, or *y*-value, is one, all through that interval.)

Then it is nonlinear and decreasing in the interval [1.1., 1.8]. Lastly, it is nonlinear and increasing from $x = 1.8$ to $x = 5$.



1. Describe this function by intervals where it is increasing, decreasing, or constant. Include also whether it is linear or nonlinear in those intervals.