math MAMMOTH

Grade 7-A Worktext
South African Version

Light Blue Series

T he language of algebra

I ntegers

S olving one–step equations

R ational numbers

E quations and inequalities

By Maria Miller

EDITION 1/2017

Contents

Chapter 4: Rational Numbers

Chapter 5: Equations and Inequalities

Foreword

Math Mammoth South African version has been customised to South Africa in the following manners:

- The names used are South African names (instead of Jack and Jill, there are Ansie and Jali).
- The currency used in word problems is rand and cents.
- The material is all metric. In other words, the US customary measuring units are not used.
- Spelling is British English instead of American English.
- Paper size is A4.

Please note that the curriculum is not following the South African official syllabus for seventh grade maths. Instead, it is a copy of the US version of Math Mammoth Grade 7, aligned to the US Common Core Standards. This decision was made because of the great amount of work that would be involved in writing new lessons and reorganizing old ones to match all the standards in the South African syllabus. For the most part, Math Mammoth is exceeding South African standards.

We start out with an introduction to basic algebra, which is in many ways a revision of the same topics from 6th grade. The first chapter revises the order of operations, the concepts of an expression and an equation, and the distributive property. Students learn about the commutative and associative properties of addition and multiplication, and they simplify expressions that do not involve negative numbers.

In chapter 2, we study integers and their operations in detail. Some of this is revision from 6th grade, and some of it is new. The four operations of integers are explained with the help of two visual models: the number line and counters, hopefully providing an intuitive understanding of the processes. Students need to be able to add, subtract, multiply, and divide integers when they learn to solve equations in chapter 3. In the end of the chapter students also learn about negative fractions.

The lesson about distance contains a formula that may look unfamiliar to the teacher. You can find the distance between two integers by taking the absolute value of their difference. In symbols, the distance between a and b is $|a - b|$. The idea behind the formula is simple, though, and most people use that idea instinctively without knowing about the formula. For example, how far apart are 14 and 92 from each other? To solve that mentally, we find their difference, but we take that difference in a positive sense. In other words, we do not calculate $14 - 92 = -78$ and state that the distance is negative 78 units, but instead, we say the distance is 78 units because distance is always positive. The absolute value takes care of that: it turns any negative quantity into a positive one.

In the next chapter (chapter 3) students study simple one-step equations. They already know the basics of how to solve this type of equation from 6th grade, but this time we use negative numbers in them.

Chapter 4 is titled *Rational Numbers*, which are simply fractions and certain decimals, so the student is already very familiar with them. The goal of the chapter is to be able to add, subtract, multiply, and divide both positive and negative fractions and decimals. We also solve simple equations involving fractions and decimals and learn about scientific notation.

The last chapter in part 7-A focuses on linear equations. The student learns to solve various types of linear equations and practises using those in simple word problems. We study linear inequalities but not to the same depth as linear equations. Lastly, the student graphs linear equations and is introduced to the concept of slope, which is the steepness of a line. The student will continue studying these topics with more details in an Algebra 1 course.

In part 7-B, students study ratios, proportions, percent, geometry, the Pythagorean Theorem, probability, and statistics.

I wish you success in teaching maths!

Maria Miller, the author

Chapter 1: The Language of Algebra
Introduction

In the first chapter of *Math Mammoth South African Version Grade 7* we revise all of the sixth grade algebra topics and also study some basic properties of the operations.

The main topics are the order of operations, expressions, and simplifying expressions in several different ways. The main principles are explained and practised both with visual models and in abstract form, and the lessons contain varying practice problems that approach the concepts from various angles.

This chapter is like an introduction that lays a foundation for the rest of the year. For example, when we study integers in the next chapter, students will once again simplify expressions, only with negative numbers. Then when we study equations in chapters 3 and 5, students will again simplify expressions, use the distributive property, and solve equations.

The Lessons in Chapter 1

Helpful Resources on the Internet

ORDER OF OPERATIONS

Otter Rush
Practise exponents in this otter-themed maths game.
http://www.mathplayground.com/ASB_Otter_Rush.html

Exponents Jeopardy Game
Practise evaluating exponents, equations with exponents, and exponents with fractional bases in this interactive Jeopardy-style game.
http://www.math-play.com/Exponents-Jeopardy/exponents-jeopardy-math-game.html

Choose A Maths Operation
Choose the mathematical operation(s) so that the number sentence is true. Practise the role of zero and one in basic operations or operations with negative numbers. Helps develop number sense and logical thinking.
http://www.homeschoolmath.net/operation-game.php

Order of Operations Quiz
A 10-question online quiz that includes two different operations and possibly parentheses in each question. You can also modify the quiz parameters yourself.
http://www.thatquiz.org/tq-1/?-j8f-lk-p0

Exploring Order of Operations (Object Interactive)
The program shows an expression, and you click on the correct operation (either +, −, ×, ÷ or exponent) to be done first. The program then solves that operation, and you click on the *next* operation to be performed, *etc*., until it is solved. Lastly, the resource includes a game where you click on the falling blocks in the sequence that the order of operations would dictate.
http://www.learnalberta.ca/content/mejhm/html/object_interactives/order_of_operations/use_it.html

Make 24 Game
Arrange the number cards, the operation symbols, and the parentheses, so that the expression will make 24.
http://www.mathplayground.com/make_24.html

Order of Operations Practice
A simple online quiz of 10 questions. Uses parentheses and the four operations.
http://www.onlinemathlearning.com/order-of-operations-practice.html

WRITING EXPRESSIONS

Algebraic Symbolism Matching Game
Match each verbal statement with its algebraic expression.
http://www.quia.com/mc/319817.html

Equivalent Algebraic Expressions
Practise determining whether or not two algebraic expressions are equivalent by manipulating the expressions. These problems require you to combine like terms and apply the distributive property.
https://www.khanacademy.org/math/algebra-basics/alg-basics-algebraic-expressions/alg-basics-equivalent-expressions/e/equivalent-forms-of-expressions-1

Visual Patterns
Click on the pattern to see a larger image and the answer to step 43. Can you solve the equation?
http://www.visualpatterns.org/

Expressions: Expressions and Variables Quiz
Choose an equation to match the word problem or situation.
http://www.softschools.com/quizzes/math/expressions_and_variables/quiz815.html

Translating Words to Algebraic Expressions
Match the correct maths expression with the corresponding English phrase, such as "7 less than a number". You can do this activity either as a matching game or as a concentration game.
https://www.quia.com/jg/1452190.html

Rags to Riches - Verbal and Algebraic Expressions
Translate between verbal and algebraic expressions in this quest for fame and fortune.
http://www.quia.com/rr/520475.html

Algebra Noodle
Play a board game against the computer while modelling and solving simple equations and evaluating simple expressions. Choose level 2 (level 1 is too easy for 7th grade).
http://www.free-training-tutorial.com/math-games/algebra-noodle.html

Matching Algebraic Expressions with Word Phrases
Five sets of word phrases to match with expressions.
http://www.mrmaisonet.com/index.php?/Algebra-Quizzes/Matching-Algebraic-Expressions-With-Word-Phrases.html

PROPERTIES OF THE OPERATIONS

Properties of Operations at Quizlet
Includes explanations, online flashcards, and a test for the properties of operations (commutative, associate, distributive, inverse, and identity properties). The inverse and identity properties are not covered in this chapter of Math Mammoth but can be learned at the website. The identity property refers to the special numbers that do not change addition or multiplication results (0 and 1).
http://quizlet.com/2799611/properties-of-operation-flash-cards/

Commutative/Associative/Distributive Properties Matching Game
Match the terms and expressions in the two columns.
http://www.quia.com/cm/61114.html?AP_rand=1554068841

Properties of Multiplication
Simple online practice about the commutative, associative, distributive, and identity properties of multiplication.
http://www.aaamath.com/pro74b-propertiesmult.html

Properties of Multiplication
Simple online practice about the commutative, associative, distributive, and identity properties of multiplication.
http://www.aaamath.com/pro74ax2.htm

Properties of the Operations Scatter Game
Drag the corresponding items to each other to make them disappear.
http://quizlet.com/763838/scatter

Associative, Distributive and Commutative Properties
Examples of the various properties followed by a simple self-test.
http://www.mathwarehouse.com/properties/associative-distributive-and-commutative-properties.php

SIMPLIFYING EXPRESSIONS

Simplifying Algebraic Expressions Quiz
An online quiz of 15 questions.
http://www.quia.com/quiz/1200540.html

BBC Bitesize - Simplifying Algebraic Terms
A 10-question online quiz on simplifying expressions.
http://www.bbc.co.uk/bitesize/quiz/q14530139

THE DISTRIBUTIVE PROPERTY

Factor the Expressions Quiz
Factor expressions such as $3x + 15$ into $3(x + 5)$.
http://www.thatquiz.org/tq-0/?-jh00-l3-p0

Distributive Property Practice
Guided practice for applying the distributive property, such as writing $-8(-7a + 10)$ as $56a - 80$.
http://www.hstutorials.net/dialup/distributiveProp.htm

Distributive Property Game
Solve questions related to the usage of the distributive property amidst playing a game. Play either a bouncing balls game or free-kick soccer game, with the same questions.
http://reviewgamezone.com/games3/bounce.php?test_id=22828&title=DISTRIBUTIVE%20PROPERTY
http://reviewgamezone.com/games3/freekick.php?test_id=22828&title=DISTRIBUTIVE%20PROPERTY

EVALUATE EXPRESSIONS

Escape Planet
Choose the equation that matches the words.
http://www.harcourtschool.com/activity/escape_planet_6/

Evaluating Expressions Quiz
Includes ten multiple-choice questions.
http://www.mrmaisonet.com/index.php?/Algebra-Quizzes/Evaluating-Expressions.html

Writing and Evaluating Expressions Quiz
This quiz has 12 multiple-choice questions and tests both evaluating and writing expressions.
http://www.quibblo.com/quiz/aWAUlc6/Writing-Evaluating-Expressions

TERMINOLOGY

Coefficients, Like Terms, and Constants
How to find and name the coefficients, like terms, and constants in expressions.
http://mathcentral.uregina.ca/QQ/database/QQ.09.07/h/maddie1.html

Identifying Variable Parts and Coefficients of Terms
After the explanations, you can generate exercises by pushing the button that says "new problem." The script shows you a multiplication expression, such as $-(3e)(3z)m$, and you need to identify its coefficient and variable part, effectively by first simplifying it.
http://www.onemathematicalcat.org/algebra_book/online_problems/id_var_part_coeff.htm#exercises

Tasty Term Treats
A lesson followed by a simple game where you drag terms into Toby's bowl and non-terms into the trash can.
http://mathstar.lacoe.edu/lessonlinks/menu_math/var_terms.html

Algebra - Basic Definitions
Clear definitions with illustrations of basic algebra terminology, including term, coefficient, constant, and expression.
http://www.mathsisfun.com/algebra/definitions.html

GENERAL

Balance Beam Activity
A virtual balance that poses puzzles where the student must think algebraically to find the weights of various figures. Includes three levels.
http://mste.illinois.edu/users/pavel/java/balance/index.html

Algebraic Reasoning
Find the value of an object based on two scales.
http://www.mathplayground.com/algebraic_reasoning.html

Algebra Puzzle
Find the value of each of the three objects presented in the puzzle. The numbers given represent the sum of the objects in each row or column.
http://www.mathplayground.com/algebra_puzzle.html

Algebraic Expressions - Online Assessment
During this online quiz you must simplify expressions, combine like terms, use the distributive property, express word problems as algebraic expressions and recognise when expressions are equivalent. Each incorrect response will allow you to view a video explanation for that problem.
http://www.mrmaisonet.com/index.php?/Algebra-Quizzes/Online-Assessment-Algebraic-Expressions.html

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Simplifying Expressions

Example 1. Simplify $2x \cdot 4 \cdot 5x$.

Notice, this expression contains only multiplications (because $2x$ and $5x$ are also multiplications).

Since we can multiply in any order, we can write this expression as $2 \cdot 4 \cdot 5 \cdot x \cdot x$.

Now we multiply 2, 4, and 5 to get 40. What is left to do? The part $x \cdot x$, which is written as x^2.

So, $2x \cdot 4 \cdot 5x = 40x^2$.

Note: The equals sign used in $2x \cdot 4 \cdot 5x = 40x^2$ signifies that the two expressions are equal no matter what value x has. That equals sign does not signify an equation that needs to be solved.

Similarly, we can simplify the expression $x + x$ and write $2x$ instead. That whole process is usually written as $x + x = 2x$.

Again, the equals sign there does not indicate an equation to solve, but just the fact that the two expressions are equal. In fact, if you think of it as an equation, *any* number x satisfies it! (Try it!)

1. Simplify the expressions.

a. $p + 8 + p + p$	**b.** $p \cdot 8 \cdot p \cdot p \cdot p$	**c.** $2p + 4p$
d. $2p \cdot 4p$	**e.** $5x \cdot 2x \cdot x$	**f.** $y \cdot 2y \cdot 3 \cdot 2y \cdot y$

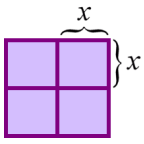
2. Write an expression for the area and perimeter of each rectangle.

a.

area =

perimeter =

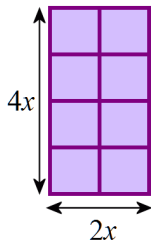
b.



area =

perimeter =

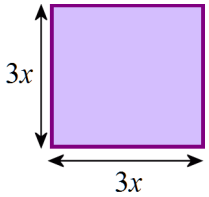
c.



area =

perimeter =

d.



area =

perimeter =

3. **a.** Sketch a rectangle with sides $2b$ and $7b$ long.

b. What is its area?

c. What is its perimeter?

4. **a.** The perimeter of a rectangle is $24s$.
Sketch one such rectangle.

What is its area?
Hint: there are many possible answers.

b. Find the area and perimeter of your rectangle in (a) if s has the value 3 cm.

5. **a.** Which expression below is for an area of a rectangle? Which one is for a perimeter?

$4a + 4b$ $2a \cdot 2b$

b. Sketch the rectangle.

6. **a.** Find the value of the expressions $3p$ and $p + 3$ for different values of p.

Value of p	$3p$	$p + 3$
0		
0,5		
1		
1,5		
2		
2,5		
3		
3,5		
4		

b. Now, look at the table. Can you tell which is larger, $3p$ or $p + 3$?

Some revision! In algebra, a **term** is an expression that consists of numbers and/or variables that are multiplied together. A single number or variable is also a term.

Examples.

- $2xy$ is a term, because it only contains multiplications, a number, and variables.
- $(5/7)z^3$ is a term. Remember, the exponent is a shorthand for repeated multiplication.
- Addition and subtraction separate the individual terms in an expression from each other. For example, the expression $2x^2 - 6y^3 + 7xy + 15$ has four terms, separated by the plus and minus signs.
- $s + t$ is *not* a term, because it contains addition. Instead, it is a sum of *two* terms, s and t.

The number by which a variable or variables are multiplied is called a ***coefficient***.

Examples.

- The term $0{,}9ab$ has the coefficient 0,9.
- The coefficient of the term m^2 is 1, because you can write m^2 as $1 \cdot m^2$.

If the term is a single number, such as 7/8, we call it a ***constant***.

Example 2. The expression $1{,}5a + b^2 + 6/7$ has three terms: $1{,}5a$ and b^2 and 6/7. The last term, 6/7, is a constant.

7. Fill in the table.

Expression	the terms in it	coefficient(s)	Constants
$(5/6)s$			
w^3			
$0{,}6x + 2{,}4y$			
$x + 3y + 7$			
$p \cdot 101$			
$x^5y^2 + 8$			

The two terms in the expression $2x^2 + 5x^2$ are **like terms**: they have the same variable part (x^2). Because of that, we can add the two terms to simplify the expression. To do that, simply add the coefficients 2 and 5 and use the same variable part: $2x^2 + 5x^2 = 7x^2$. It is like adding 2 apples and 5 apples.

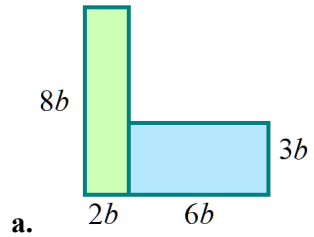
However, you cannot add (or simplify) $2x + 7y$. That would be like adding 2 apples and 7 oranges.

Example 3. Simplify $6x - x - 2x + 9x$. The terms are like terms, so we simply add or subtract the coefficients: $6 - 1 - 2 + 9 = 12$ and tag the variable part x to it. The expression simplifies to $12x$.

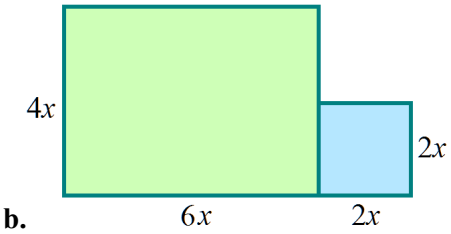
8. Simplify the expressions.

a. $5p + 8p - p$	**b.** $p^2 + 8p^2 + 3p^2$	**c.** $12a^2 - 8a^2 - 3a^2$

9. Write an expression for the total area.



a.



b.

In problems 10-12, write an expression for part (a). For part (b), write an equation and solve it. Don't skip writing the equation, even if you can solve the problem without it, because we are practising writing equations! You don't have to use algebra to solve the equations—you can solve them in your head or by guessing and checking.

10. **a.** The length of a rectangle is 4 metres and its width is *w*.
What is its perimeter? Write an expression.

b. Let's say the perimeter has to be 22 metres. How wide is the rectangle then?
Write an *equation* for this situation, using your expression from (a).

Remember, you do not have to use algebra to solve the equation—you can solve it in your head or by "guess and check." But **do** write the equation.

11. **a.** Suraya borrows six books from the library each week, and her mum borrows two.
How many books, in total, do both of them borrow in *w* weeks? Write an expression.

b. How many weeks will it take them to have borrowed 216 books? Write an equation.

12. **a.** Inga buys *y* containers of mints for R6 apiece. A fixed shipping cost of R5 is added to her order. What is the total cost? Write an expression.

b. The total cost for what Inga bought was R155. How many containers of mints did she buy? Write an equation.

Puzzle Corner

a. What is the total value, in cents, if Ashley has *n* ten-cent pieces and *m* twenty-cent pieces? Write an expression.

b. The total value of Ashley's coins is 490 cents. How many ten-cent pieces and twenty-cent pieces can she have? *Hint: make a table to organize the possibilities.*

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Chapter 2: Integers
Introduction

This chapter deals with integers, which are signed (positive or negative) whole numbers. We begin with a revision of addition and subtraction of integers from 6th grade. Then we study in detail multiplication and division of integers and conclude with negative fractions and the order of operations.

The first lesson revises the concepts of integers, absolute value, the opposite of an integer, and simple inequalities on a number-line. The next lessons present the addition and subtraction of integers through two visual models: first as movements on a number-line, and then using positive and negative counters. These lessons also endeavour to connect the addition and subtraction of integers with various situations from real life.

The lesson *Subtraction of Integers* includes this important principle: Any subtraction can be converted into an addition (of the number of opposite sign) and vice versa. This principle allows us to calculate not only subtractions such as $5 - (-7)$ but also problems that contain both addition and subtraction. These mixed problems become simple sums after the subtractions have been converted into additions. Converting subtractions into additions or vice versa is also important when simplifying expressions. For example, $5 + (-x)$ can be simplified to $5 - x$.

Next, we study the distance between two integers. This can be found by taking the absolute value of their difference: the distance between x and y is $|x - y|$. Students learn to use this formula to find distances between integers, and they also compare the result the formula gives to the answer they get by logical thinking.

The lesson *Multiplying Integers* not only teaches the mechanics of how to multiply integers, but also gives both intuitive understanding and formal justification for the shortcut, "a negative times a negative makes a positive." This formal justification using the distributive property introduces and illustrates the type of careful and precise reasoning that mathematicians use in proofs.

The next lesson, on the division of integers, leads into the topic of negative fractions in the following lesson. The final topic is a revision of the order of operations where we perform several operations at a time with integers.

The Lessons in Chapter 2

Helpful Resources on the Internet

Integers Video Lessons by Maria
A set of free videos that teach the topics in this book—by the author.
http://www.mathmammoth.com/videos/prealgebra/pre-algebra-videos.php#integers

Free Downloadable Integer Fact Sheets
http://www.homeschoolmath.net/download/Add_Subtract_Integers_Fact_Sheet.pdf
http://www.homeschoolmath.net/download/Multiply_Divide_Integers_Fact_Sheet.pdf

ORDERING INTEGERS

Number Balls
Click the balls in the ascending order of numbers.
http://www.sheppardsoftware.com/mathgames/numberballs/numberballsAS2.htm

Plot Inequalities Quiz
Practise plotting simple inequalities, such as $x < -7$, on a number line in this 10-question interactive quiz.
https://www.thatquiz.org/tq-o/?-j18-l3-p0

Compare Integers Quiz
A 10-question online quiz where you compare two integers. You can also modify the quiz parameters to include sums, differences, products, and quotients, which makes it more challenging.
http://www.thatquiz.org/tq-8/?-j11-l1i-p0

Diamond Drop
Drag integers to the empty spaces in comparison sentences (such as ___ < ____, ___ = ____ , and ___ > ____) as they fall from the top of the screen. (The link does not work when clicked from the PDF file; please copy and paste it into your browser window.)
http://oame.on.ca/CLIPS/swfPlayer.html?swfURL=lib/CL005_IntegersRepresentCompareOrder/CL005_C02_A05_C_DiamondDrop/CL005_C02_A05_C_DiamondDrop.swf

ABSOLUTE VALUE AND OPPOSITES

Number Opposites Challenge
Practise solving challenging problems finding the opposites of numbers.
https://www.khanacademy.org/math/arithmetic-home/negative-numbers/number-opposites/e/opposites-on-the-number-line

Number Balls - Absolute Value
Click the balls in the ascending order of numbers.
http://www.onlinemathlearning.com/absolute-value-game.html

Absolute Value Boxes
You are shown expressions with absolute value (such as $|-11|$, $-|8|$ and $|3|$). Choose the one that has either the greatest or the least value.
http://www.xpmath.com/forums/arcade.php?do=play&gameid=96

Absolute Value to Find Distance
Practise taking the absolute value of the difference of two numbers to find the distance between those numbers. Apply this principle to solve word problems.
https://www.khanacademy.org/math/arithmetic-home/negative-numbers/abs-value/e/absolute-value-to-find-distance

Absolute Value Quiz
Find the absolute value of each integer or sum.
http://www.softschools.com/quizzes/math/absolute_value/quiz1035.html

Absolute Value Exercises
Click on "new problem" to get randomly generated practice-problems that practise absolute value.
http://www.onemathematicalcat.org/algebra_book/online_problems/intro_abs_val.htm#exercises

Absolute Value Quiz - harder
This quiz includes mixed operations and absolute value.
http://www.softschools.com/testing/math/theme2.html

ADDITION AND SUBTRACTION

Number-Line Jump Maker
Practise making different-sized jumps on a customisable number line with this interactive tool.
http://www.ictgames.com/numberlineJumpMaker/

Number-Line Addition
Click on the addition sentence on the fruit that matches the number line. Choose level 3.
http://www.sheppardsoftware.com/mathgames/integers/FS_NumberLine_integer.htm

Number-Line Integer Subtraction
Click on the subtraction sentence on the fruit that matches the number line. Choose level 3.
http://www.sheppardsoftware.com/mathgames/integers/FS_NumberLine_int_minus.htm

Add Negative Numbers on a Number Line
Practise matching number-line diagrams to addition expressions involving negative numbers.
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-negative-numbers-add-and-subtract/cc-7th-add-sub-neg-number-line/e/adding-negative-numbers-on-the-number-line

Maths Lines Integers
Combine positive and negative balls to make the target number.
http://www.mathplayground.com/math_lines_integers.html

Spider Match
Choose pairs of numbers that add to the given integer. Can be played as a multi-player game or against the computer.
http://www.arcademics.com/games/spider-match/spider-match.html

Subtracting Integers Activity
Model subtraction problems by dragging individual + or − signs off the board. You may need to add neutral pairs (a positive-negative pair) to the working space to solve the problem.
http://mathstar.lacoe.edu/newmedia/integers/subtract/activities/activities.html

Interpreting Negative Number Statements
Practise matching addition and subtraction equations to real-world scenarios in this interactive exercise.
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-negative-numbers-add-and-subtract/cc-7th-add-sub-word-problems-w-negatives/e/negative-number-addition-and-subtraction--interpretation-problems

Add Integers Quiz
Practise adding integers in this 10-question online quiz.
https://www.thatquiz.org/tq-1/?-j4101-lk-p0

Subtracting Integers by Adding the Opposite
Practise your integer skills with this interactive quiz.
http://www.buzzmath.com/Docs#CC07E10485

Subtracting Negative Integers
Practise subtracting positive and negative single-digit numbers in this interactive online activity.
https://www.khanacademy.org/math/algebra-basics/basic-alg-foundations/alg-basics-negative-numbers/e/adding_and_subtracting_negative_numbers

Adding and Subtracting Several Negative Numbers
Practise solving addition and subtraction problems with several integers.
https://www.khanacademy.org/math/arithmetic/arith-review-negative-numbers/arith-review-add-and-sub-integersss/e/integer-addition-and-subtraction-2

Find the Missing Integers
Fill in the missing integer in addition equations such as −23 + ____ = −8.
http://www.aaamath.com/g8_65_x3.htm

Find the Missing Number
Practise finding the missing value in an addition or subtraction equation involving negative numbers.
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-negative-numbers-add-and-subtract/cc-7th-add-and-sub-integers/e/integer-addition-and-subtraction

Space Coupe to the Rescue
By choosing a positive or negative number, the player controls the vertical position of a spaceship. If the spaceship reaches the same vertical position as a virus pod, the pod is destroyed.
http://pbskids.org/cyberchase/games/negativenumbers

Red and Black Triplematch Game for Adding Integers
This is a fun card game with 2-5 people to practice adding integers.
http://mathmamawrites.blogspot.com/2010/07/black-and-red-triplematch-card-game-for.html

Different Ways to Play "Integer War" Card Game
This page explains various ways to play a common card game that is used to revise integers.
http://www.fortheloveofteachingmath.com/2011/09/13/the-different-ways-we-play-integer-war/

Combine Like Terms with Negative Coefficients
Simplify algebraic expressions by combining like terms. Coefficients on some terms are negative.
https://www.khanacademy.org/math/algebra/introduction-to-algebra/alg1-manipulating-expressions/e/combining_like_terms_1

MULTIPLICATION AND DIVISION

Integers Multiplication Blocks
Click on numbers whose product is equal to the target number.
http://www.xpmath.com/forums/arcade.php?do=play&gameid=58

Integers Multiplication -144 to 144
Solve as many integer multiplication problems as you can within one minute with these online flashcards.
http://www.xpmath.com/forums/arcade.php?do=play&gameid=17

Integers Division -81 to 81
Solve as many integer division problems as you can within one minute with these online flashcards.
http://www.xpmath.com/forums/arcade.php?do=play&gameid=16

Fruit Splat Multiplication of Integers
Click on the fruit that has the correct answer to the integer multiplication problem.
http://www.sheppardsoftware.com/mathgames/integers/FS_Integer_multiplication.htm

Multiplying and Dividing Negative Numbers Word Problems
Practise matching situations to multiplication and division expressions and equations in this online activity.
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-negative-numbers-multiply-and-divide/cc-7th-mult-div-neg-word-problems/e/negative-number-word-problems-1

Find the Missing Number Quiz
Practise integer multiplication with missing numbers in this interactive 10-question quiz.
https://www.thatquiz.org/tq-1/?-j114-la-p0

ALL OPERATIONS / GENERAL

Rational Numbers on the Number Line
Practise placing positive and negative fractions and decimals on the number line in this interactive online activity.
https://www.khanacademy.org/math/6th-engage-ny/engage-6th-module-3/6th-module-3-topic-a/e/fractions_on_the_number_line_3

Integer Operations Quiz
Practise several operations with integers in this 10-question interactive quiz.
https://www.thatquiz.org/tq-1/?-jh8f-la-p0

Solving Problems with Integers
Use this set of interactive word problems to reinforce your knowledge of integers.
http://www.buzzmath.com/Docs#F6KME533

Order of Operations with Negative Numbers
Practice evaluating expressions using the order of operations in this interactive online activity.
https://www.khanacademy.org/math/algebra-basics/basic-alg-foundations/alg-basics-order-of-operations/e/order_of_operations

Integers Quiz
This interactive multiple-choice quiz revises absolute value, ordering integers and more.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-829633-1&chapter=3&headerFile=4

Numerate Game with Integers
In this game, two players take turns forming equations using the available tiles.
http://www.transum.org/Maths/Game/Numerate/Default.asp?Level=2

Integers Jeopardy
A jeopardy game where the questions involve adding, subtracting, multiplying, and dividing integers.
http://www.math-play.com/Integers-Jeopardy/Integers-Jeopardy.html

Integers Mystery Picture Game
Solve expressions with many operations.
http://www.dositey.com/2008/addsub/Mystery11.htm

Solve For Unknown Variable - Integer Revision
Find the value of an unknown variable in a given addition or subtraction equation with integers.
http://www.mrmaisonet.com/index.php?/Algebra-Quizzes/Solve-For-Unknown-Variable-Integer-Review.html

Fruit Shoot Game: Mixed Integer Operations
Practise all four operations with integers while shooting fruits. You can choose the difficulty level and the speed.
http://www.sheppardsoftware.com/mathgames/fruitshoot/FS_Mixed_Integers.htm

Student CLIPS in Mathematics
Activities, video clips, and games for middle-school math topics, including integers.
http://oame.on.ca/CLIPS/

The History of Negative Numbers
While they seem natural to us now, in the past negative numbers have spurred controversy and have been called "fictitious" and other names.
http://nrich.maths.org/public/viewer.php?obj_id=5961
http://www.classzone.com/books/algebra_1/page_build.cfm?content=links_app3_ch2&ch=2

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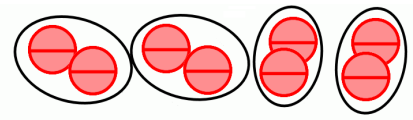
Dividing Integers

Divide a negative number by a positive



The image illustrates (−8) ÷ 4, or eight negatives divided into four groups. We can see the answer is −2.

Any time a negative integer is divided by a positive integer, we can illustrate it as so many negative counters divided evenly into groups. The answer will be negative.

Divide a positive integer by a negative. For example, 24 ÷ (−8) = ?

Remember, multiplication is the opposite operation to division. Let's write the answer to 24 ÷ (−8) as *s*. Then from that we can write a multiplication:

$24 \div (-8) = s \quad \Rightarrow \quad (-8)s = 24$

(You could use an empty line instead of *s*, if the variable *s* confuses you.)

The only number that fulfils the equation $(-8)s = 24$ is $s = -3$. So, $24 \div (-8) = -3$.

Similarly, each time you divide a positive integer by a negative integer, the answer is negative.

Divide a negative integer by a negative. For example, (−24) ÷ (−8) = ?

Again, let's denote the answer to −24 ÷ (−8) with *y*, and then write a multiplication sentence.

$-24 \div (-8) = y \quad \Rightarrow \quad (-8)y = -24$

The only number that fulfils the equation $(-8)y = -24$ is $y = 3$. Therefore, $-24 \div (-8) = 3$.

Similarly, each time you divide a negative integer by a negative integer, the answer is positive.

Summary. The symbols below show whether you get a positive or negative answer when you multiply or divide integers. Notice that the rules for multiplication and division are the same!

Multiplication	Examples	Division	Examples
⊕ · ⊖ = ⊖	4 · (−5) = −20	⊕ ÷ ⊖ = ⊖	20 ÷ (−5) = −4
⊖ · ⊕ = ⊖	−4 · 5 = −20	⊖ ÷ ⊕ = ⊖	−20 ÷ 5 = −4
⊖ · ⊖ = ⊕	−4 · (−5) = 20	⊖ ÷ ⊖ = ⊕	−20 ÷ (−5) = 4
⊕ · ⊕ = ⊕	4 · 5 = 20	⊕ ÷ ⊕ = ⊕	20 ÷ 5 = 4

Here is a shortcut for *multiplication* and *division* (NOT for addition or subtraction):

- If both numbers have the same sign (both are positive *or* negative), the answer is positive.
- If the numbers have different signs, the answer is negative.

1. Divide.

a. −50 ÷ (−5) = _____	**b.** (−8) ÷ (−1) = _____	**c.** 81 ÷ (−9) = _____
−12 ÷ 2 = _____	14 ÷ (−2) = _____	−100 ÷ (−10) = _____

2. Multiply. Then use the same numbers to write an equivalent division equation.

a. $-5 \cdot (-5) =$ _____ _____ ÷ _____ = _____	**b.** $9 \cdot (-6) =$ _____ _____ ÷ _____ = _____	**c.** $-80 \cdot 8 =$ _____ _____ ÷ _____ = _____

3. Four people shared a debt of R280 equally. How much did each owe? Write an integer division.

4. In a maths game, you get a negative point for every wrong answer and a positive point for every correct answer. Additionally, if you answer in 1 second, your negative points from the past get slashed in half!

Noluthando had accumulated 14 negative and 25 positive points in the game. Then she answered a question correctly in 1 second. Write an equation for her current "point balance."

5. Complete the patterns.

a.	**b.**	**c.**
$12 \div 4 =$ _____	_____ $\div (-7) = -3$	$60 \div$ _____ $= 2$
$8 \div 4 =$ _____	_____ $\div (-7) = -2$	$40 \div$ _____ $= 2$
$4 \div 4 =$ _____	_____ $\div (-7) = -1$	$20 \div$ _____ $= 2$
$0 \div 4 =$ _____	_____ $\div (-7) = 0$	$-20 \div$ _____ $= 2$
$(-4) \div 4 =$ _____	_____ $\div (-7) = 1$	$-40 \div$ _____ $= 2$
$(-8) \div 4 =$ _____	_____ $\div (-7) = 2$	$-60 \div$ _____ $= 2$
$(-12) \div 4 =$ _____	_____ $\div (-7) = 3$	$-80 \div$ _____ $= 2$
$(-16) \div 4 =$ _____	_____ $\div (-7) = 4$	$-100 \div$ _____ $= 2$

6. Here is a funny riddle. Solve the maths problems to uncover the answer.

E ____ $\div (-8) = 2$

E $3 \cdot (-12) =$ ____

N $-15 \div$ ____ $= -5$

G $-1 \cdot (-9) =$ _____

S $-48 \div 6 =$ _____

N $-12 \cdot (-5) =$ _____

H ____ $\div 12 = -5$

E ____ $\cdot (-6) = 0$

I $-27 \div 9 =$ _____

N $3 \cdot$ _____ $= -24$

E $(-144) \div 12 =$ _____

T $-4 \cdot (-9) =$ ____

V $-45 \div$ _____ $= 5$

I $-7 \cdot$ ____ $= -84$

Why is six afraid of seven? Because....

−8	−12	−9	−36	60		0	12	9	−60	36		3	−3	−8	−16

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Chapter 3: One-Step Equations
Introduction

The goal of this chapter is that students learn to solve one-step equations that involve integers.

The first lesson revises the concept of an equation and how to model equations using a pan balance (scale). The basic principle for solving equations is that, when you perform the same operation on both sides of an equation, the two sides remain equal.

The chapter presents two alternatives for keeping track of the operations to be performed in an equation. The one method, writing the operation under each side of the equation, is common in the United States. The other method, writing the operation in the right margin, is common in Finland. Either is adequate, and the choice is just a matter of the personal preference of the teacher.

The introduction to solving equations is followed by a lesson on addition and subtraction equations and another on multiplication and division equations. All the equations are easily solved in only one step of calculations. The two-fold goal is to make the student proficient in manipulating negative integers and also to lay a foundation for handling more involved equations in Chapter 5.

In the next lesson, students write equations to solve simple word problems. Even though they could solve most of these problems without using the equations, the purpose of the lesson is to make the student proficient in writing simple equations before moving on to more complex equations from more difficult word problems.

The last topic, in the lesson *Constant Speed*, is solving problems with distance (d), rate or velocity (v), and time (t). Students use the equivalent formulas $d = vt$ and $v = d/t$ to solve problems involving constant or average speed. They learn an easy way to remember the formula $v = d/t$ from the unit for speed that they already know, "kilometres per hour."

The Lessons in Chapter 3

Helpful Resources on the Internet

The Simplest Equations - Video Lessons by Maria
A set of free videos that teach the topics in this chapter - by the author.
http://www.mathmammoth.com/videos/prealgebra/pre-algebra-videos.php#equations

Stable Scales Quiz
In each picture, the scales are balanced. Can you find the weight of the items on the scales?
http://www.transum.org/software/SW/Starter_of_the_day/Students/Stable_Scales_Quiz.asp

Model Algebra Equations
Model an equation on a balance using algebra tiles (tiles with numbers or the unknown x). Then, solve the equation by placing −1 tiles on top of +1 tiles or vice versa. Includes one-step and two-step equations.
http://www.mathplayground.com/AlgebraEquations.html

Balance When Adding and Subtracting
Click on the buttons above the scales to add or subtract until you can determine the value of x in the equation.
http://www.mathsisfun.com/algebra/add-subtract-balance.html

One-Step Equations Quizzes
Practise one-step equations in these timed quizzes.
http://crctlessons.com/One-Step-Equations/one-step-equations.html

http://crctlessons.com/One-Step-Equation-Test/one-step-equation-test.html

Modelling with One-Step Equations
Practise writing basic equations to model real-world situations in this interactive activity from Khan Academy.
https://www.khanacademy.org/math/pre-algebra/pre-algebra-equations-expressions/pre-algebra-equation-word-problems/e/equations-in-one-variable-1

Exploring Equations E-Lab
Choose which operation to do to both sides of an equation in order to solve one-step equations.
http://www.harcourtschool.com/activity/elab2004/gr6/12.html

Algebra Four
A connect four game with equations. For this level, choose difficulty "Level 1" and "One-Step Problems".
http://www.shodor.org/interactivate/activities/AlgebraFour/

Algebra Meltdown
Solve simple equations using function machines to guide atoms through the reactor. Do not keep the scientists waiting too long or they blow their tops.
http://www.mangahigh.com/en/games/algebrameltdown

One-Step Equation Game
Choose the correct root for the given equation (multiple-choice), and then you get to attempt to shoot a basket.
http://www.math-play.com/One-Step-Equation-Game.html

Arithmagons
Find the numbers that are represented by question marks in this interactive puzzle.
http://www.transum.org/Software/SW/Starter_of_the_day/starter_August20.ASP

Distance, Speed, and Time from BBC Bitesize
Instruction, worked out exercises, and an interactive quiz relating to constant speed, time, and distance. A triangle with letters D, S, and T helps students remember the formulas for distance, speed, and time.
http://www.bbc.co.uk/bitesize/standard/maths_i/numbers/dst/revision/1/

Absorb Advanced Physics - Speed
An online tutorial that teaches the concept of average speed with the help of interactive simulations and exercises.
http://www.absorblearning.com/advancedphysics/demo/units/010101.html#Describingmotion

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Multiplication and Division Equations

Do you remember **how to show simplification**? Just cross out the numbers and write the new numerator above the fraction and the new denominator below it. Notice that the number you divide by (the 5 in the fraction at the right) isn't indicated in any way!	$\frac{\overset{7}{\cancel{35}}}{\underset{11}{\cancel{55}}} = \frac{7}{11}$
We can simplify expressions involving variables in exactly the same way. In the examples on the right, we cross out the *same number* from the numerator and the denominator. That is based on the fact that a number divided by itself is 1. We could write a little "1" beside each number that is crossed out, but that is usually omitted.	$\frac{\cancel{2}x}{\cancel{2}} = x \qquad \frac{-\cancel{5}s}{-\cancel{5}} = s$ $\frac{4\cancel{x}}{\cancel{x}} = 4$
In this example, we simplify the fraction 3/6 into 1/2 the usual way.	$\frac{\overset{1}{\cancel{3}x}}{\underset{2}{\cancel{6}}} = \frac{1}{2}x \text{ or } \frac{x}{2}$
Notice: We divide both the numerator and the denominator by 8, but this leaves −1 in the denominator. Therefore, the whole expression simplifies to $-z$ instead of z.	$\frac{\cancel{8}z}{-\cancel{8}} = \frac{z}{-1} = -z$

1. Simplify.

a. $\frac{8x}{8}$	**b.** $\frac{8x}{2}$	**c.** $\frac{2x}{8}$
d. $\frac{-6x}{-6}$	**e.** $\frac{-6x}{6}$	**f.** $\frac{6x}{-6}$
g. $\frac{6w}{2}$	**h.** $\frac{6w}{w}$	**i.** $\frac{6w}{-2}$

2. Draw the fourth and fifth steps of the pattern and answer the questions.

a. How would you describe the growth of this pattern?

b. How many flowers will there be in step 39?

c. In step n?

Now you should be ready to use multiplication and division to solve simple equations.

Example 1. Solve $-2x = 68$.

The unknown is being multiplied by −2. To isolate it, we need to divide both sides by −2.
(See the solution on the right.)

We get $x = -34$. Lastly we check the solution by substituting −34 in the place of x in the original equation:

$-2(-34) \stackrel{?}{=} 68$

$68 = 68$ It checks.

$-2x = 68$ This is the original equation.

$\frac{-2x}{-2} = \frac{68}{-2}$ We divide both sides by −2.

$\frac{\cancel{-2}x}{\cancel{-2}} = \frac{68}{-2}$ Now it is time to simplify. We cross out the −2 factors on the left side. On the right side, we do the division.

$x = -34$ This is the final answer.

Note: Most people combine the first 3 steps into one when writing the solution. They are written out here for clarity.

3. Solve. Check your solutions.

a. $5x = -45$	**b.** $-3y = -21$
c. $-4 = 4s$	**d.** $72 = -6y$

4. Solve. Simplify the one side first.

a. $-5q = -40 - 5$	**b.** $2 \cdot 36 = -6y$
c. $3x = -4 + 3 + (-2)$	**d.** $5 \cdot (-4) = -10z$

Example 2. Solve $\frac{x}{-6} = -5$.

Here the unknown is divided by −6. To undo that division, we need to *multiply* both sides by −6. (See the solution on the right.)

We get $x = 30$. Lastly we check the solution:

$$\frac{30}{-6} \stackrel{?}{=} -5$$

$$-5 = -5 \quad \checkmark$$

$\frac{x}{-6} = -5$	This is the original equation.
$\frac{x}{-6} \cdot (-6) = -5 \cdot (-6)$	We multiply both sides by −6.
$\frac{x}{\cancel{-6}} \cdot (\cancel{-6}) = 30$	Now it is time to simplify. We cross out the −6 factors on the left side, and multiply on the right.
$x = 30$	This is the final answer.

When writing the solution, most people would combine steps 2 and 3. Here both are written out for clarity.

5. Solve. Check your solutions.

a. $\frac{x}{2} = -45$	**b.** $\frac{s}{-7} = -11$
c. $\frac{c}{-7} = 4$	**d.** $\frac{a}{-13} = -9 + (-11)$

6. Write an equation for each situation. Then solve it. Do not write just the answer, as the main purpose of this exercise is to practice writing equations.

a. A submarine was located at a depth of 180 m.
There was a shark swimming at 1/6 of that depth.
At what depth is the shark?

b. Three towns equally divided the cost to repair a highway.
Each town ended up paying R21 200.
How much did the repairs cost in total?

Example 3. Solve $-\frac{1}{5}x = 2$. The unknown is multiplied by a negative fraction, but do not panic!

You see, you can *also* write this equation as $\frac{x}{-5} = 2$, where the unknown is simply divided by negative 5.

So what should we do in order to isolate x?

That is correct! Multiplying by −5 will isolate x. In the boxes below, this equation is solved in two slightly different ways, though both are doing essentially the same thing: multiplying both sides by −5.

Multiplying a fraction by its reciprocal:

$$-\frac{1}{5}x = 2 \quad \big| \cdot (-5)$$

$$(-5)\cdot\left(-\frac{1}{5}\right)x = (-5)\cdot 2$$

Note that −5 times −1/5 is 1.

$$1x = -10$$

$$x = -10$$

Cancelling a common factor:

$$-\frac{1}{5}x = 2$$ **rewrite the equation**

$$\frac{x}{-5} = 2 \quad \big| \cdot (-5)$$

$$\frac{x}{\cancel{-5}} \cdot (\cancel{-5}) = 2 \cdot (-5)$$

$$x = -10$$

Lastly we check the solution by substituting −10 in place of x in the original equation:

$$-\frac{1}{5}(-10) \stackrel{?}{=} 2$$

$$2 = 2$$

It checks.

7. Solve. Check your solutions.

a. $\frac{1}{3}x = -15$	**b.** $-\frac{1}{6}x = -20$	**c.** $-\frac{1}{4}x = 18$
d. $-2 = -\frac{1}{9}x$	**e.** $-21 = \frac{1}{8}x$	**f.** $\frac{1}{12}x = -7 + 5$

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Chapter 4: Rational Numbers
Introduction

In this chapter we study *rational* numbers, which are numbers that can be written as a *ratio* of two integers. All fractions and whole numbers are rational numbers, and so are percents and decimals (except non-ending non-repeating decimals). Students should already know a lot about rational numbers and how to calculate with them. Our focus in this chapter is to extend that knowledge to negative fractions and negative decimals.

The first lesson presents the definition of a rational number, how to convert rational numbers back and forth between their fractional and decimal forms, and a bit about repeating decimals (most fractions become repeating decimals when written as decimals). The next lesson deals with adding and subtracting rational numbers, with an emphasis on adding and subtracting negative fractions and decimals.

The next two lessons are about multiplying and dividing rational numbers. The first of the two focuses on basic multiplication and division with negative fractions and decimals. The second of the two compares multiplying and dividing in decimal notation to multiplying and dividing in fraction notation. Students come to realise that, though the calculations—and even the answers—may look very different, the answers are equal. The lesson also presents problems that mix decimals, fractions, and percents, and deals with real-life contexts for the problems and the importance of pre-estimating what a reasonable answer would be.

The lesson *Multiple Operations with Rational Numbers* revises the order of operations and applies it to fraction and decimal problems with more than one operation. It also presents a simple method to solve complex fractions, which are fractions that contain another fraction, either in the numerator, in the denominator, or in both.

After a lesson on scientific notation, the instructional portion of the chapter concludes with two lessons on solving simple equations that involve fractions and decimals.

The Lessons in Chapter 4

Rational Numbers - Video Lessons by Maria
A set of free videos that teach the topics in this book - by the author.
http://www.mathmammoth.com/videos/prealgebra/pre-algebra-videos.php#rational

RATIONAL NUMBERS

Compare Positive and Negative Decimals
Choose the correct inequality or equals sign from the drop-down box between each set of two numbers.
http://www.transum.org/software/SW/Starter_of_the_day/Students/Inequalities.asp?Level=2

Compare Rational Numbers
Practise comparing decimals, percents, fractions, and mixed numbers in this interactive exercise.
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-fractions-decimals/cc-7th-add-sub-rational-numbers/e/comparing-rational-numbers

Recurring Decimals and Fractions
Grade or No Grade where you answer multiple-choice questions about repeating decimals.
https://sites.google.com/a/revisemaths.org.uk/revise/number-files/recdecfrac-gong.swf?attredirects=0

Terminating Versus Repeating Decimals Game
A card game that practices repeating and terminating decimals. Several fun twists to score extra points! This game costs $1 (per download).
http://www.teacherspayteachers.com/Product/Terminating-VS-Repeating-Decimals-Game-425199

Terminating and Repeating Decimals Worksheet
A 10-question online quiz about repeating decimals.
http://worksheets.tutorvista.com/terminating-and-repeating-decimals-worksheet.html

Converting Repeating Decimals to Fractions
A lesson that explains the method for writing repeating decimals as fractions.
http://www.basic-mathematics.com/converting-repeating-decimals-to-fractions.html

Classifying Numbers
Drag the given numbers to the correct sets. This chapter of Math Mammoth does not teach about square roots and irrational numbers, but you can probably do these activities, if you note that most square roots are irrational, and that the set of whole numbers is {0, 1, 2, 3, 4, ...}.
http://www.softschools.com/math/classifying_numbers/
http://www.softschools.com/math/classifying_numbers/real_rational_integer_whole_natural_irrational_number_table/

Number-System Muncher
"Munch" or select all the numbers from the grid that are in the specified set. Again, this chapter of Math Mammoth does not teach about square roots and irrational numbers, but you can probably play the game, if you note the following: Finding a square root is the opposite operation of squaring. For example, $\sqrt{25} = 5$ because $5^2 = 25$. Therefore, $\sqrt{25}$ is actually a natural number (5). However, most square roots, such as $\sqrt{5}$ and $\sqrt{13}$ are irrational.
http://staff.argyll.epsb.ca/jreed/math9/strand1/munchers.htm

ADD AND SUBTRACT RATIONAL NUMBERS

Adding and Subtracting Rational Numbers Test
A 15-question test about adding, subtracting, and comparing rational numbers.
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/Documents/examviewweb/ev2-2.htm

Adding and Subtracting Rational Numbers Worksheets
Generate a worksheet for adding and subtracting negative fractions and decimals.
http://www.math-aids.com/Algebra/Algebra_1/Basics/Add_Sub_Rational.html

Add Decimals Quiz
Reinforce your decimal addition skills with this 10-question online quiz.
https://www.thatquiz.org/tq-3/?-j1i1-lk-p0

Add Fractions Quiz
Add the fractions, and express the answer as a simple fraction in lowest terms in this 10-question quiz.
https://www.thatquiz.org/tq-3/?-j1gh-la-p0

Adding and Subtracting Rational Numbers
Practise addition and subtraction of rational numbers with this interactive quiz that practises equivalent fractions, addition on the number line, and more.
http://www.buzzmath.com/Docs#CC07E11776

Adding and Subtracting Negative Fractions
Practise adding and subtracting positive and negative fractions with this interactive online activity.
https://www.khanacademy.org/math/algebra-basics/basic-alg-foundations/alg-basics-fractions/e/adding_and_subtracting_fractions

Adding and Subtracting Rational Numbers
Practise adding and subtracting negative fractions, decimals, and percents with this interactive online exercise.
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-fractions-decimals/cc-7th-add-sub-rational-numbers/e/adding_and_subtracting_rational_numbers

Add and Subtract Fractions Quiz
Add and subtract negative fractions and solve simple equations. Refresh the page to get a different set of questions.
http://www.phschool.com/webcodes10/index.cfm?fuseaction=home.gotoWebCode&wcprefix=asa&wcsuffix=0204

MULTIPLY AND DIVIDE RATIONAL NUMBERS

Multiply and Divide Fractions Quizzes
Multiple-choice quizzes of five questions. Refresh the page to get a different set of questions.
http://www.phschool.com/webcodes10/index.cfm?fuseaction=home.gotoWebCode&wcprefix=asa&wcsuffix=0205

http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-829635-8&chapter=2&lesson=4

Multiply and Divide Rational Numbers Quiz
A multiple-choice quiz of five questions.
http://www.softschools.com/quizzes/math/multiply_rational_numbers/quiz3285.html

Multiply and Divide Fractions
Practise multiplying and dividing fractions with positive numbers with this interactive exercise.
http://www.onemathematicalcat.org/algebra_book/online_problems/md_fractions.htm#exercises

Divide Positive and Negative Fractions
Practise dividing fractions. The fractions in these problems may be positive or negative.
https://www.khanacademy.org/math/algebra-basics/basic-alg-foundations/alg-basics-fractions/e/dividing_fractions_2

Multiply Decimals by Fractions - Short Quiz
Practise multiplying decimals by fractions with this multiple-choice quiz.
https://www.sophia.org/concepts/multiplying-decimals-by-fractions

Divide Fractions by Decimals - Short Quiz
Practise dividing fractions by decimals with this multiple-choice quiz.
https://www.sophia.org/concepts/dividing-fractions-by-decimals

Multiplying and Dividing Negative Numbers Word Problems
Practise matching situations to multiplication and division expressions and equations.
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-negative-numbers-multiply-and-divide/cc-7th-mult-div-neg-word-problems/e/negative-number-word-problems-1

Decimals Quiz
Reinforce your decimal skills with this interactive online quiz.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-02-833051-X&chapter=2&

Simplify Complex Fractions
Practice simplifying complex fractions with this interactive online activity from Khan Academy.
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-negative-numbers-multiply-and-divide/cc-7th-mult-div-fractions-2/e/complex-fractions

Complex Fractions Quiz
Practise simplifying complex fractions with this interactive quiz.
http://www.saddleback.edu/faculty/LPerez/Algebra2go/prealgebra/fractions/compFrac.html

SCIENTIFIC NOTATION

Scientific Notation
Interactive practice where you write the given number in scientific notation.
http://www.xpmath.com/forums/arcade.php?do=play&gameid=21

Scientific Notation Quiz
Write numbers in scientific notation, and vice versa. You can also modify the quiz parameters.
http://www.thatquiz.org/tq-c/?-j820-l6-p0

Scientific Notation Quizzes
Short, multiple-choice quizzes on scientific notation.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-02-833051-X&chapter=2&lesson=9

EQUATIONS / GENERAL

Equations Quizzes
Five-question quizzes on simple equations with decimals or fractions. Refresh the page to get different questions.
http://www.phschool.com/webcodes10/index.cfm?fuseaction=home.gotoWebCode&wcprefix=ara&wcsuffix=0404

http://www.phschool.com/webcodes10/index.cfm?fuseaction=home.gotoWebCode&wcprefix=bja&wcsuffix=0508

Fraction Four Game
Choose "algebra" as the question type to solve equations that involve fractions in this connect-the-four game.
http://www.shodor.org/interactivate/activities/FractionFour/

One-Step Equations with Fractions
This algebra 1 worksheet will produce one step problems containing fractions.
http://www.math-aids.com/Algebra/Algebra_1/Equations/One_Step_Fractions.html

One-Step Equations: Fractions and Decimals
Practise solving equations in one step by multiplying or dividing a number from both sides.
https://www.khanacademy.org/math/algebra/one-variable-linear-equations/alg1-one-step-mult-div-equations/e/one-step-mult-div-equations-2

7th Grade Numbers and Operations Jeopardy Game
The questions in this game range from absolute value to different operations with rational numbers.
http://www.math-play.com/7th-Grade-Numbers-and-Operations-Jeopardy/7th-Grade-Numbers-and-Operations-Jeopardy.html

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Multiply and Divide Rational Numbers 1

In real life we often combine **fractions, decimals, ratios, and percents**—rational numbers in different forms—in the same situation. You need to be able to easily calculate with them in their different forms.

In this lesson, we will concentrate on multiplying and dividing *decimals* and *fractions* because percentages are usually rewritten as decimals and ratios as fractions before calculating with them.

To multiply decimals

Shortcut: First multiply as if there were no decimal points. Then put the decimal point in the answer so that the number of decimal digits in the answer is the SUM of the number of the decimal digits in all the factors.

Example 1. Solve $-0{,}2 \cdot 0{,}09$.

Multiply $2 \cdot 9 = 18$. The answer will have three decimals *and* be negative (Why?), so the answer is $-0{,}018$.

Multiply fractions and mixed numbers

1. Change any mixed numbers to fractions.
2. Multiply using the shortcut (multiply the numerators; multiply the denominators).

Example 2. $-\frac{4}{5} \cdot \left(-5\frac{1}{8}\right)$

$= -\frac{4}{5} \cdot \left(-\frac{41}{8}\right)$ A negative times a negative makes a positive, so we can drop the minus signs in the next step.

$= \frac{4 \cdot 41}{5 \cdot 8} = \frac{1 \cdot 41}{5 \cdot 2} = \frac{41}{10} = 4\frac{1}{10}$

1. Write the rational numbers in their four forms.

	ratio	fraction	decimal	percent
a.	2:5	$\frac{2}{5}$	0,4	40%
b.	3:4			
c.	4:25			

	ratio	fraction	decimal	percent
d.		$\frac{7}{20}$		
e.				55%
f.			0,85	

2. Multiply these in your head.

a. $0{,}1 \cdot 6{,}5$	**b.** $-0{,}08 \cdot 0{,}006$	**c.** $-0{,}09 \cdot 0{,}02$
d. $-0{,}2 \cdot (-1{,}6)$	**e.** $-0{,}8 \cdot 1{,}1 \cdot (-0{,}02)$	**f.** $0{,}8^2$
g. $(-0{,}5)^2$	**h.** $(-0{,}2)^3$	**i.** $(-0{,}1)^5$

3. Multiply

a. $-\frac{1}{7} \cdot \left(-\frac{3}{8}\right)$	**b.** $\frac{1}{5} \cdot \left(-2\frac{1}{2}\right)$	**c.** $-\frac{2}{9} \cdot \frac{5}{6} \cdot \frac{3}{10}$
d. $-3\frac{1}{4} \cdot \frac{5}{2}$	**e.** $\frac{7}{18} \cdot \left(-\frac{12}{27}\right)$	**f.** $\frac{8}{7} \cdot \left(-\frac{3}{10}\right) \cdot 1\frac{1}{2}$

4. Multiply using the regular multiplication algorithm (write one number under the other).

a. 12,5 · 2,5	**b.** −0,088 · 0,16
c. −9,08 · (−0,006)	**d.** 24 · (−0,0087)

To divide decimals

1. If the divisor has no decimal digits, you can divide using long division "as is."
2. If the divisor does have decimal digits, multiply *both* the dividend and the divisor by the same number (usually a power of ten) to make the divisor into a whole number. Now with that whole number divisor, performing the long division has become straightforward.

Example 3. Solve 6 ÷ 0,5 without a calculator.

Since 0,5 fits into 6 exactly twelve times, the answer is 12. So mental maths was sufficient in this case.

Example 4. Solve −92,91 ÷ 0,004 without a calculator.

It may be easier to write the problem using a fraction line:

$$\frac{-92{,}91}{0{,}004} = \frac{-929{,}1}{0{,}04} = \frac{-9291}{0{,}4} = \frac{-92910}{4}$$

Notice how we multiply both the dividend and the divisor repeatedly by 10 until the divisor becomes a whole number (4). (You could, of course, simply multiply them both by 1 000 to start with.) Then we use long division.

The long division gives us the absolute value of the final answer, but we still need to apply the correct sign. So −92,91 ÷ 0,004 = −23 227,5.

Does this make sense? Yes. The answer has a very large absolute value because 0,004 is a very tiny number, and it "fits" into 92,91 multitudes of times.

```
  23227,5
4)92910,0
 -8
  12
 -12
   09
   -8
    11
    -8
     30
    -28
      20
     -20
       0
```

5. Divide in your head.

a. −0,88 ÷ 4	**b.** 8,1 ÷ 9	**c.** 72 ÷ 10000
d. −1,6 ÷ (−0,2)	**e.** 8 ÷ 0,1	**f.** 0,8 ÷ (−0,04)

6. Multiply both the dividend and the divisor by the same number so that you get a divisor that is a *whole number*. Then divide using long division. If necessary, round your answer to three decimal digits.

a. 27,6 ÷ 0,3	**b.** 2,088 ÷ 0,06

To divide fractions and mixed numbers 1. Change any mixed numbers to fractions. 2. Divide using the shortcut. (Change the division into a multiplication by the reciprocal of the divisor.)	**Example 5.** $\frac{4}{5} \div \left(-2\frac{1}{2}\right)$ $= \frac{4}{5} \div \left(-\frac{5}{2}\right)$ $= \frac{4}{5} \cdot \left(-\frac{2}{5}\right) = -\frac{8}{25}$ The answer makes sense, because 2 1/2 does not fit into 4/5, not even half-way.

7. Divide.

a. $-\frac{2}{9} \div \frac{6}{7}$	**b.** $\frac{9}{8} \div \left(-1\frac{1}{2}\right)$
c. $-10 \div \frac{5}{6}$	**d.** $-\frac{1}{9} \div \left(-\frac{1}{3}\right)$
e. $10\frac{1}{5} \div \left(-2\frac{1}{3}\right)$	**f.** $10 \div \frac{1}{6}$

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Chapter 5: Equations and Inequalities

Introduction

In this chapter we delve deeper into our study of equations. Now the equations require two or more steps to solve and may contain brackets. The variable may appear on both sides of the equation. Students will also write equations to solve simple word problems.

There is also another lesson on patterns of growth, which may simply seem to be a fascinating topic, but in reality presents the fundamentals of a very important concept in algebra—that of linear functions (although they are not mentioned by that name)—and complements the study of lines in the subsequent lessons.

After the section about equations, the text briefly presents the basics of inequalities and how to graph them on a number line. Students apply the principles for solving equations to solve simple inequalities and word problems that involve inequalities.

The last major topic is graphing. Students begin the section by learning to graph linear equations and continue on to the concept of slope, which in informal terms is a measure of the inclination of a line. More formally, slope can be defined as the ratio of the change in y-values to the change in x-values. The final lesson applies graphing to the previously-studied concepts of speed, time, and distance through graphs of the equation $d = vt$ in the coordinate plane.

The Lessons in Chapter 5

Helpful Resources on the Internet

Pre-algebra Videos
A set of videos matching this pre-algebra course, made by the author.
http://www.mathmammoth.com/videos/prealgebra/pre-algebra-videos.php

TWO-STEP EQUATIONS

Model Algebraic Equations with a Scale
Model and solve algebraic equations using a pan balance and tiles. Choose "2-Step Equations" for this level.
http://www.mathplayground.com/AlgebraEquations.html

Two-Step Equations Game
Choose the correct root for the given equation (multiple-choice), and then you get to attempt to shoot a basket. The game can be played alone or with another student.
http://www.math-play.com/Two-Step-Equations-Game.html

Solving Two-Step Equations
Type the answer to two-step-equations such as $-4y + 9 = 29$, and the computer checks it. If you choose "Practice Mode," it is not timed.
http://www.xpmath.com/forums/arcade.php?do=play&gameid=64

Two-Step Equations
Practise solving equations that take two steps to solve in this interactive exercise from Khan Academy.
https://www.khanacademy.org/math/algebra/one-variable-linear-equations/alg1-two-steps-equations-intro/e/linear_equations_2

Two-Step Equations Word Problems
Practise writing equations to model and solve real-world situations in this interactive exercise.
https://www.khanacademy.org/math/algebra-basics/alg-basics-linear-equations-and-inequalities/alg-basics-two-steps-equations-intro/e/linear-equation-world-problems-2

Visual Patterns
Hundreds of growing patterns. The site provides the answer to how many elements are in step 43 of the pattern.
http://www.visualpatterns.org/

SIMPLIFYING EXPRESSIONS

Factor the Expressions Quiz
Factor expressions. For example, $-4x + 16$ factors into $-4(x - 4)$.
http://www.thatquiz.org/tq-0/?-jh00-l4-p0

Simplifying Algebraic Expressions Practice Problems
Practise simplifying expressions such as $4(2p - 1) - (p + 5)$ with these 10 questions. Answer key included.
http://www.algebra-class.com/algebraic-expressions.html

Distributive Property with Negative Numbers
Use the distributive property to remove the parentheses in this interactive exercise. Click to see an example.
http://www.hstutorials.net/dialup/distributiveProp.htm

Simplifying Algebraic Expressions (1)
Eight practise problems that you can check yourself about combining like terms and using the distributive property.
http://www.algebralab.org/lessons/lesson.aspx?file=Algebra_BasicOpsSimplifying.xml

Simplifying Algebraic Expressions (2)
You can check this five-question quiz from Glencoe yourself.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-825200-8&chapter=3&lesson=2&&headerFile=4

MORE EQUATIONS

Balance When Adding and Subtracting Game
An interactive balance where you add or subtract *x*'s and 1's until you leave *x* alone on one side.
http://www.mathsisfun.com/algebra/add-subtract-balance.html

Solve Equations Quiz
A 10-question online quiz where you need to solve equations with an unknown on both sides.
http://www.thatquiz.org/tq-0/?-j102-l4-p0

Equations Level 3 Online Exercise
Practise solving equations with an unknown on both sides in this self-check online exercise.
http://www.transum.org/software/SW/Starter_of_the_day/Students/Equations.asp?Level=3

Missing Lengths
Try to figure out the value of the letters used to represent the missing numbers.
http://www.transum.org/software/SW/Starter_of_the_day/Students/Missing_Lengths.asp

Equations Level 4 Online Exercise
Practise solving equations which include brackets in this self-check online exercise.
http://www.transum.org/software/SW/Starter_of_the_day/Students/Equations.asp?Level=4

Equations Level 5 Online Exercise
This exercise includes more complex equations requiring multiple steps to find the solution.
http://www.transum.org/software/SW/Starter_of_the_day/Students/Equations.asp?Level=5

Solving Equations Quizzes
Here are some short online quizzes that you can check yourself.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-825200-8&chapter=3&lesson=5&&headerFile=4

http://www.phschool.com/webcodes10/index.cfm?wcprefix=bja&wcsuffix=0701

Rags to Riches Equations
Choose the correct root to a linear equation.
http://www.quia.com/rr/4096.html

Algebra Four Game
To practise the types of equations we study in this chapter, choose "Level 1," and tick the boxes "Variable on both sides," "Distributive Property," and "Two-Step Problems" (do not check "Quadratic Equations").
http://www.shodor.org/interactivate/activities/AlgebraFour/

Solve Equations Exercises
Click "new problem" (down the page) to get a randomly generated equation to solve. This exercise includes an optional graph which the student can use as a visual aid.
http://www.onemathematicalcat.org/algebra_book/online_problems/solve_lin_int.htm#exercises

Equation Word Problems Quiz
Solve word problems which involve equations and inequalities in this multiple-choice online quiz.
http://www.phschool.com/webcodes10/index.cfm?wcprefix=bja&wcsuffix=0704

Whimsical Windows - Equation Game
Write an equation for the relationship between *x* and *y* based on a table of *x* and *y* values. Will you discover the long lost black unicorn stallion?
http://mrnussbaum.com/whimsical-windows/

INEQUALITIES

Inequality Quiz
A 10-question multiple choice quiz on linear inequalities (like the ones studied in this chapter).
http://www.mrmaisonet.com/index.php?/Inequality-Quiz/Inequality-Quiz.html

Inequalities
Here is another five-question quiz from Glencoe that you can check yourself.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-825200-8&chapter=7&lesson=3&&headerFile=4

Plot Simple Inequalities
Practise plotting simple inequalities on a number line in this 10-question interactive quiz.
https://www.thatquiz.org/tq-o/?-j18-l1-p0

Match Inequalities and Their Plots
Match the statements with the corresponding diagrams in this interactive online activity.
http://www.transum.org/software/SW/Starter_of_the_day/Students/InequalitiesB.asp?Level=5

Solve Simple Inequalities
For each inequality, find the range of values for x which makes the statement true. An example is given.
http://www.transum.org/software/SW/Starter_of_the_day/Students/InequalitiesC.asp?Level=6

Two-Step Inequality Word Problems
Practise constructing, interpreting, and solving linear inequalities that model real-world situations.
https://www.khanacademy.org/math/algebra/one-variable-linear-inequalities/alg1-two-step-inequalities/e/interpretting-solving-linear-inequalities

SPEED, TIME, AND DISTANCE

Understanding Distance, Speed, and Time
An interactive simulation of two runners. You set their starting points and speeds, and observe their positions as the tool runs the simulation. It graphs the position of both runners in relation to time.
http://illuminations.nctm.org/Activity.aspx?id=6378

Representing Motion
A tutorial an interactive quiz with various questions about speed, time, and distance.
http://www.bbc.co.uk/schools/gcsebitesize/science/add_aqa_pre_2011/forces/represmotionrev1.shtml

Absorb Advanced Physics - Speed
An online tutorial that teaches the concept of average speed with the help of interactive simulations and exercises.
http://www.absorblearning.com/advancedphysics/demo/units/010101.html#Describingmotion

Distance-Time Graphs
An illustrated tutorial about distance-time graphs. Multiple-choice questions are included.
http://www.absorblearning.com/advancedphysics/demo/units/010103.html

Distance-Time Graph
Click the play button to see a distance-time graph for a vehicle which moves, stops, and then changes direction.
http://www.bbc.co.uk/schools/gcsebitesize/science/add_aqa_pre_2011/forces/represmotionrev5.shtml

Distance Versus Time Graph Puzzles
Try to move the stick man along a number line in such a way as to illustrate the graph that is shown.
http://davidwees.com/graphgame/

GRAPHING AND SLOPE

Graph Linear Equations
A ten-question online quiz where you click on three points on the coordinate grid to graph the given equation.
http://www.thatquiz.org/tq-0/?-j10g-l4-p0

Find the Slope
A ten-question online quiz that asks for the slope of the given line.
http://www.thatquiz.org/tq-0/?-j300-l4-p0

Slope Slider
Use the sliders to change the slope and the *y*-intercept of a linear equation to see what effect they have on the graph of the line.
http://www.shodor.org/interactivate/activities/SlopeSlider/

Graphing Equations Match
Match the given equations to their corresponding graphs.
http://www.math.com/school/subject2/practice/S2U4L3/S2U4L3Pract.html

Find Slope from the Graph
Find the slope of a line on the coordinate plane in this interactive online activity.
https://www.khanacademy.org/math/algebra-basics/alg-basics-graphing-lines-and-slope/alg-basics-slope/e/slope-from-a-graph

Slope - Exercises
Practise finding the slope in this interactive online exercise.
http://www.onemathematicalcat.org/algebra_book/online_problems/compute_slope.htm#exercises

Graphs Quiz
Check your knowledge of graphing with this interactive self-check quiz.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-860467-2&chapter=5&lesson=2&headerFile=4

Equations and Graphing Quiz
Practise linear equations and functions in this interactive online test.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-829633-1&chapter=4&headerFile=4

GENERAL

Algebra Quizzes
A variety of online algebra quizzes from MrMaisonet.com.
http://www.mrmaisonet.com/index.php?/Algebra-Quizzes/

Pre-algebra Quizzes
Reinforce the concepts studied in this chapter with these interactive online quizzes.
http://www.phschool.com/webcodes10/index.cfm?fuseaction=home.gotoWebCode&wcprefix=bjk&wcsuffix=0099

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A Variable on Both Sides

Example 1. Solve $2x + 8 = -5x$.

Notice that the unknown appears on both sides of the equation. To isolate it, we need to

- either subtract $2x$ from both sides—because that makes $2x$ disappear from the left side
- or add $5x$ to both sides—because that makes $-5x$ disappear from the right side.

$$\begin{aligned} 2x + 8 &= -5x && \mid + 5x \\ 2x + 8 + 5x &= 0 && \text{(now add } 2x \text{ and } 5x \text{ on the left side)} \\ 7x + 8 &= 0 && \mid - 8 \\ 7x &= -8 && \mid \div 7 \\ x &= -8/7 \end{aligned}$$

Check:

$$\begin{aligned} 2 \cdot (-8/7) + 8 &\stackrel{?}{=} -5 \cdot (-8/7) \\ -16/7 + 8 &\stackrel{?}{=} 40/7 \\ -2\ 2/7 + 8 &= 5\ 5/7 \quad \checkmark \end{aligned}$$

Example 2. Solve $10 - 2s = 4s + 9$.

To isolate s, we need to

- either add $2s$ to both sides
- or subtract $4s$ from both sides.

The choice is yours. Personally, I like to keep the unknown on the left side and eliminate it from the right.

$$\begin{aligned} 10 - 2s &= 4s + 9 && \mid - 4s \\ 10 - 2s - 4s &= 9 && \text{(now simplify } -2s - 4s \text{ on the left side)} \\ 10 - 6s &= 9 && \mid - 10 \\ -6s &= -1 && \mid \div (-6) \\ s &= 1/6 \end{aligned}$$

Check:

$$\begin{aligned} 10 - 2 \cdot (1/6) &\stackrel{?}{=} 4 \cdot (1/6) + 9 \\ 10 - 2/6 &\stackrel{?}{=} 4/6 + 9 \\ 9\ 4/6 &= 9\ 4/6 \quad \checkmark \end{aligned}$$

1. Solve. Check your solutions (as always!).

a. $3x + 2 = 2x - 7$

b. $9y - 2 = 7y + 5$

2. Solve. Check your solutions (as always!).

a. $11 - 2q = 7 - 5q$	**b.** $6z - 5 = 9 - 2z$
c. $8x - 12 = -1 - 3x$	**d.** $-2y - 6 = 20 + 6y$
e. $6w - 6{,}5 = 2w - 1$	**f.** $5g - 5 = -20 - 2g$

Combining like terms

Remember, in algebra, a *term* is an expression that consists of numbers, fractions, and/or variables that are multiplied. This means that the expression $-2y + 7 + 8y$ has three terms, separated by the plus signs.

In the expression $-2y + 7 + 8y$, the terms $-2y$ and $8y$ are called **like terms** because they have the same variable part (in this case a single y). We can **combine** (add or subtract) like terms.

To do that, it helps to organize the terms in the expression in alphabetical order according to the variable part and write the constant terms last. We get $-2y + 8y + 7$ ($8y - 2y + 7$ is correct, too).

Next, we add $-2y + 8y$ and get $6y$. So the expression $-2y + 7 + 8y$ simplifies to $6y + 7$.

Example 3. Simplify $6y - 8 - 9y + 2 - 7y$.

First, we organize the expression so that the terms with y are written first, followed by the constant terms.

For that purpose, we **view each operation symbol** (+ or −) **in front of the term as the *sign* of each term.** In a sense, you can imagine each plus or minus symbol as being "glued" to the term that follows it. Of course the first term, $6y$, gets a "+" sign.

$$+6y - 8 - 9y + 2 - 7y$$

Why can we do it this way?

Because subtracting a term is the same as adding its opposite. In symbols,

$$\begin{aligned} & 6y \quad -8 \quad -9y \quad +2 \quad -7y \\ = \ & 6y + (-8) + (-9y) + 2 + (-7y). \end{aligned}$$

In other words, the expression $6y - 8 - 9y + 2 - 7y$ is the SUM of the terms $6y$, -8, $-9y$, 2, and $-7y$.

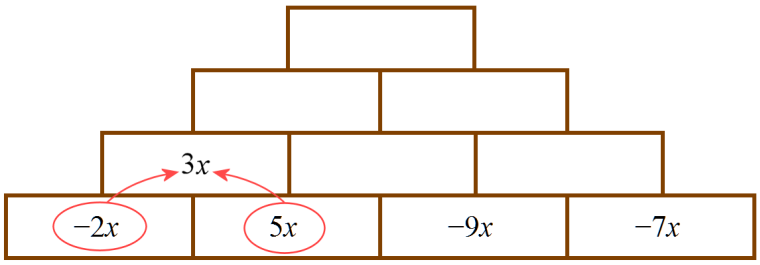
After reordering the terms, the expression becomes $6y - 9y - 7y - 8 + 2$.

Now we need to combine the like terms $6y$, $-9y$, and $-7y$. We do that by finding the sum of their coefficients 6, −9, and −7. Since $6 - 9 - 7 = -10$, we know that $6y - 9y - 7y = -10y$.

Similarly, we combine the two constant terms: $-8 + 2 = -6$.

Our expression simplifies to $-10y - 6$.

3. Fill in the pyramid! Add each pair of terms in neighbouring blocks and write its sum in the block above it.



4. Organize the expressions so that the variable terms are written first, followed by constant terms.

a. $6 + 2x - 3x - 7 + 11$

b. $-s - 12 + 15s + 9 - 7s$

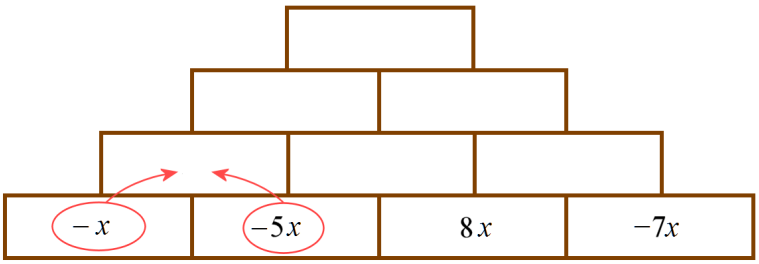
c. $-8 + 5t - 2 - 6t$

5. Simplify the expressions in the previous exercise.

6. Simplify.

a. $5x - 8 - 7x + 1$	**b.** $-6a - 15a + 9a + 7a$
c. $-8 + 7c - 11c + 8 - c$	**d.** $10 - 5x - 8x - 9 + x$

7. Fill in the pyramid! Add each pair of terms in neighbouring blocks and write its sum in the block above it.



8. Find what is missing from the sums.

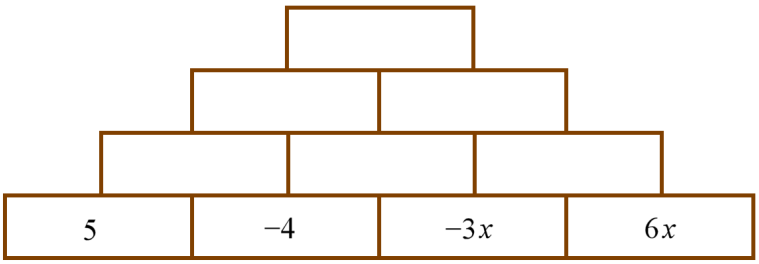
a. $8x + 2 +$ _____ $= 5x + 8$

b. $5b - 2 +$ _____ $= 2b + 7$

c. $-2z +$ _____ $= 1 - 5z$

d. $-4f + 3 +$ _____ $= -f - 1$

9. Fill in the pyramid! Add each pair of terms in neighbouring blocks and write its sum in the block above it.



10. Simplify.

a. $0{,}5y + 1{,}2y - 0{,}6y$	**b.** $-1{,}6v - 1 - v$	**c.** $-0{,}8k + 3 + 0{,}9k$

11. A challenge! Solve the equation $(-1/2)x - 6 + 8x + 7 - x = 0$.

Example 4. One or both sides of an equation may have several terms with the unknown. In that case, we need to combine the like terms (simplify) before continuing with the actual solution.

$$
\begin{aligned}
3x + 7 - 5x &= 6x + 1 - 5x && \text{On the left side, combine } 3x \text{ and } -5x. \\
& && \text{On the right side, combine } 6x \text{ and } -5x. \\
-2x + 7 &= x + 1 && \mathbf{- x} \\
-3x + 7 &= 1 && \mathbf{- 7} \\
-3x &= -6 && \mathbf{\cdot (-1)} \\
3x &= 6 && \mathbf{\div 3} \\
x &= 2
\end{aligned}
$$

12. Solve. Check your solutions.

a. $6x + 3x + 1 = 9x - 2x - 7$

b. $16y - 4y - 3 = -4y - y$

c. $-26x + 12x = -18x + 8x - 6$

d. $-9h + 4h + 7 = -2 + 5h + 9h + 8h$

13. Solve. Check your solutions.

a. $2x - 4 - 7x = -8x + 5 + 2x$	**b.** $-6 - 4z - 3z = 5z + 8 - z$
c. $8 - 2m + 5m - 8m = 20 - m + 5m - 2m$	**d.** $-x - x + 2x = 5 - 5x + 9x$
e. $-q + 2q - 5q - 6q = 20 - 7 - 9 + q$	**f.** $9 - s + 7 - 9s = 2 - 2s - 11$

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math MAMMOTH

Grade 7-B Worktext
South African Version

Light Blue Series

- **R**atios and proportions
- **P**ercent
- **G**eometry
- **P**ythagorean theorem
- **P**robability
- **S**tatistics

By Maria Miller

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EDITION 1/2017

Contents

Chapter 8: Geometry

Chapter 9: Pythagorean Theorem

Chapter 10: Probability

Chapter 11: Statistics

Foreword

Math Mammoth South African version has been customised to South Africa in the following manners:

- The names used are South African names (instead of Jack and Jill, there are Ansie and Jali).
- The currency used in word problems is rand and cents.
- The material is all metric. In other words, the US customary measuring units are not used.
- Spelling is British English instead of American English.
- Paper size is A4.

Please note that the curriculum is not following the South African official syllabus for seventh grade maths. Instead, it is a copy of the US version of Math Mammoth Grade 7, aligned to the US Common Core Standards. This decision was made because of the great amount of work that would be involved in writing new lessons and reorganizing old ones to match all the standards in the South African syllabus. For the most part, Math Mammoth is exceeding South African standards.

Part B starts out with a study of ratios and proportions (chapter 6). Students study unit rates, proportions, proportional relationships and graphing, scaling geometric figures, floor plans, and maps

The next chapter provides thorough lessons on the concept of percent. Students learn to solve a wide variety of problems involving percentages, including percentage of change, percentages of comparison, and simple interest problems.

Geometry is our focus in chapter 8. Students draw geometric figures using a a protractor and a ruler, and they also learn some basic geometric constructions. The other themes of this chapter are various angle relationships, area and the perimeter of a circle, conversions between units of area and of volume, surface area, volume, and cross-sections when solids are sliced with a plane.

Chapter 9 covers square roots, the Pythagorean Theorem, and its applications. I have included the Pythagorean Theorem in order to make the curriculum work as a pre-algebra course, and you can omit the entire chapter 9 if you are following the Common Core Standards for grade 7.

Chapter 10 is an introduction to probability. Besides learning the basic idea behind probability as the ratio of favourable events to all possible events, students compare experimental probabilities to theoretical ones in probability simulations and even design some on their own.

Lastly, in chapter 11, the curriculum covers statistical concepts. The major areas of study are random sampling and learning to compare two populations using some basic statistical measures and graphs.

Part 7-A covers an introduction to basic concepts of algebra, integers, one-step equations, fractions and decimals, and linear equations.

I wish you success in teaching maths!

Maria Miller, the author

Chapter 6: Ratios and Proportions
Introduction

Chapter 6 revises the concept, which has already been presented in previous grades, of the ratio of two quantities. From this concept, we develop the related concepts of a rate (so much of one thing per so much of another thing) and a proportion (an equation of ratios). We also study how tables of equivalent ratios can help to solve problems with rates, and how cross-multiplying can help to solve problems with proportions.

The lesson *Unit Rates* defines the concept of the unit rate, shows how to calculate one, and gives practice at doing so, including practise with complex fractions. We also consider rates as two quantities that vary, graph the corresponding equation in the coordinate grid, and tie in the concept of unit rate with the concept of slope.

The concept of direct variation is introduced in the lesson *Proportional Relationships*. Writing and graphing equations gives a visual understanding of proportionality. In two following lessons on proportions, students also practise solving rate problems in different ways, using the various methods they have learned throughout the chapter.

The lessons *Scaling Figures, Floor Plans*, *and Maps* give useful applications and more practice to master the concepts of proportions.

Before the *Chapter 6 Revision* there is also an optional lesson, *Significant Digits*, which deals with the concept of the accuracy of a measurement and how it limits the accuracy of the solution. It is optional because significant digits is not a standard topic for seventh grade, yet the concept in it is quite important, especially in science.

The Lessons in Chapter 6

Helpful Resources on the Internet

Ratios and Proportions—Video Lessons by Maria
A set of free videos about ratios, rates, and proportions—by the author.
http://www.mathmammoth.com/videos/proportions.php

RATIOS AND RATES

Ratio Pairs Matching Game
Match cards representing equivalent ratios.
Easy: **http://nrich.maths.org/4824** Challenge: **http://nrich.maths.org/4821**

All About Ratios - Quiz
An interactive five-question quiz about equivalent ratios presented with pictures
http://math.rice.edu/~lanius/proportions/quiz1.html

Rate Module from BrainingCamp
A comprehensive interactive lesson on the concepts of ratio, rate, and constant speed (for 6th and 7th grades). Includes an animated lesson, a virtual manipulative, and questions and problems to solve.
http://www.brainingcamp.com/lessons/rates/

Ratios Activity from BBC Bitesize
An animated tutorial about dividing in a given ratio and scale models with some quiz questions along the way.
http://www.bbc.co.uk/education/guides/znnycdm/activity

Self-Check Quiz from Glencoe
A five-question multiple-choice quiz about comparing with ratios and rates. By reloading the page you will get different questions.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=1-57039-855-0&chapter=8&lesson=1

Ratio Quiz from BBC Skillswise
A multiple-choice quiz about the concept of ratio. You can take the quiz online or download it as a PDF or doc file.
http://www.bbc.co.uk/skillswise/quiz/ma19rati-e1and2-quiz

Ratio Quiz from Syvum
A 10-question online quiz about ratios and problem solving.
http://www.syvum.com/cgi/online/mult.cgi/gmat/math_review/arithmetic_5.tdf?0

Unit Rates Involving Fractions
Practise computing rates associated with ratios of fractions or decimals in this interactive activity.
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-ratio-proportion/cc-7th-rates/e/rate_problems_1

Ratio Word Problems
Reinforce your ratios skills with these interactive word problems.
https://www.khanacademy.org/math/pre-algebra/pre-algebra-ratios-rates/pre-algebra-ratio-word-problems/e/ratio_word_problems

Comparing Rates
Practise completing rate charts in this interactive online activity.
https://www.khanacademy.org/math/pre-algebra/pre-algebra-ratios-rates/pre-algebra-rates/e/comparing-rates

Free Ride
An interactive activity about bicycle gear ratios. Choose the front and back gears, which determines the gear ratio. Then choose a route, pedal forward, and make sure you land exactly on the five flags.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=178

Exploring Rate, Ratio and Proportion (Video Interactive)
The video portion of this resource illustrates how these maths concepts play a role in photography. The interactive component allows students to explore ratio equivalencies by enlarging and reducing images.
http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.RATE

Three-Term Ratios
Practise the equivalency of ratios by filling in the missing numbers in three-term ratios (for example, 2:7:5 = __ : 105 : ___) where the numbers represent the amounts of three colours in different photographs. Afterwards you get to assemble a puzzle from the nine photographs.
http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.RATE&lesson=html/object_interactives/3_term_ratio/use_it.html

If the two links above do not work, use this link:
http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB
First choose Rate/Ratio/Proportion, and then either *Exploring Rate, Ratio, and Proportion* or *3-Term Ratios*.

PROPORTIONS

Ratios and Proportions
A tutorial with interactive practice exercises about ratios and proportions.
https://www.wisc-online.com/learn/formal-science/mathematics/gem2004/ratios-and-proportions

Solving Proportions Practice
In this interactive practice, you can choose to show a hint "vertically", "horizontally", or algebraically.
http://www.xpmath.com/forums/arcade.php?do=play&gameid=97

Solving Proportions
Practise solving basic proportions with this interactive exercise from Khan Academy.
https://www.khanacademy.org/exercise/proportions_1

Proportions Quiz
Use this multiple-choice self-check quiz to test your knowledge about proportions.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-829633-1&chapter=7&lesson=3&headerFile=4

Proportions: Short Quiz
This short multiple-choice quiz reinforces basic proportions skills.
http://www.phschool.com/webcodes10/index.cfm?wcprefix=bja&wcsuffix=0602&area=view

Challenge Board
Choose questions from the challenge board about rates, ratios, and proportions.
http://www.quia.com/cb/158527.html
http://www.quia.com/cb/101022.html

Write a Proportion for a Problem
Practise writing proportions to describe real-world situations in this interactive exercise.
https://www.khanacademy.org/math/pre-algebra/pre-algebra-ratios-rates/pre-algebra-write-and-solve-proportions/e/writing_proportions

Proportion Word Problems
Practise setting up and solving proportions to solve word problems in this interactive online activity.
https://www.khanacademy.org/math/pre-algebra/pre-algebra-ratios-rates/pre-algebra-write-and-solve-proportions/e/constructing-proportions-to-solve-application-problems

Rags to Riches—Proportions
Solve proportions and advance towards more and more difficult questions.
http://www.quia.com/rr/35025.html

How Much Is a Million?
This is a lesson plan for a hands-on activity where students count grains of rice in a cup, weigh that amount of rice, and then build a proportion to figure out the weight of 1 million grains of rice.
http://illuminations.nctm.org/Lesson.aspx?id=2674

PROPORTIONAL RELATIONSHIPS

Proportional Relationships and Graphs
Practise plotting points on a graph and reading graphs in this interactive online activity.
http://www.buzzmath.com/Docs#CC07E11805

Proportional Relationships Activity
Answer true or false questions, practise reading charts, and more in this interactive online activity.
http://www.buzzmath.com/Docs#CC07E11806

Proportional Relationships
Practise telling whether or not the relationship between two quantities is proportional by reasoning about equivalent ratios.
https://www.khanacademy.org/math/pre-algebra/pre-algebra-ratios-rates/pre-algebra-proportional-rel/e/analyzing-and-identifying-proportional-relationships

Proportional Relationships: Graphs
Practise telling whether or not the relationship between two quantities is proportional by looking at a graph of the relationship.
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-ratio-proportion/cc-7th-graphs-proportions/e/analyzing-and-identifying-proportional-relationships-2

Interpreting Graphs of Proportional Relationships
Practise reading and analyzing graphs of proportional relationships in this interactive online exercise.
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-ratio-proportion/cc-7th-equations-of-proportional-relationships/e/interpreting-graphs-of-proportional-relationships

Writing Proportional Equations
Practise writing equations to describe proportional relationships in this interactive online activity.
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-ratio-proportion/cc-7th-equations-of-proportional-relationships/e/writing-proportional-equations

SCALE DRAWINGS AND MAPS

Similar Figures Activity
Practise finding the missing length, answer questions about proportions, and more with this interactive activity.
http://www.buzzmath.com/Docs#F6KME681

Similar Shapes Exercises
Answer questions about the scale factors of lengths, areas, and volumes of similar shapes.
http://www.transum.org/Maths/Activity/Similar/

Scale Drawings Exercise
Measure line segments and angles in geometric figures, including interpreting scale drawings.
http://www.transum.org/software/Online_Exercise/ScaleDrawing/

Scale Drawings Quizzes
Interactive self-check quizzes about scale drawings. By reloading the page you will get different questions.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-860467-2&chapter=4&lesson=1&headerFile=4

http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-02-833050-1&chapter=8&lesson=3

Ratio and Scale
An online unit about scale models, scale factors, and maps with interactive exercises and animations.
http://www.absorblearning.com/mathematics/demo/units/KCA024.html

Use Proportions to Solve Problems Involving Scale Drawings
A set of word problems. You can choose how they are presented: as flashcards, as a quiz where you match questions and answers, as a multiple choice quiz, or a true/false quiz. You can also play a game (Jewels) .
http://www.cram.com/flashcards/use-proportions-to-solve-problems-involving-scale-drawings-3453121

Scale Drawings - Problem Solving and Constructing Scale Drawings Using Various Scales
A comprehensive lesson with several completed examples concerning scale drawings.
http://www.ck12.org/user:c2ZveDJAb3N3ZWdvLm9yZw../book/Oswego-City-School-District---Grade-7-Common-Core/section/12.0/

Constructing Scale Drawings
Practise making scale drawings on an interactive grid. The system includes hints and the ability to check answers.
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-geometry/cc-7th-scale-drawings/e/constructing-scale-drawings

Interpreting Scale Drawings
Solve word problems involving scale drawings in an online practice environment.
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-geometry/cc-7th-scale-drawings/e/interpreting-scale-drawings

Scale Drawings and Maps Quiz
Answer questions about scales on maps and scale drawings in these five self-check word problems.
http://www.math6.org/ratios/8.6_quiz.htm

Maps
A tutorial with completed examples and interactive exercises about how to calculate distances on the map or in real life based on the map's scale.
http://www.cimt.org.uk/projects/mepres/book7/bk7i19/bk7_19i3.htm

Short Quiz on Maps
Practise map-related concepts in this multiple-choice quiz.
http://www.proprofs.com/quiz-school/story.php?title=map-scales

SIGNIFICANT DIGITS

Sig Fig Rules
Drag Sig J. Fig to cover each significant digit in the given number.
http://www.sigfig.dreamhosters.com/

Practice with Significant Figures
A multiple-choice quiz that also reminds you of the rules for significant digits.
http://www.chemistrywithmsdana.org/wp-content/uploads/2012/07/SigFig.html

Significant digits quiz
A 10-question multiple-choice quiz about significant digits.
http://www.quia.com/quiz/114241.html?AP_rand=1260486279

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Unit Rates

Remember that a rate is a ratio where the two terms have different units, such as 2 kg/R45 and 600 km/5 h.

In a **unit rate, the second term of the rate is one** (of some unit).

For example, 80 km/1 h and R14,95/1 kg are unit rates. The number "1" is nearly always omitted so those rates are usually written as 80 km/h and R14,95/kg.

To convert a rate into an equivalent unit rate simply divide the numbers in the rate.

Example 1. Mark can ride his bike 35 km in 1 ½ hours. What is the unit rate?

To find the unit rate, we use the principles of division by fractions to divide 35 km by 1 ½ h. The units "km" and "hours" are divided, too, and become "km per hour" or "km/hour."

$$\frac{35 \text{ km}}{1\ 1/2 \text{ h}} = 35 \div \frac{3}{2} \text{ km/h} = 35 \times \frac{2}{3} \text{ km/h}$$

$$= \frac{70}{3} \text{ km/h} = 23\ ⅓ \text{ km/h}.$$

We could also use decimal division:
35 km ÷ 1,5 h = 23,333... km/h.

So the unit rate is 23 ⅓ km per hour.

Example 2. A snail can slide through the mud 50 cm in 6 minutes. What is the unit rate?

Here, it is actually not clear whether we should give the unit rate as cm/min or cm/hr. Let's do both.

(1) To get the unit rate in cm/min, we simply divide 50 cm ÷ 6 min. We get the fraction 50/6. We also divide the units to get "cm/min." So we get

50 cm ÷ 6 min = 50/6 cm/min = 8 1/3 cm/min

= 8,3 cm/min.

(2) For centimetres per hour, we multiply both terms of the rate by 10 to get an equivalent rate of 500 cm in 60 minutes, which is 5 m in 1 hour.

1. Find the unit rate.

a. R125 for 5 packages

b. R300 for 500 envelopes

c. R33,70 for ½ hour

d. 5 cm per 4 minutes

e. 6 m^2 per 3/4 L

2. A person is walking 3/4 km every 1/4 hour. Choose the correct fraction for the unit rate and simplify it.

$$\frac{\frac{1}{4}}{\frac{3}{4}} \text{ km per hour} \quad \text{or} \quad \frac{\frac{3}{4}}{\frac{1}{4}} \text{ km per hour}$$

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Chapter 7: Percent
Introduction

In this chapter we revise the concept of percent as "per hundred" or as hundredth parts and how to convert between fractions, decimals, and percents. *Solving Basic Percentage Problems* is another lesson intended for revision of sixth grade topics, focusing on finding a known percentage of a number (such as 21% of 56) or finding a percentage when you know the part and the total.

We take a little different perspective of these concepts in the lesson *Percent Equations*. Students write simple equations for situations where a price increases or decreases (discounts). This lesson also explains what a percent proportion is. Personally, I prefer *not* to use percent proportion but to write the percentage as a decimal and then write an equation. I feel that approach adapts better to solving complex problems than using percent proportion.

Here is a quick example to show the difference between the two methods. Let's say an item is discounted by 22% and it now costs R28. Then, the new price is 78% of the original. If we let p be the price of the item before the discount, we can write the percent proportion $R28/p = 78/100$ and solve for p. If, we write the percentage 78% as the decimal 0,78, we get the equation $0{,}78p = R28$. Personally, I consider percent proportion to be an optional topic, and the reason I have included it here is to make this curriculum fully meet the Common Core Standards for seventh grade.

The lesson *Circle Graphs* provides students a break from new concepts and allows them to apply the concept of percent in a somewhat familiar context. Next, we delve into the percentage of change. Students sometimes view the percentage of change as a totally different concept as compared to other percentage topics, but it is not that at all. To calculate the percentage of change, we still use the fundamental idea of *percentage = part/total*, only this time, the "part" is how much the quantity in question changes (the difference) and the "total" is the original quantity.

Tying in with percentage of change, students also learn to compare values using percentages, such as how many percent more or less one thing is than another. Once again, this is not really a new concept but is based on the familiar formula *percentage = part/total*. The percentage difference (or relative difference) is the fraction (*actual difference*)/(*reference value*).

Simple Interest is a lesson on the important topic of interest, using as a context both loans and savings accounts. Students learn to use the formula $I = prt$ in a great variety of problems and situations.

The text concludes with a revision lesson of all of the concepts taught in the chapter.

The Lessons in Chapter 7

Helpful Resources on the Internet

Percent videos by Maria
Videos on percent-related topics that match the lessons in this chapter.
http://www.mathmammoth.com/videos/prealgebra/pre-algebra-videos.php#percent

Percent worksheets
Create an unlimited number of free customizable percent worksheets to print.
http://www.homeschoolmath.net/worksheets/percent-decimal.php
http://www.homeschoolmath.net/worksheets/percent-of-number.php
http://www.homeschoolmath.net/worksheets/percentages-words.php

PERCENTAGES, FRACTIONS, AND DECIMALS

Mission: Magnetite
Hacker tries to drop magnetite on Motherboard. To unlock a code to stop him, match up percentages, fractions, and images showing fractional parts in five different sets of items.
http://pbskids.org/cyberchase/media/games/percent/index.html

Decention Game
Build teams of three players by matching fractions, decimals, and percentages with the same value.
http://www.mathplayground.com/Decention/Decention.html

Fractions and Percent Matching Game
A simple matching game: match fractions and percentages.
http://www.mathplayground.com/matching_fraction_percent.html

What percentage is shaded?
Practise guessing what percentage of the pie chart has been shaded yellow in this interactive activity.
http://www.interactivestuff.org/sums4fun/pietest.html

Pie Chart and Questions
First, read a short illustrated lesson about pie charts. Then, click on the questions at the bottom of the page to practise.
https://www.mathsisfun.com/data/pie-charts.html

Flower Power
Grow flowers and harvest them to make money in this addictive order-'em-up game. Practise ordering decimals, fractions, and percentages.
https://www.mangahigh.com/en/games/flowerpower

Matching Fractions, Decimals, and Percentages
A simple matching memory game.
http://nrich.maths.org/1249

Sophie's Dominoes
Order dominoes that contain either numbers or a percentage of a number (such as 15% of 300).
http://www.bsquaredfutures.com/pluginfile.php/212/mod_resource/content/1/doms.swf

Percent Goodies: Fraction-Decimal-Percent Conversions
Practise conversions between fractions, decimals and percents. There are three levels of difficulty and instant scoring for each. Note that fractions must be written in lowest terms.
http://www.mathgoodies.com/games/conversions/

BASIC PERCENTAGE CALCULATIONS

Penguin Waiter
A simple game where you calculate the correct tip to leave the waiter (levels "easy" and "medium"), the percentage that the given tip is (level "hard"), or the original bill (level "Super Brain").
http://www.funbrain.com/penguin/

Percent Jeopardy
An interactive jeopardy game where the questions have to do with a percentage of a quantity.
http://www.quia.com/cb/42534.html

Matching Percentage of a Number
Match cards that ask for a percentage of a number (such as 75% of 40) with the values. The game is fairly easy and can be completed using mental maths.
http://www.sheppardsoftware.com/mathgames/percentage/MatchingPercentNumber.htm

Discount Doors
Calculate the price after the discount.
http://www.bsquaredfutures.com/pluginfile.php/214/mod_resource/content/1/doors.swf

The Percentage Game
This is a printable board game for 2-3 players that practises questions such as
"20 percent of ___ is 18" or "___ is 40 percent of 45".
http://nzmaths.co.nz/resource/percentage-game

A Conceptual Model for Solving Percent Problems
A lesson plan that uses a 10 x 10 grid to explain the basic concept of percent and to solve various types of percentage problems. The lesson includes seven different word problems to solve. Please note their solutions are included on the same page.
http://illuminations.nctm.org/LessonDetail.aspx?id=L249

PERCENT OF CHANGE

Percent of Change Matching
Match five flashcards with given increases or decreases (such as "25 is decreased to 18") with five percentages of increase/decrease.
https://www.studystack.com/matching-182854

Percent Shopping
Choose toys to purchase. In level 1, you find the sale price when the original price and percent discount are known. In level 2, you find the percent discount (percent of change) when the original price and the sale price are known.
http://www.mathplayground.com/percent_shopping.html

Rags to Riches: Percent Increase or Decrease
Answer simple questions about percent increase or decrease and see if you can win the grand prize in the game.
http://www.quia.com/rr/230204.html

Percentage Change 1
A self-marking quiz with 10 questions about percentage change. The link below goes to level 1 quiz, and at the bottom of that page you will find links to level 2, 3, 4, 5 and 6 quizzes.
http://www.transum.org/software/SW/Starter_of_the_day/Students/PercentageChange.asp

Percent of Change Quiz
Practise determining the percent of change in this interactive multiple-choice quiz.
http://www.phschool.com/webcodes10/index.cfm?wcprefix=bja&wcsuffix=0607&area=view

Percentage Increase and Decrease 4 in a Line
The web page provides a game board to print. Players take turns picking a number from the left column, and increase or decrease it by a percentage from the right column. They cover the answer on the grid with a counter. The first player to get four counters in a line wins.
https://www.tes.co.uk/teaching-resource/percentage-increase-and-decrease-4-in-a-line-6256320

Percentage of Increase Exercises
Find the percentage increase given the original and final values in this self-check quiz about percentage change.
http://www.transum.org/software/SW/Starter_of_the_day/Students/PercentageChange.asp?Level=3

Percentage of Decrease Exercises
Find the percentage decrease given the original and final values in this self-check quiz about percentage change.
http://www.transum.org/software/SW/Starter_of_the_day/Students/PercentageChange.asp?Level=4

Treasure Hunt - Percentage Increase and Decrease
The clues of this treasure hunt are printable percentage increase/decrease questions.
https://www.tes.co.uk/teaching-resource/treasure-hunt--percentage-increase-and-decrease-6113809

Percent Change Practice
Interactive flash cards with simple questions about percentage of change with three difficulty levels.
http://www.thegreatmartinicompany.com/percent-percentage/percent-change.html

Percentage Increase and Decrease
Multiple-choice questions about percentage of change to be solved without a calculator (mental math).
https://www.mangahigh.com/en/maths_games/number/percentages/percentage_increase_and_decrease_-_no_calculator

Percent of Change Jeopardy
This is an online jeopardy game that provides you the game board, questions for percent increase, percent decrease, sales tax, discounts, and mark-ups, the answers, and a scoreboard where you can enter the teams' points. However, it does not have a place to enter answers and requires someone to supervise the play and the teams' answers.
https://www.superteachertools.net/jeopardyx/jeopardy-review-game.php?gamefile=1396523998

Percentage Difference
A short lesson about percentage difference, followed by practice questions at the bottom of the page.
http://www.mathsisfun.com/percentage-difference.html

INTEREST

Interest (An Introduction)
Read an introduction to interest, and then click on the questions at the bottom of the page to practise.
https://www.mathsisfun.com/money/interest.html

Quiz: Simple Interest
A quiz with five questions that ask for the interest earned, final balance, interest rate, or the principal.
http://www.cliffsnotes.com/math/algebra/algebra-ii/word-problems/quiz-simple-interest

Simple Interest
Another quiz where you need to find the principal, the amount of time, interest earned, or the final amount in an account earning interest. Four out of nine questions have to do with terminology and the rest are maths problems.
http://www.proprofs.com/quiz-school/story.php?title=simple-interest

Simple Interest Game
Answer questions about simple interest by clicking on the correct denominations in the cash register.
http://www.math-play.com/Simple-Interest/Simple-Interest.html

Simple Interest Quiz
Test your knowledge of simple interest with this interactive self-check quiz.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-860103-7&chapter=8&lesson=6&headerFile=4

Simple Interest Practice Problems
Practise using the formula for simple interest in this interactive online activity.
http://www.transum.org/Maths/Activity/Interest/

Calculating simple interest
This page includes several video tutorials plus a short three-question quiz on simple interest.
https://www.sophia.org/concepts/calculating-simple-interest

Compound interest
A simple introduction to compound interest with many examples.
http://www.mathsisfun.com/money/compound-interest.html

GENERAL

Percents Quiz
Revise basic percent calculations with this short multiple-choice quiz.
http://www.phschool.com/webcodes10/index.cfm?wcprefix=bja&wcsuffix=0605&area=view

Percent Equation Quiz
Test your knowledge of percent equations with this self-check multiple-choice quiz.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-860103-7&chapter=8&lesson=2&headerFile=4

Maths at the Mall
Practise percentages while shopping at a virtual mall. Find the percentage of discount and the sales price, calculate the interest earned at the bank, compare health memberships at the gym and figure out how much to tip your waiter at the Happy Hamburger.
http://www.mathplayground.com/mathatthemall2.html

Percent Word Problems
Practise solving word problems involving percents in this interactive online activity.
https://www.khanacademy.org/math/algebra-basics/basic-alg-foundations/alg-basics-decimals/e/percentage_word_problems_1

Percents Quiz 2
Practise answering questions about percent in this multiple-choice online quiz.
http://www.phschool.com/webcodes10/index.cfm?wcprefix=bja&wcsuffix=0606&area=view

Percentages Multiple-Choice Test
Test your understanding of percentages with this self-check multiple choice quiz.
http://www.transum.org/Software/Pentransum/Topic_Test.asp?ID_Topic=28

Solving Problems with Percent
Reinforce your skills with these interactive word problems.
http://www.buzzmath.com/Docs#CC07E253

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Percentage of Change

Percent(age of) change is a way to describe how much a price or some other quantity is increasing or decreasing (changing). Let's look at how to calculate the percentage a quantity is changing.

Example 1. A phone used to cost R400. Now it has been discounted to R360. What percentage was the discount?

Since this problem is asking for the *percentage,* we will use our basic formula $\frac{part}{total} = percentage.$

Because the change is relative to the *original* price, that original price becomes the "total" in our equation. The "part" is the actual amount by which the quantity changes, in this case R40. So we get:

$$percentage = \frac{part}{total} = \frac{\text{R40}}{\text{R400}} = 1/10 = 10\%$$

Essentially, we wrote **what fraction the R40 discount is of the original R400 price,** and converted that fraction into a percentage.

In summary: To calculate the percentage of change, use the same basic formula that defines a percentage: *part*/*total*. Since the change is relative to the original price, the original price is the "total," and the change in price is the "part."

$$\text{percentage of change} = \frac{part}{total} = \frac{difference}{original}$$

1. Write an equation and calculate the percentage of change.

a. A toy construction set costs R360. It is now discounted and costs only R240. What percentage is the discount?

$\frac{difference}{original} =$

b. A sewing kit cost R100. It is discounted and costs only R80 now. What percentage is the discount?

c. A bouquet of roses used to cost R150, but now it costs R200. What is the percentage of increase?

d. The price of a blanket was R160. The price has increased, and now it costs R200. What is the percentage of increase?

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Chapter 8: Geometry
Introduction

The main topics we study in this chapter are:

- various angle relationships
- drawing geometric figures, including basic geometric constructions
- pi and the area of a circle
- slicing 3-dimensional solids
- surface area and volume.

In the first lesson of the chapter, we examine various angle relationships: angles that are formed when several rays originate from the same starting point, vertical angles (formed when two lines intersect), and corresponding angles (formed when a line intersects two parallel lines). Then, the lesson *Angles in a Triangle* presents and proves the well-known result that the angles in a triangle sum to 180 degrees. With this knowledge, students are now able to solve various problems that involve unknown angles.

Next, students practise drawing geometric figures. Basic geometric constructions are done just like in ancient times: with only a compass and straightedge (a ruler without measurement units). These constructions help students to think about the main defining features of a figure. Personally I have always enjoyed geometric constructions because they are like little puzzles to solve.

Students also draw figures using a normal ruler and compass in the lesson *Drawing Problems*. They especially determine whether the given information defines a unique figure (triangle or a parallelogram).

Then we turn our attention to pi. Students first learn the definition of pi as a ratio of a circle's circumference to its diameter in the lesson *Circumference of a Circle*. Then they learn and practise how to calculate the area of a circle in a wide variety of word problems and applications. We also briefly study the proof for the formula for the area of a circle. I feel it is important that students encounter justifications for mathematical formulas and procedures and even read some proofs before high school. We don't want students to think that mathematics is only a bag of magic tricks or formulas to memorise that seemingly came out of nowhere. Proofs and logical thinking are foundations to mathematics and school mathematics should not be left without them.

After this, we slice three-dimensional solids with a plane, and learn that the result is always a two-dimensional shape. Students see that in a concrete way by slicing cubes and pyramids made of modelling clay. Some Internet links (provided in the lesson) will also help students to visualise what happens when a solid is cut with a plane.

In this chapter, students also solve a variety of problems concerning surface area and volume and practise converting between various units of area and volume. While these topics tend to involve lots of calculations and less possibilities for hands-on activities, they are very important in real life.

The Lessons in Chapter 8

Helpful Resources on the Internet

ANGLE RELATIONSHIPS

Math Warehouse - Angles
Find interactive demonstrations, examples and practice problems concerning various types of angles.
http://www.mathwarehouse.com/geometry/angle/complementary-angles.php

http://www.mathwarehouse.com/geometry/angle/supplementary-angles.php

http://www.mathwarehouse.com/geometry/angle/vertical-angles.php

http://www.mathwarehouse.com/geometry/triangles/

Angles at a Point
Drag the points of the angles to see how the angle measurements of the model change.
http://www.transum.org/software/SW/Angle_Theorems/ShowOne.asp?T=1

Angles Around a Point
A short lesson showing that angles around a point add up to 360 degrees, followed by self-check questions.
http://www.mathsisfun.com/angle360.html

Angle Points Exercises
Apply the properties of angles at a point, angles at a point on a straight line, and vertically opposite angles in this interactive online exercise.
http://www.transum.org/software/SW/Starter_of_the_day/Students/AnglePoints.asp?Level=4

Complementary and Supplementary Angles from Maths Is Fun
Each page includes a clear explanation, an interactive exploration, and self-check interactive questions.
http://www.mathsisfun.com/geometry/complementary-angles.html

http://www.mathsisfun.com/geometry/supplementary-angles.html

Complementary and Supplementary Angles
Find missing angles in this interactive exercise.
https://www.khanacademy.org/math/geometry/hs-geo-foundations/hs-geo-angles/e/complementary_and_supplementary_angles

Working with Angles
Online lessons with interactive self-check questions from Absorb Mathematics course. The lessons cover measuring angles, the types of angles, angles on a straight line, and other angle topics.
http://www.absorblearning.com/mathematics/demo/units/KCA003.html

Identifying Supplementary, Complementary, and Vertical Angles
Practise telling whether two angles are supplementary, complementary, or vertical.
https://www.khanacademy.org/math/basic-geo/basic-geo-angle/vert-comp-supp-angles/e/identifying-supplementary-complementary-vertical

Finding Missing Angles
Use your knowledge of vertical, complementary, and supplementary angles to find missing angles in this exercise.
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-geometry/cc-7th-unknown-angle-algebra/e/find-missing-angles

Angle Relationships Quizzes
Test your knowledge about angle relationships with these interactive online quizzes.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-860103-7&chapter=10&lesson=3&headerFile=4

http://www.thatquiz.org/tq-C/?-j7-l8-p1ug

Solving for Unknown Angles from Khan Academy
Use your knowledge of supplementary and complementary angles to solve questions of varying difficulty. Some questions involve writing and solving an equation.
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-geometry/cc-7th-angles/e/solving-for-unknown-angles

Angle Sums
Examine the angles in a triangle, quadrilateral, pentagon, hexagon, heptagon or octagon. Can you find a relationship between the number of sides and the sum of the interior angles?
http://illuminations.nctm.org/Activity.aspx?id=3546

Angles in a Triangle Quiz
Find the size of the angle marked with a letter in each triangle.
http://www.transum.org/software/SW/Starter_of_the_day/Students/AnglesInTriangle/Quiz.asp

CONSTRUCTIONS

Geometric Construction
These lessons cover constructions for perpendicular lines, an equilateral triangle, angle bisection, parallel lines, and copying an angle. They include explanations, interactive animations, and self-check questions.
http://www.absorblearning.com/mathematics/demo/units/KCA006.html

Animated Geometric Constructions
Simple animations that show how to do basic geometric constructions.
http://www.mathsisfun.com/geometry/constructions.html

Constructing A Triangle With Three Known Sides
A short demonstration showing how to construct a triangle with three known sides using a compass and a ruler.
http://www.mathsisfun.com/geometry/construct-ruler-compass-1.html

Constructing Triangles
Practise constructing triangles in this interactive activity from Khan Academy.
https://www.khanacademy.org/math/7th-engage-ny/engage-7th-module-6/7th-module-6-topic-b/e/constructing-triangles

Triangle Inequality Theorem
Answer questions about the third side of a triangle when the lengths of two sides are given.
https://www.khanacademy.org/math/7th-engage-ny/engage-7th-module-6/7th-module-6-topic-b/e/triangle_inequality_theorem

PI AND THE CIRCUMFERENCE OF A CIRCLE

A Rolling Circle Illustrating Pi
Two animations where a circle with diameter of 1 rolls on a number line one complete roll. Of course having rolled once around its circumference, it now lands at pi.
http://www.mathwarehouse.com/animated-gifs/images/demonstration-of-PI-in-math-animation.gif

http://i.imgur.com/dsCw0.gif

Approximating Pi
How did Archimedes find the approximate value of pi? This interactive tool illustrates Archimedes' basic approach with inscribed or circumscribed polygons.
http://www.pbs.org/wgbh/nova/physics/approximating-pi.html

5 Trillion Digits of Pi
As of 2014, the world record for computing digits of pi was over 13 trillion digits.
http://www.numberworld.org/digits/Pi/

Radius, Diameter and Circumference
Practise finding the radius, diameter, or circumference of a circle in this interactive online activity.
https://www.khanacademy.org/math/geometry/hs-geo-foundations/hs-geo-area/e/radius_diameter_and_circumference

Circles and Circumference Quiz
Test your skills with this interactive self-check quiz.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-860103-7&chapter=6&lesson=9&headerFile=4

Amazing History of Pi
A short and simple introduction to the history of pi.
http://ualr.edu/lasmoller/pi.html

AREA OF CIRCLE

Interactive Area of a circle
Explore and discover the relationship between the area, radius, and graph of a circle. Just click and drag the points.

http://www.mathwarehouse.com/geometry/circle/interactive-area.php

Circle Tool from Illuminations
How do the area and circumference of a circle compare to its radius and diameter? Investigate these relationships in the Intro and Investigation sections and then hone your skills in the Problems section.
http://illuminations.nctm.org/Activity.aspx?id=3547

Area of a Circle and Its Formula
Seven practice problems concerning the area of the circle with full solutions.
http://www.mathwarehouse.com/geometry/circle/area-of-circle.php

Quiz: Area of Circle
Practise finding the area, diameter and radius of a circle in this interactive online quiz.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-829633-1&chapter=11&lesson=6&headerFile=4

Area and Perimeter Practice
A 10-question quiz that will let you practise finding the area and circumference of a circle.
http://www.thatquiz.org/tq-4/?-j201g-la-p1ug

Area and circumference of a circle - Test from BBC Bitesize
A 10-question quiz about the area and circumference of a circle where the questions increase with difficulty.
http://www.bbc.co.uk/bitesize/quiz/q90581037

Mangahigh.com - Shape
Questions on the area of a circle, including the area of a semi-circle and simple compound shapes.
https://www.mangahigh.com/en/maths_games/shape/circles_and_cylinders/area_of_a_circle

Area and Circumference of a Circle
A 15-question multiple-choice quiz.
http://www.proprofs.com/quiz-school/story.php?title=area-circumference--circle

Area of Circle
An interactive activity where you cut a circle into wedges in order to determine its area.
http://www.learner.org/courses/learningmath/measurement/session7/part_b/index.html

Area of Circles
Cut a circle into sectors and rearrange the sectors to form a figure close to a parallelogram in this interactive activity. By increasing the number of sectors, the figure gets closer and closer to a perfect parallelogram.
https://www.geogebra.org/m/fyqAUV22

The Area of a Circle as a Limit
An animation that illustrates how we can find the area of a circle by drawing triangles into it. The area of the circle is then the limit of the sum of the areas of the interior triangles as the number of triangles goes to infinity.
http://www.learnerstv.com/animation/animation.php?ani=96&cat=physics

AREA AND PERIMETER

Area Tool
Use this tool to determine how the length of the base and the height of a figure can be used to determine its area. How are the area formulas for trapezoids, parallelograms, and triangles similar? How do they differ?
http://illuminations.nctm.org/Activity.aspx?id=3567

Free Worksheets for the Area of Triangles, Quadrilaterals, and Polygons
Generate customisable worksheets for the area of triangles, parallelograms, trapezoids, or polygons in the coordinate grid. Options include scaling, image size, workspace, border, and more.
http://www.homeschoolmath.net/worksheets/area_triangles_polygons.php

BBC Bitesize - Area
Brief revision "bites," including a few interactive questions, about the area of triangles, parallelograms, and compound shapes.
http://www.bbc.co.uk/bitesize/ks3/maths/measures/area/revision/4/

Geometry Area/Perimeter Quiz from ThatQuiz.org
Find the area and perimeter of rectangles, triangles, parallelograms, and trapezoids. You can modify the quiz parameters, for example to omit a shape, or to solve for an unknown side when perimeter/area is given.
http://www.thatquiz.org/tq-4/?-j1i00f-lc-p0

Find Areas of Trapezoids
Practise finding the areas of trapezoids in this interactive online activity from Khan Academy.
https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-geometry-topic/cc-6th-area/e/areas_of_trapezoids_rhombi_and_kites

Find Areas of Compound Shapes
Practise finding the areas of complex shapes that are composed of smaller shapes in this interactive online exercise.
https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-geometry-topic/cc-6th-area/e/area-of-quadrilaterals-and-polygons

Area of Composite Shapes
Find the areas of combined shapes made up of one or more simple polygons and circles.
http://www.transum.org/software/SW/Starter_of_the_day/Students/Areas_of_Composite_Shapes.asp?Level=4

Metric Area
Read a short, illustrated lesson about metric area. Then, click on the questions at the bottom of the page to practise.

http://www.mathsisfun.com/measure/metric-area.html

Metric Unit Conversions: Area
Use these printable worksheets for revision or to brush up on metric conversion skills.
http://www.dadsworksheets.com/worksheets/metric-si-unit-conversions-metric-si-area.html

SURFACE AREA

2-D and 3-D Shapes
Learn about different solids: rotate them and see their nets.
http://www.tgfl.org.uk/tgfl/custom/resources_ftp/netmedia_ll/ks2/maths/3d

Exploring Nets of 3D-Shapes
This interactive resource allows you to explore the concepts of surface area, volume, 3D shapes, and nets.
http://learnalberta.ca/content/mejhm/?l=0&ID1=AB.MATH.JR.SHAP&ID2=AB.MATH.JR.SHAP.SURF

Geometric Solids
Manipulate (rotate) geometric solids by dragging with the mouse and see their nets.
http://illuminations.nctm.org/Activity.aspx?id=3521

Surface Area of a Cylinder
This animation demonstrates the formula for finding the surface area of a cylinder.
http://www.mathwarehouse.com/animated-gifs/images/surface-area-of-cylinder-animation.gif

Interactives - Surface area - Cylinders
A lesson that includes an interactive portion where you calculate the surface area of a cylinder step-by-step.
http://www.learner.org/interactives/geometry/area_surface2.html

Surface Area Quiz
A 10- question quiz about the surface area of prisms, pyramids, and cylinders. Note: you need to input the surface area of cylinders as a multiple of pi · cm. For example, a cylinder with height 7 cm and bottom radius of 2 cm has the surface area of $(4 \text{ cm} \cdot \pi \cdot 7 \text{ cm}) + (2 \cdot (2 \text{ cm})^2 \cdot \pi) = 28\pi \text{ cm} + 8\pi \text{ cm} = 36\pi \text{ cm}$.
http://www.thatquiz.org/tq-4/?-j824a0-l6-p1ug

CROSS-SECTIONS OF SOLIDS

Cross Sections
An interactive activity that allows you to slice a cube with a plane and find its different cross sections.
http://www.learner.org/courses/learningmath/geometry/session9/part_c/index.html

Cross-Sections of 3-D Objects
Match 3D objects with their 2D cross-sections in this interactive online exercise.
https://www.khanacademy.org/math/geometry/hs-geo-solids/hs-geo-2d-vs-3d/e/slicing-3d-figures

Cross-Section Flyer from Shodor
Examine cross-sections of a cone, cylinder, pyramid, prism, and a double-cone. You can rotate and move the cutting plane, plus decide the number of faces for the pyramid and prism. An excellent tool!
http://www.shodor.org/interactivate/activities/CrossSectionFlyer/

VOLUME (AND SURFACE AREA)

What's in a Cube, Level 2
A demonstration of the size of a cubic metre and practise estimating the volume of irregular cuboid objects.
https://fuse.education.vic.gov.au/Resource/LandingPage?ObjectId=76c2f71b-5dba-4da6-b905-52d0f458b14f

Interactive Illustration of a Volume of a Cylinder
Drag the orange dot to resize the cylinder, and its volume is calculated as you drag it.
http://www.mathopenref.com/cylindervolume.html

Volume of a Cylinder
Interactive and guided questions about the volume of a cylinder. Students also find the height or the radius of a cylinder when given the volume. Some problems leave the answer in terms of pi.
https://www.mangahigh.com/en/math_games/shape/circles_and_cylinders/volume_of_a_cylinder

Find the Volume of a Cylinder
Practise finding the volume of a cylinder in this interactive exercise.
https://www.learner.org/interactives/geometry/area_volume2.html

Interactivate: Surface Area and Volume
Explore or calculate the surface area and volume of rectangular and triangular prisms. You can change the base, height, and depth interactively.
http://www.shodor.org/interactivate/activities/SurfaceAreaAndVolume

Surface Area and Volume of 3-D Shapes
This page contains three worked out examples about the volume and surface area of cylinders and prisms, followed by over a dozen exercises and word problems. You can self-check your answers.
http://www.cimt.org.uk/projects/mepres/book9/bk9i9/bk9_9i4.html

Geometry Volume Quiz
Find the volume of prisms and cylinders. Note: you need to input the volume of cylinders as a multiple of pi · cm. For example, a cylinder with height 7 cm and bottom radius of 2 cm has the volume of $(2\text{ cm})^2 \cdot \pi \cdot 7\text{ cm} = 28\pi$ cm.
http://www.thatquiz.org/tq-4/?-j28g0-la-p1ug

MathGuide's Volume of Cylinders Quizmaster
Interactive questions about the area of the bottom and the volume of a cylinder, given its radius and height.
http://www.mathguide.com/cgi-bin/quizmasters/CylindersV.cgi

Volume of Cylinders Quiz
Practise finding the volume, diameter, and depth of a cylinder in this interactive self-check quiz.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-860103-7&chapter=12&lesson=3&headerFile=4

Area and Volume Quiz
Revise area and volume with this interactive online quiz.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-860059-6&chapter=14&headerFile=4

Volume and Surface Area Quiz
Practise finding volume and surface area in this interactive online quiz.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-829633-1&chapter=12&headerFile=4

GENERAL

Formula Pairs Matching Game
Match each formula with the picture that illustrates it.
http://www.transum.org/software/SW/Starter_of_the_day/Students/Pairs.asp?Topic=3

Online Kaleidoscope
Create your own kaleidoscope pattern with this interactive tool.
http://www.zefrank.com/dtoy_vs_byokal/

Interactivate! Tessellate
An interactive tool for creating your own tessellations. Choose a shape, then edit its corners or edges.
http://www.shodor.org/interactivate/activities/Tessellate

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Area of a Circle

The area of a circle is given by this formula: $A = \pi r^2$, where r is the radius of the circle.
Read the formula as: "Area equals pi r squared." It means that you first multiply the radius by itself and then multiply the result by π.

You can remember the formula by thinking, "Pie are square." Of course pies are usually round, not square! It is bad English, as well, but the purpose of this silly mnemonic is just to help you remember the formula.

Example 1. The radius of a circle measures 8 cm. What is its area?

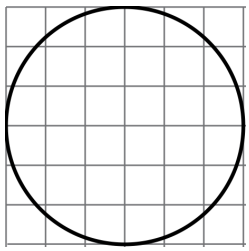
We use the formula: $A = \pi r^2 = \pi \cdot 8 \text{ cm} \cdot 8 \text{ cm} \approx 3{,}14 \cdot 64 \text{ cm}^2 = 200{,}96 \text{ cm}^2$, or about 200 cm^2.

Remember to always give your answer for an area in <u>square</u> units, such as, square centimetres, square metres, square kilometres, *etc*. If no measuring unit is given, use "square units."

You can use a calculator for all the problems in this lesson.

1. Estimate the area of the circles by counting squares and parts of squares. After that, calculate the area to the nearest tenth of a square unit.

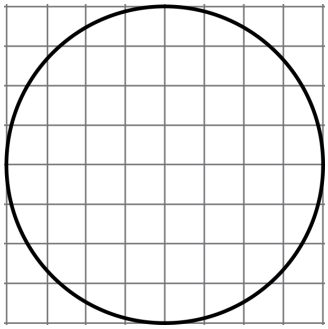
a.



Estimation: _____ square units

Calculation: _____ square units

b.



Estimation: _____ square units

Calculation: _____ square units

2. Find the areas of these circles.

a. A circle with a radius of 7,0 cm. Round the answer to the nearest ten square centimetres (to 2 significant digits). Area =	**b.** A circle with a radius of 2,34 m. Round the answer to one decimal digit (to 3 significant digits). Area =
c. A circle with a *diameter* of 75,0 cm. Round the answer to the nearest ten square centimetres (to 3 significant digits). Area =	**d.** A circle with a diameter of 22,58 km. Round the answer to the nearest tenth of a square kilometre (to 4 significant digits). Area =

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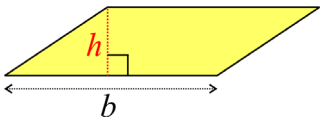
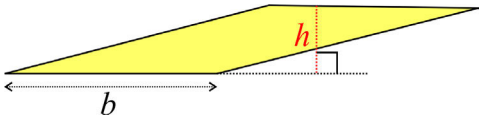
Area and Perimeter Problems



The area of a parallelogram is

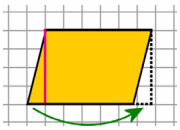
$$A = bh$$

where b is the base and h is the altitude (height).



The **altitude** of a parallelogram is a perpendicular line segment from the base, or the extension of the base, to the top. So, the altitude might not be inside the parallelogram.

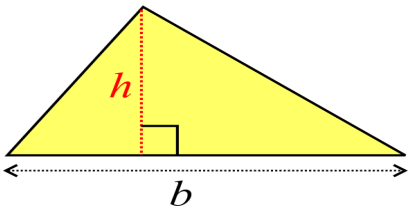
Recall that from any parallelogram we can cut off a triangular piece and move it to the other side to make it a rectangle. That is why the formula for the area of a parallelogram is so similar to the formula for the area of a rectangle.



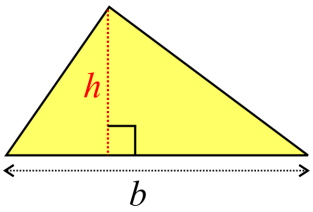
Since any triangle is half of its corresponding parallelogram, the area of a triangle is half the area of that parallelogram:

$$A = \frac{bh}{2}$$

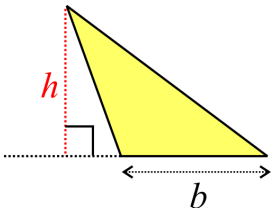
where b is the base and h is the altitude of the triangle.



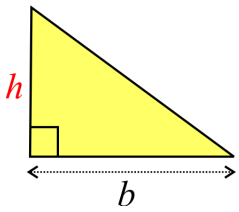
The **altitude** of a triangle is a line from one vertex to the opposite side that is perpendicular to that side. It can:



(1) fall inside the triangle;



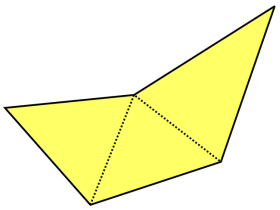
(2) fall outside the triangle;

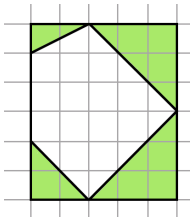


(3) be one of the sides of a right triangle.

To calculate the area of a polygon:

(1) Divide it into rectangles, triangles, and other simple shapes.
(2) Calculate the area of each part separately.
(3) Add the area of each of the parts.





Sometimes we can use another strategy to find the area of a polygon.

(1) Draw a rectangle around the polygon.
(2) Calculate the areas of the triangles and quadrilaterals that are outside the polygon but inside the rectangle.
(3) Subtract those areas from the area of the entire rectangle.

You may use a calculator for all the problems in this lesson.

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Chapter 9: The Pythagorean Theorem
Introduction

This is a relatively short chapter focusing on the Pythagorean Theorem and its applications. The Pythagorean Theorem is actually not part of the Common Core Standards for seventh grade. The Common Core places it in eighth grade. However, I have included it in this curriculum because it is a traditional topic in pre-algebra. That way, Math Mammoth Grade 7 International Version works as a full pre-algebra curriculum. If you are following the Common Core Standards strictly, you can safely omit this entire chapter, because your student(s) will encounter these topics in eighth grade.

First, students need to become familiar with square roots, so they can solve the equations that result from applying the Pythagorean Theorem. The first lesson of the chapter introduces taking a square root as the opposite operation to squaring a number. The lesson includes both applying a guess-and-check method and using a calculator to find the square root of a number.

Next, students learn how to solve simple equations that include taking a square root. This makes them fully ready to study the Pythagorean Theorem and apply it.

The Pythagorean Theorem is introduced in the lesson by that name. Students learn to verify that a triangle is a right triangle by checking if it fulfils the Pythagorean Theorem. They apply their knowledge about square roots and solving equations to solve for an unknown side in a right triangle when two of the sides are given.

Next, students solve a variety of geometric and real-life problems that require the Pythagorean Theorem. This theorem is extremely important in many practical situations. Students should show their work for these word problems to include the equation that results from applying the Pythagorean Theorem to the problem and its solution.

There are literally hundreds of proofs for the Pythagorean Theorem. In this chapter, we present one easy proof based on geometry (not algebra). As an exercise, students are asked to supply the steps of reasoning to another geometric proof of the theorem, and for those interested the lesson also provides an Internet link that has even more proofs of this theorem.

The Lessons in Chapter 9

Helpful Resources on the Internet

The Pythagorean Theorem - Video Lessons by Maria
A set of free videos that teach the topics in this chapter - by the author.
http://www.mathmammoth.com/videos/prealgebra/pre-algebra-videos.php#pythagorean

SQUARE ROOTS

Squares and Square Roots
A fun lesson about squares and square roots with lots of visuals and little tips. It is followed by 10 interactive multiple-choice questions.
http://www.mathsisfun.com/square-root.html

The Roots of Life
Practise finding square roots of perfect squares and help the roots of a tree grow.
http://www.hoodamath.com/games/therootsoflife.html

Square Root Game
Match square roots of perfect squares with the answers. Includes several levels.
http://www.math-play.com/square-root-game.html

Approximating Square Roots
Practise finding the approximate value of square roots by thinking about perfect squares.
https://www.khanacademy.org/math/pre-algebra/pre-algebra-exponents-radicals/pre-algebra-square-roots/e/square_roots_2

Square Roots Quiz
Test your knowledge of square roots with this interactive self-check quiz.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-829635-8&chapter=3&lesson=1&headerFile=4

Pyramid Maths
Choose "SQRT" to find square roots of perfect squares. Drag the correct answer to the jar on the left. This game is pretty easy.
http://www.mathnook.com/math/pyramidmath.html

Rags to Riches Square Root Practice
Answer multiple-choice questions that increase in difficulty. The questions include finding a square root of perfect squares, determining the two nearest whole numbers to a given square root, and finding square roots of numbers that are not perfect squares to one decimal digit.
http://www.quia.com/rr/382994.html

THE PYTHAGOREAN THEOREM

Pythagorean Theorem - Braining Camp
This learning module includes a lesson, an interactive manipulative, multiple-choice questions, real-life problems, and interactive open-response questions.
https://www.brainingcamp.com/content/pythagorean-theorem/

Pythagoras' Theorem from Maths Is Fun
A very clear lesson about the Pythagorean Theorem and how to use it, followed by 10 interactive practice questions.
http://www.mathsisfun.com/pythagoras.html

Pythagorean Triplets
Move the two orange points in this activity to find Pythagorean Triplets, sets of three whole numbers that fulfil the Pythagorean Theorem.
http://www.interactive-maths.com/pythagorean-triples-ggb.html

The Pythagorean Theorem Quiz
A 10-question quiz that asks for the length of the third side of a right triangle when the two sides are given.
http://www.thatquiz.org/tq-A/?-j10-la-p1ug

Interactivate: Pythagorean Theorem
Interactive practice problems for calculating the third side of a right triangle when two sides are given.
http://www.shodor.org/interactivate/activities/PythagoreanExplorer/

Exploring the Pythagorean Theorem
This multimedia mathematics resource shows how the Pythagorean Theorem is an important maths concept used in the structural design of buildings. Using an interactive component, students construct right triangles of various sizes to explore calculations of the Pythagorean Theorem.
http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.SHAP&ID2=AB.MATH.JR.SHAP.PYTH

Using the Pythagorean Theorem - Quiz
Revise the Pythagorean Theorem in this interactive self-check quiz.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-829635-8&chapter=3&lesson=5&headerFile=4

Self-Check Quiz
Test your knowledge of the Pythagorean Theorem in this interactive online quiz.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-829633-1&chapter=11&lesson=3&headerFile=4&

Pythagoras in 3D
A challenge problem: can you find the longest dimension of a box?
http://www.interactive-maths.com/pythagoras-in-3d-ggb.html

PROOF

Proving the Pythagorean Theorem
See if you can figure out two more proofs of the Pythagorean theorem. Only the pictures are given to you. Tips and Solutions are available.
http://www.learner.org/courses/learningmath/geometry/session6/part_b/more.html

Annotated Animated Proof of the Pythagorean Theorem
Watch the animation to learn a proof of the Pythagorean Theorem.
http://www.davis-inc.com/pythagor/proof2.html

Many Proofs of the Pythagorean Theorem
A list of animated proofs.
http://www.takayaiwamoto.com/Pythagorean_Theorem/Pythagorean_Theorem.html

Pythagorean Theorem and Its Many Proofs
A collection of 111 approaches to prove this theorem.
http://cut-the-knot.com/pythagoras/

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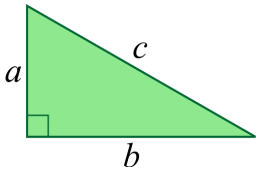
The Pythagorean Theorem

You will now learn a very famous mathematical result, the Pythagorean Theorem, which has to do with the lengths of the sides in a right triangle. First, we need to study some terminology.

In a right triangle, the two sides that are perpendicular to each other are called **legs**. The third side, which is always the longest, is called the **hypotenuse**.

In the image on the right, the sides a and b are the legs, and c is the hypotenuse.



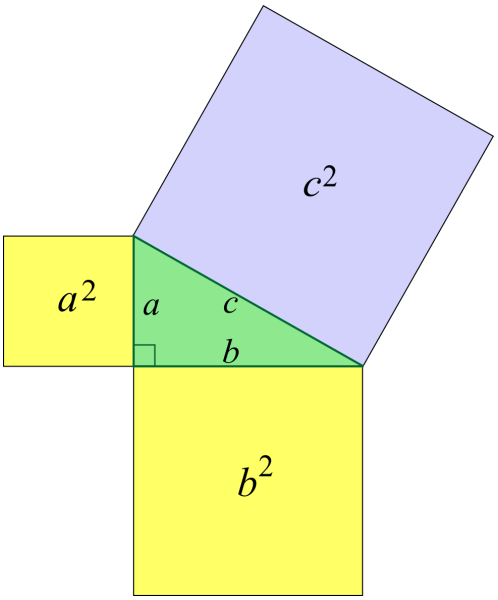
Note: We do not use the terms "leg" and "hypotenuse" to refer to the sides of an acute or obtuse triangle—this terminology is restricted to *right* triangles.

The Pythagorean Theorem states that **the sum of the squares of the legs equals the square of the hypotenuse.**

In symbols it looks much simpler:

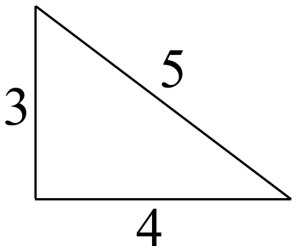
$$a^2 + b^2 = c^2$$

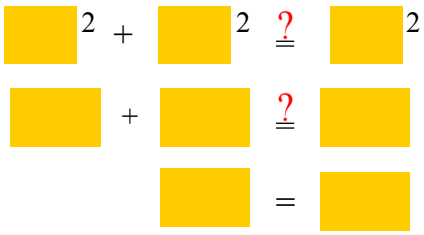
The picture shows squares drawn on the legs and on the hypotenuse of a right triangle. Verify visually that the total combined area of the two yellow squares drawn on the legs looks about equal to the area of the blue square on the hypotenuse.



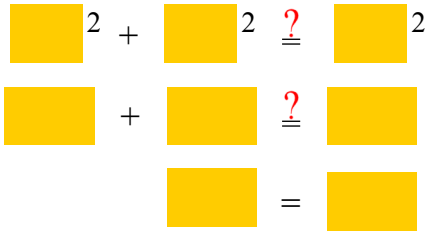
We will prove this theorem in another lesson.
For now, let's get familiar with it and learn how to use it.

1. This is the famous 3-4-5 triangle: its sides measure 3, 4, and 5 units. It is a right triangle. Check that the Pythagorean Theorem holds true for it by filling in the numbers below.





2. **a.** Check that the Pythagorean Theorem holds for a triangle with sides 6, 8, and 10 units long by filling in the numbers at the right.



b. Use a compass and a ruler to draw a triangle with sides 6, 8, and 10 cm long. You can revise the box, "A Triangle with Three Given Sides," on page 127. Measure its angles: did you get a right triangle?

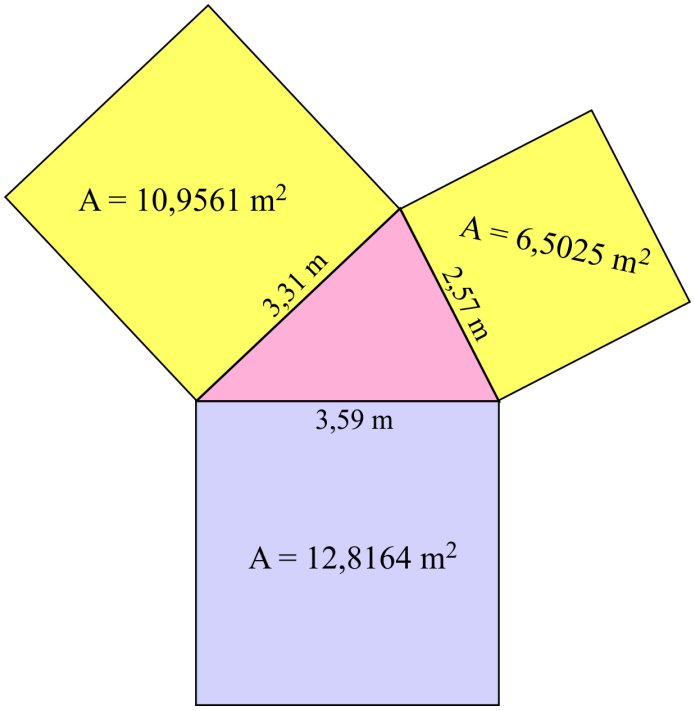
Example 1. This triangle is *not* a right triangle, so the Pythagorean Theorem does *not* hold:

$$2{,}55^2 + 3{,}31^2 \stackrel{?}{=} 3{,}58^2$$

$$6{,}5025 + 10{,}9561 \stackrel{?}{=} 12{,}8164$$

$$17{,}4586 > 12{,}8164$$

The sum of the areas of the squares drawn on the two shortest sides are more than the area of the square drawn on the longest side. As you can see, the triangle is acute.



Example 2. Is a triangle with sides 4 cm, 5 cm, and 7 cm a right triangle?

We check if 4, 5, and 7 fulfil the Pythagorean Theorem (on the right). They don't. In fact, $4^2 + 5^2 < 7^2$ and the triangle is obtuse. (You can check that by drawing it.)

$$4^2 + 5^2 \stackrel{?}{=} 7^2$$

$$16 + 25 \stackrel{?}{=} 49$$

$$41 < 49$$

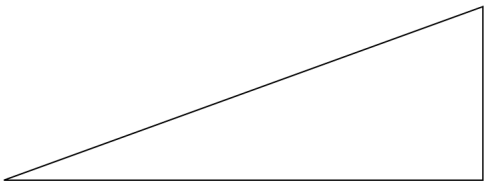
This triangle is obtuse.

3. For each set of lengths, determine whether they form a right triangle using the Pythagorean Theorem. Notice carefully which length is the hypotenuse.

a. 6, 9, 13

b. 12, 13, 5

4. **a.** Measure each side of this triangle to the nearest millimetre.



b. Verify that the sum of the areas of the squares on the legs is *very close* to the area of the square on the hypotenuse. I say "very close" because the process of measuring is always inexact, and therefore your calculations and results will probably not yield true equality, just something close.

5. For each set of lengths below, determine whether the lengths form an acute, right, or obtuse triangle—or *no* triangle. You can construct the triangles using a compass and a ruler and also use the Pythagorean theorem.

a. 9, 6, 4

b. 13, 11, 10

c. 12, 14, 28

d. 15, 20, 25

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Chapter 10: Probability
Introduction

Probability is new to Math Mammoth students, as the topic does not appear at all in grades 1-6. However, most students have an intuitive understanding of probability based on hearing the terms "probably" and "likely," from listening to weather forecasts, and so on.

In the past, probability was not taught until high school—for example, I personally encountered it for the first time in 12th grade. However, since probability is such a useful and easily accessible field in maths, it was felt that it should be introduced sooner, so during the 1990s and 2000s it "crept" down the grade levels until many states required probability even in elementary school. The Common Core Standards include probability starting in 7th grade. I feel that is good timing because by 7th grade students have studied fractions, ratios, and proportions, so they have the tools they need to study probability. Moreover, they will need an understanding of the basic concepts of probability in order to understand the statistical concepts that they will study in middle school and high school.

In this chapter we start with the concept of simple (classic) probability, which is defined as the ratio of the number of favourable outcomes to the number of all possible outcomes. Students calculate probabilities that involve common experiments, to include flipping a coin, tossing a pair of dice, picking marbles, and spinning a spinner.

The lesson *Probability Problems from Statistics* introduces probability questions involving the phrase "at least," which are often solved by finding the probability of the complement event. For example, it might be easier to count the number of students who got at most D+ on a test than to count the number of students who got at least C-.

In the next lesson, *Experimental Probability*, students conduct experiments, record the outcomes, and calculate both the theoretical and experimental probabilities of events, in order to compare the two. They will draw a card from a deck or roll a dice hundreds of times. The download version of this curriculum includes spreadsheet files for some of the lengthier probability simulations. You can also access those simulations on the web page http://www.mathmammoth.com/lessons/probability_simulations.php

Next, we study compound events, which combine two or more individual simple events. Tossing a dice twice or choosing first a girl then a boy from a group of people are compound events. Students calculate the probabilities of compound events by using the complete sample space (a list of all possible outcomes). They construct the sample space in several ways: by drawing a tree diagram, by making a table, or simply by using logical thinking to list all the possible outcomes.

The last major topic in this chapter is simulations. Students design simulations to find the probabilities of events. For example, we let heads represent "female" and tails represent "male," so we can toss a coin to simulate the probability of choosing a person of either sex at random. Later in the lesson, students design simulations that use random numbers. They generate those numbers by using either the free tool at http://www.random.org/integers or a spreadsheet program on a computer.

In the last lesson of the chapter, *Probabilities of Compound Events*, we learn to calculate the probability of a compound event by *multiplying* the probabilities of the individual events (assuming the outcomes of the individual events are independent of each other). This topic exceeds the Common Core Standards for 7th grade and is optional. I have included it here because the idea studied in the lesson is very simple and I feel many students will enjoy it.

The Lessons in Chapter 10

Helpful Resources on the Internet

Probability Videos by Maria
These video lessons cover topics that have been chosen to complement the lessons in this chapter.
http://www.mathmammoth.com/videos/probability/probability_lessons.php

SIMPLE PROBABILITY

Probability Game with Coco
A multiple-choice online quiz on simple probability.
http://www.math-play.com/Probability-Game.html

Probability Circus
Choose the spinner that matches the probability in this interactive online activity.
http://www.hbschool.com/activity/probability_circus/

Simple Probability
Practise finding probabilities of events, such as rolling dice, drawing marbles out of a bag, and spinning spinners.
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-probability-statistics/cc-7th-basic-prob/e/probability_1

Card Sharks Game
Use your knowledge of probability to bet on whether the next card is higher or lower than the last one.
http://mrnussbaum.com/cardsharks/

Mystery Spinners
In this activity, you need to find the hidden mystery spinner working from only one clue.
http://www.scootle.edu.au/ec/viewing/L2384/index.html

Simple Events - Probability Quiz
Reinforce your probability skills with this interactive self-check quiz.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-860103-7&chapter=9&lesson=1&headerFile=4

EXPERIMENTAL PROBABILITY

Virtual Experimental Probability
Roll one or two dice, spin a spinner, choose a card, or toss a coin virtually many times. The results are recorded so you can compare them to the theoretical probability.
http://staff.argyll.epsb.ca/jreed/math9/strand4/probability_display.htm

Adjustable Spinner
Create a virtual spinner with variable-sized sectors to compare experimental results to theoretical probabilities. You can choose the sizes of the sectors, the number of sectors, and the number of trials.
http://www.shodor.org/interactivate/activities/AdjustableSpinner/

Interactive Customisable Spinners
Build coloured spinners, test them a number of spins. Compare actual results with expected results.
https://fuse.education.vic.gov.au/Resource/LandingPage?ObjectId=8eb446b6-bd1e-446a-848a-935dce8b0b70

Dice Duels Tool
Explore the experimental probability distributions when you roll between two and five dice. Add, subtract, or multiply the numbers. The tool graphs the results, and can do up to 9,999 rolls.
https://fuse.education.vic.gov.au/Resource/LandingPage?ObjectId=f8341288-4733-4604-abf3-7c1d9de7fc4b

Dice Roll
Choose the number of virtual dice and how many times you want to roll them. It shows both the actual results and expected (theoretical) probabilities, and the simulation works for a large numbers of rolls.
http://www.btwaters.com/probab/dice/dicemain3D.html

Coin Flip
This virtual coin toss shows the results numerically and can generate at least 100,000 flips.
http://www.btwaters.com/probab/flip/coinmainD.html

Coin Toss Simulation
Another virtual coin toss. This one shows the results both using images of coins and numerically.
http://syzygy.virtualave.net/multicointoss.htm

Rocket Launch
A three-stage rocket is about to be launched. In order for a successful launch to occur, all three stages of the rocket must successfully pass their pre-takeoff tests. By default, each stage has a 50% chance of success, however, this can be altered by dragging the bar next to each stage. Observe how many tries it takes until there is a successful launch.
http://mste.illinois.edu/activity/rocket/

Find the Bias
A dice has been weighted (loaded) to favour one of the six numbers. Roll the dice to work out which is the favoured face. Explore how many rolls are needed for you to be reasonably sure of a conclusion.
https://fuse.education.vic.gov.au/Resource/LandingPage?ObjectId=0ec1fe96-7c91-4975-9863-766a1fe9c1c5

Load One Dice (Biased dice)
Make a biased dice by loading it to favour one of the six numbers. Then roll the loaded dice, and compare the shape of theoretical data distributions with experimental results.
https://fuse.education.vic.gov.au/Resource/LandingPage?ObjectId=b60d5135-608d-4fd6-8e65-a81fa1dc172a

Racing Game with One Dice
Explore how experimental probability relates to fair and unfair games with this two-car race. You choose which and how many numbers of the dice make each of the cars move. Other options include the number of trials and the length of the race (in segments). The program calculates the percentage of wins for each car and draws a pie chart.
http://www.shodor.org/interactivate/activities/RacingGameWithOneDie/

COMPOUND EVENTS AND TREE DIAGRAMS

Tree Diagrams and Spinners Quiz
Practise reading tree diagrams in this interactive 10-question quiz.
https://www.thatquiz.org/tq/practicetest?8x24p16y3o5t

Lucky 16 Game
You place counters on the game board, and then they will be removed based on the sum of two dice that are rolled. Try to predict the best positions for the counters before the game starts.
https://fuse.education.vic.gov.au/Resource/LandingPage?ObjectId=31ee684d-742e-4fe3-bb55-e8e6e61e0d6d

Airport Subtraction Game
This game is based on rolling two dice and subtracting the results. Your task is to place your plane at the back of the queue on one of the runways. Try to predict which lane is most likely to clear quickly.
https://fuse.education.vic.gov.au/Resource/LandingPage?ObjectId=0c1b063f-52ec-4564-ac6e-265ddcbdcada

Quiz: Compound Probability
Test your knowledge of compound probability with this interactive self-check quiz.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-860467-2&chapter=2&lesson=3&headerFile=4

How could I send the check and not pay the bill?
What is the probability that Tessellation will put each of the three checks into the correct envelopes if she does it randomly? The page includes a hint and a complete solution (click "answer" at the bottom of the page).
http://figurethis.nctm.org/challenges/c69/challenge.htm

Flippin' Discs
In this interactive activity, you throw two discs. You win if they both show the same colour. You can run the game 100 times and see the detailed results. Can you explain why you win approximately half the time? Explore the situation also with 3, 4, and even 5 discs. The solution is found in a link near the top left of the page.
http://nrich.maths.org/4304

"Data Analysis & Probability Games" from MathWire
A list of board and dice games to help to teach topics appropriate for beginners in probability.
http://mathwire.com/games/datagames.html

Cross the Bridge
This is a printable board game based on throwing two dice and the probabilities for the sum of the dice.
http://www.mathsphere.co.uk/downloads/board-games/board-game-17-crossing-the-river.pdf

At Least One...
The tree diagram and related discussion on this page guides students' thinking to help them answer probability questions like, "What is the probability of getting at least one head by flipping a coin ten times?" A link near the top left of the page leads to the solution.
http://nrich.maths.org/7286

SIMULATIONS

Probability Simulations in Excel
These spreadsheet files match some of the lengthier probability simulations in this chapter.
http://www.mathmammoth.com/lessons/probability_simulations.php

Random Integer Generator
Choose the numbers, how many columns, and the values of the integers; then click to generate.
https://www.random.org/integers/

Coin-Toss Simulation
Another virtual coin toss. This one shows the results both using images of coins and numerically.
http://syzygy.virtualave.net/multicointoss.htm

Marbles
Experiment by drawing 1, 2, or 3 marbles from a set of blue, red, purple, and green marbles. The results shown include the frequencies of each possible outcome, the experimental probabilities, and the theoretical probabilities. This activity can also be used to perform simulations.
http://www.shodor.org/interactivate/activities/Marbles/

Interactivate: *Fire!!* and *Directable Fire!!*
In these two activities, you first set the probability that a fire will spread from tree to tree in a forest of 100 trees. Then you click the tree where the fire starts and watch it spread. In the Directable Fire activity, you can set the probabilities for each direction to be different. Repeat the activity several times to see that the amount of forest that burns varies (for any set probability of fire spreading).
http://www.shodor.org/interactivate/activities/Fire/
http://www.shodor.org/interactivate/activities/DirectableFire/

COMPOUND PROBABILITY

Self-Check Quiz
Test your knowledge about probability in this interactive self-check quiz.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-860467-2&chapter=2&lesson=3&headerFile=4

Quiz: Probability of Compound Events
Revise probability of compound events with this multiple-choice quiz.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-860139-8&chapter=8&lesson=5&headerFile=4

Probability Quiz
This quiz covers topics such as theoretical and experimental probability and compound events.
http://mrmaisonet.com/index.php?/Probability-Quizzes/Probability-Quiz.html

FOR FUN

Monty Hall Paradox
Try this interactive version of the famous Monty Hall problem. Behind which door is the car? If you choose the wrong one, you will win a goat instead.
http://www.math.ucsd.edu/~crypto/Monty/monty.html

What Does Random Look Like?
This problem challenges our thinking about randomness. Make up a sequence of twenty H's and T's that *could* represent a sequence of heads and tails generated by a fair coin. Then use the animation to generate truly random sequences of 20 coin flips. Can you learn how to spot fakes?
http://nrich.maths.org/7250

Same Number
Imagine that the teacher asks 20 students to secretly write a whole number between 1 and 225. How likely is it for each number to be different? It provides an interactive simulation to experiment with this problem. The following discussion also leads students to the classic birthday problem. The solution is found in a link near the top left of the page.
http://nrich.maths.org/7221

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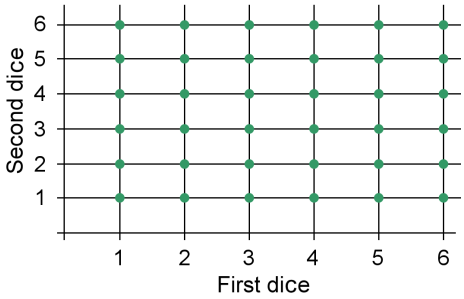
Counting the Possibilities

A **sample space** is a list of all possible outcomes of an experiment.

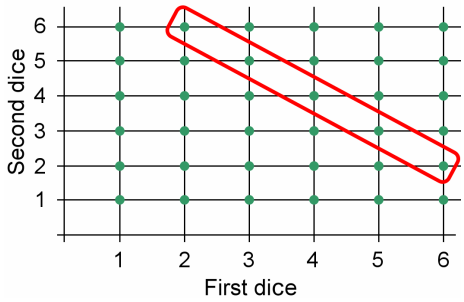
Example 1. We roll two dice. The sample space for this experiment is shown in the grid on the right. Each dot represents one outcome. For example, the point (1, 4) means that the first dice shows 1 and the second dice shows 4.



Notice that there are a total of $6 \cdot 6 = 36$ possible outcomes.

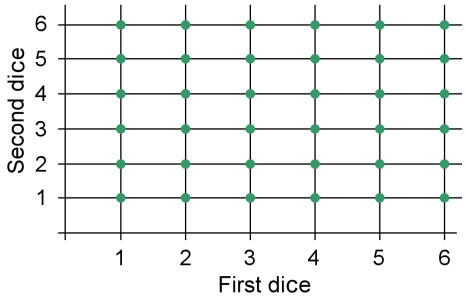
What is the probability of getting the sum of 8 when rolling two dice? The chart helps answer that question. First we find out and count how many outcomes give you the sum 8:

You could roll 2 + 6, 3 + 5, 4 + 4, 5 + 3, or 6 + 2. Those number pairs are circled in the second graphic.



So there are five favourable outcomes out of 36 possible. Therefore, the probability of getting 8 as a sum is 5/36.

1. **a.** How many outcomes are there for rolling the same number on both dice (such as (5, 5))?



b. What is the probability of rolling the same number on both dice?

2. **a.** What is the probability of rolling 5 on the first dice and 6 on the second?

b. What is the probability of rolling 5 on one dice and 6 on the other?

c. What is the probability of getting a sum of 7 when rolling two dice?

d. What is the probability of getting a sum of at least 6 when rolling two dice?

3. You roll a six-sided dice twice. Find the probabilities.

a. P(1; 5)

b. P(2; 5 or 6)

c. P(even; odd)

d. P(6; not 6)

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Chapter 11: Statistics
Introduction

Math Mammoth Grade 7 International Version ends with a study of statistics. The chapter begins with a revision lesson to remind students how to make a boxplot and a histogram and how to calculate the mean absolute deviation—all topics that were studied in 6th grade.

The first focus area of the chapter is random sampling. Students learn that sampling methods vary and that random sampling is likely to produce an *unbiased sample*—a sample that represents the population well. In the lesson *Using Random Sampling*, students choose several random samples from a population of 100 geometric shapes, and they see first hand that random samples can vary—even a lot. Yet if the sample size is sufficiently large, or if we have several random samples, we can be relatively confident in concluding something about the population itself. Students will also practise making inferences about populations based on several random samples.

The second major topic is comparing two populations, either directly or by using samples from the populations. Students learn to use the overall distributions and the measures of centre and variability to compare two sets of data in various ways. While some of the ways in which we compare the data are informal only, all of the concepts presented are fundamental to the use of statistics in various sciences. Students also do a project where they gather data on their own from two populations and compare them.

The Lessons in Chapter 11

Helpful Resources on the Internet

Statistics Videos by Maria
Videos on statistics topics that are helpful for the lessons in this chapter.
http://www.mathmammoth.com/videos/statistics/statistics_lessons.php

GRAPHS AND PLOTS

Statistics Interactive Activities from Shodor
A set of interactive tools for exploring and creating boxplots, histograms, dot plots, and stem-leaf plots. You can enter your own data or explore the examples.

http://www.shodor.org/interactivate/activities/BoxPlot/

http://www.shodor.org/interactivate/activities/Histogram/

http://www.shodor.org/interactivate/activities/PlopIt/

http://www.shodor.org/interactivate/activities/StemAndLeafPlotter/

Analyzing and Displaying Data Gizmos from ExploreLearning
Gizmos are interactive online tools with lesson plans that allow you to explore and learn about the topic in a virtual, dynamic environment. This page includes gizmos for box-and-whisker plots, histograms, stem-and-leaf plots, polling, and more. The gizmos work for 5 minutes for free. You can also sign up for a free trial account.
https://www.explorelearning.com/index.cfm?method=cResource.dspResourceExplorer&browse=Math/Grade+6-8/Data+Analysis+and+Probability

Box-and-Whisker Plots Quiz
Revise box-and-whisker plots with this interactive self-check quiz.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-825200-8&chapter=12&lesson=3&&headerFile=4

Create A Box and Whisker Chart
An online tool for creating a box-and-whisker plot from your own data. Includes lots of options.
https://www.meta-chart.com/box-and-whisker

Stem-and-Leaf Plots Quiz
An online multiple-choice quiz that is created randomly. Refresh the page (or press F5) to get another quiz.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-825200-8&chapter=12&lesson=1&&headerFile=4

Make Your Own Stem-and-Leaf Plot
Enter values from your own data, and this web page creates your stem-and-leaf plot for you.
http://www.mrnussbaum.com/graph/sl.htm

Comparing Data Displays
Practise interpreting and comparing dot plots, histograms, and box plots in this interactive online activity.
https://www.khanacademy.org/math/pre-algebra/pre-algebra-math-reasoning/pre-algebra-frequency-dot-plot/e/comparing-data-displays

Double Box-and-Whisker Plots Practice
Interactive practice questions about double box-and-whisker plots. The site may require a login with your Google or Facebook account or a free registration.
http://www.ck12.org/statistics/Double-Box-and-Whisker-Plots/asmtpractice/Double-Box-and-Whisker-Plots-Practice/

Displaying Univariate Data Practice
Interactive practice questions about distributions and various types of plots.
http://www.ck12.org/statistics/Displaying-Univariate-Data/asmtpractice/Displaying-Univariate-Data-Practice/

STATISTICAL MEASURES

Mean, Median, Mode, and Range
A lesson that explains how to calculate the mean, median, and mode for a set of data given in different ways. It also has interactive exercises.
http://www.cimt.org.uk/projects/mepres/book8/bk8i5/bk8_5i2.htm

Measures of Centre and Quartiles Quiz from ThatQuiz.org
An online quiz about the measures of centre and quartiles in boxplots, stem-and-leaf plots, and dot plots.
http://www.thatquiz.org/tq-5/?-jr0t0-l1-p0

GCSE Bitesize Mean, Mode and Median
Lessons with simple explanations and examples.
http://www.bbc.co.uk/schools/gcsebitesize/maths/statistics/measuresofaveragerev1.shtml

Measures of Variation — Self-Check Quiz
An online multiple-choice quiz about range, quartiles, and interquartile range that is created randomly. Refresh the page (or press F5) to get another quiz.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-825200-8&chapter=12&lesson=2&&headerFile=4

Mean Deviation
A simple explanation about what the mean absolute deviation is, how to find it, and what it means.
http://www.mathsisfun.com/data/mean-deviation.html

Mean Absolute Deviation
Several videos explaining how to calculate the mean absolute deviation of a data set.
http://www.onlinemathlearning.com/measures-variability-7sp3.html

Working with the Mean Absolute Deviation (MAD)
A tutorial and questions where you are asked to create line plots with a specified mean absolute deviation.
http://www.learner.org/courses/learningmath/data/session5/part_e/working.html

SAMPLING

Random and Biased Sampling
A comprehensive lesson to read that explains about unbiased types of sampling.
http://www.ck12.org/na/Random-and-Biased-Sampling-in-a-Population---7.SP.1,2-1/lesson/user%3Ac2ZveDJAb3N3ZWdvLm9yZw../Random-and-Biased-Sampling-in-a-Population---7.SP.1%252C2/

Identify a Random Sample
A video lesson about what a sample is and what makes a sample random. On the left of the web page are links to three other videos about representative (unbiased) samples and biased samples.
https://learnzillion.com/student/lessons/1844

Analyze Numbers of Jubes
Test a machine that randomly packages three types of fruit jubes. Look at patterns in numbers of jube types, manually choose jube types for a new packet, and much more.
https://fuse.education.vic.gov.au/Resource/LandingPage?ObjectId=c8dad524-5a86-4cf3-8ff4-cfe81813e230

Random or not? Analyse Runs of Jubes
Test a machine that randomly packages two types of fruit jube. Look at patterns in sequences of jube types, analyse the results of large samples, and much more.
https://fuse.education.vic.gov.au/Resource/LandingPage?ObjectId=2a4c7dd8-2319-46b2-a4d9-797174223fec

What Does Random Look Like?
Use the animation to generate several sequences of twenty coin flips, and try to get a feel for the features you would expect a random sequence to have.
http://nrich.maths.org/7250

Making Inferences about a Population by Analyzing Random Samples
This is a short instructional video about random sampling.
https://learnzillion.com/lesson_plans/6910-make-inferences-about-a-population-by-analyzing-random-samples

Random Sampling
Multiple-choice questions that test your understanding of the basics of random sampling.
https://www.khanacademy.org/math/recreational-math/math-warmup/random-sample-warmup/e/random-sample-warmup

Valid Claims
Multiple-choice questions to practice figuring out whether we took a random sample and whether we are able to draw valid conclusions from the data.
https://www.khanacademy.org/math/probability/statistical-studies/statistical-questions/e/valid-claims

Making Inferences from Random Samples
Multiple-choice questions about what can reasonably be inferred, from a random sample, about an entire population
https://www.khanacademy.org/quetzalcoatl/content-improvement/middle-school-content/e/making-inferences-from-random-samples

Matchbox Machine: Take a Sample
Check whether a machine is packing most matchboxes with an acceptable number of matches, take a sample of 100 matchboxes and make a boxplot to analyse the results, and much more.
https://fuse.education.vic.gov.au/Resource/LandingPage?ObjectId=38397b5d-25b8-4523-a0cd-0fc627a8a241

Fix the Matchbox Machine: Scoop Size and Speed
Check whether a machine is packing most matchboxes with an acceptable number of matches, look at boxplots made after taking samples of 100 matchboxes, and much more.
https://fuse.education.vic.gov.au/Resource/LandingPage?ObjectId=4087511a-60a5-48ec-b866-998882cc83cb

Polling: Neighborhood Gizmo
Conduct a phone poll of citizens in a small neighborhood to determine their response to a yes-or-no question. Use the results to estimate the sentiment of the entire population. Investigate how the error of this estimate changes as more people are polled. The gizmo works for 5 minutes for free. A free trial account is also available.
http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=507

Make inferences about a population by analyzing random samples
A video lesson that teaches how to make inferences about a population based on random samples.
https://learnzillion.com/student/lessons/1848

Capture-Recapture Method
The first resource below explains an activity you can do to learn how this method for estimating population size works. The second link is an actual interactive simulation.
https://nrich.maths.org/9609

http://www.cengage.com/biology/discipline_content/animations/capture_recapture.html

COMPARING TWO POPULATIONS

Comparing Populations - Khan Academy
Multiple-choice questions to practice comparing centres of distributions in terms of their spread.
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-probability-statistics/cc-7th-population-sampling/e/comparing-populations

Grade 7 Mathematics Module 5, Topic D, Lesson 22
In this lesson, students express the difference in sample means as a multiple of a measure of variability, and learn what a difference in sample means can show about the population means.
https://www.engageny.org/resource/grade-7-mathematics-module-5-topic-d-lesson-22

Grade 7 Mathematics Module 5, Topic D, Lesson 23
In this lesson, students use data from random samples to draw informal inferences about the difference in population means.
https://www.engageny.org/resource/grade-7-mathematics-module-5-topic-d-lesson-23

FACTS AND FIGURES

GapMinder
Visualising human development trends (such as poverty, health, gaps, income on a global scale) via interactive statistical graphs. This is an interactive, dynamic tool and not just static graphs.
https://www.gapminder.org/tools

WorldOdometers
World statistics updated in real time. Useful for general educational purposes—for some stunning facts.
http://www.worldometers.info

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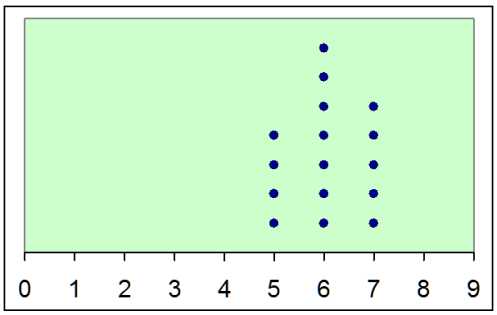
Comparing Two Populations

In this lesson, we will use measures of centre and variability to compare two sets of data.

Example 1. The two dot plots on the right show the ages of two different groups of children. The top plot shows a group of children of ages 5 to 7, and the bottom plot a group of ages 3 to 5.

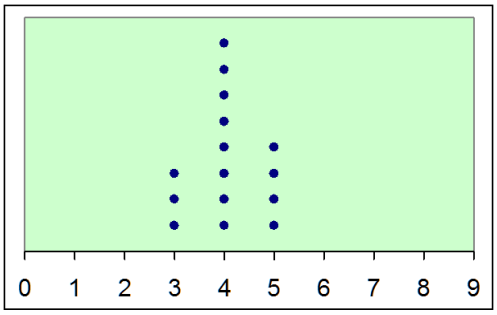


median 6 years IQR 1,5 years

The centre of the top plot is its median at 6 years old. The centre of the bottom plot is its median at 4 years old. Each distribution is clustered about its median with little variability (spread). Although both groups include 5-year-old children, there is otherwise no overlap in ages.

Intuitively we notice that the ages of these two groups are distinctly different. In statistical terms, we would say that there is a **significant difference** in the ages of the two groups.

There is a way to quantify the significance of the difference numerically: **compare the difference in the measures of centre to the measure of variability**.

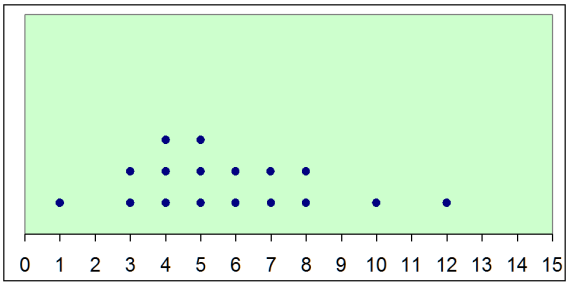


median 4 years IQR 1 year

The measures of centre—the medians—are 6 years old and 4 years old, so the difference between them is 2 years. The measure of variability—the interquartile range or IQR—of the first group is 1,5 years and of the second, 1 year. Let's round to 1 year in the comparison.

The difference in the medians (2 years) is about twice the measure of variability (about 1 year). This means that the difference is indeed significant. If the difference in centres had been only a fraction (1/2 or less) of the measure of variability, then the difference would not have been significant.

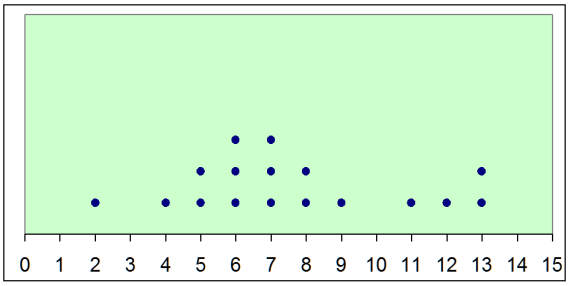
Example 2. The two dot plots on the right again show the ages of two different groups of children.



median 5 years IQR 3,5 years

This time, we can see from the plots that there is great variability in the ages in both groups. The data is very spread out. The data sets also overlap a lot: the first group has children from 1 to 12 years old, and the second from 2 to 13 years old, which means the overlap is from 2 to 12 years old.

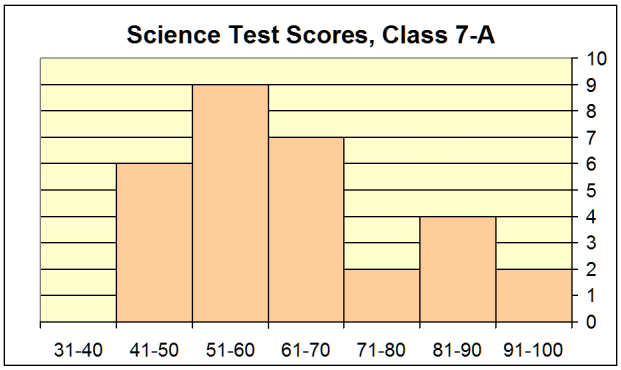
The difference in the medians is 2 years. However, the interquartile range is much bigger now (3,5 and 4,5 years; we can round it to 4 years). Therefore, the difference in the medians (2 years) is only about 1/2 of the measure of variability.



median 7 years IQR 4,5 years

This fact, along with the large overlap, helps us to see that the difference in the medians is not very significant. The ages of these groups of children are not greatly different.

Example 3. The two graphs show science test scores for two classes, 7-A and 7-B. Which class did better, generally speaking?



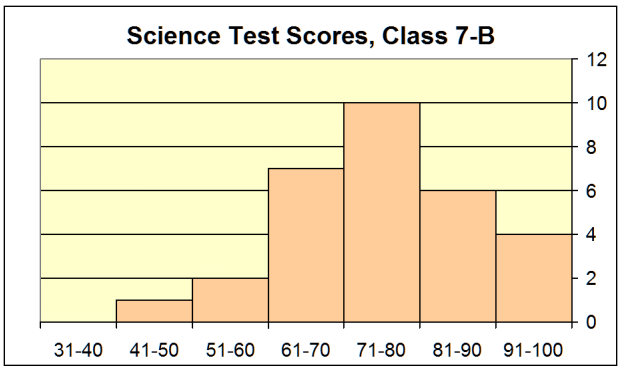
mean 64,2 MAD 11,8

We can see the answer just by looking at the distributions: The bars in the graph for 7-B are more skewed towards the right than the bars in the graph for 7-A. So class B did better.

To find out *how much* better, we will compare the means of both data sets. The mean test score for class 7-A was 64,2 points and for class 7-B it was 74,8 points. The difference is 10,6 points.

Is that a significant difference?

To check that, we compare the difference of 10,6 points to how variable or spread out the distributions are. The more variability there is in the two distributions, the bigger the difference in the means has to be for it to be significant.

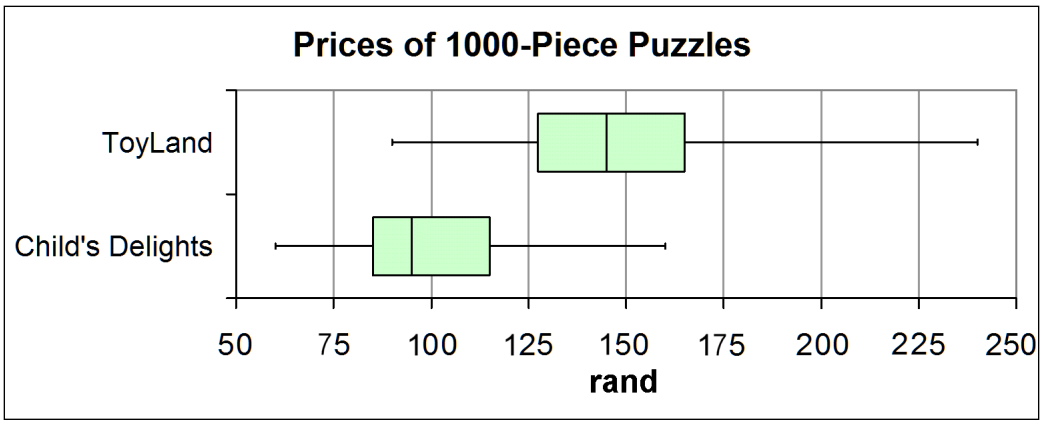


mean 74,8 MAD 10,3

We will use mean absolute deviation as a measure of variability. The mean absolute deviation of each data set is close to 11 points. So the difference in the two means (10,6 points) is about one time the measure of variability. That tells us that the difference *is* significant.

If the difference in the means had been, say, 0,3 times the measure of variability (only 3 points), then the difference wouldn't have been significant.

Example 4. The boxplots show the prices of 1000-piece puzzles in two stores, ToyLand and Child's Delights.

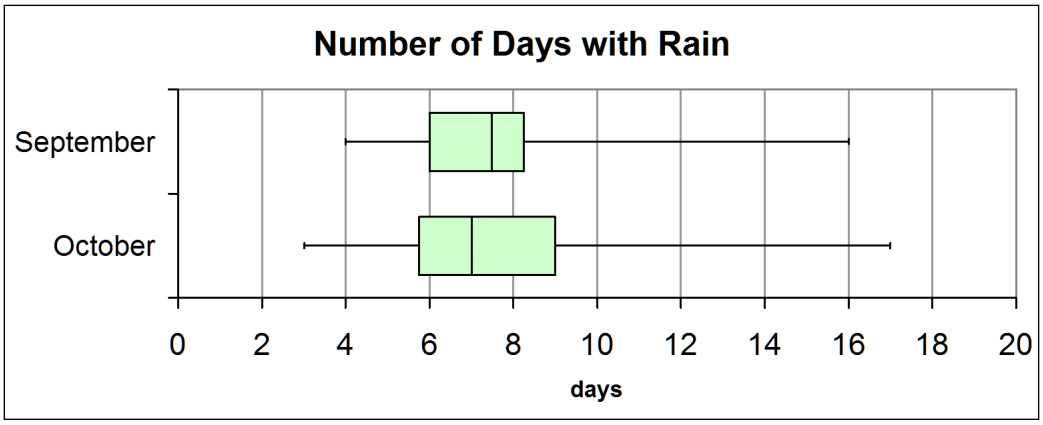


Boxplots make comparing sets of data very easy, since you can immediately see both the centres (medians) and the spread (interquartile range) of the data sets from the plot.

We can see from the medians that, overall, the puzzles in Child's Delights are cheaper. The prices in Toyland vary more, though, so you can find some inexpensive puzzles there, as well.

While the ranges of the prices are quite different, the interquartile ranges (the lengths of the boxes) are similar: about 40 rand for ToyLand and about 30 rand for Child's Delights. The difference in the medians is about 50 rand, which is about 1,5 times the interquartile range. That is a significant difference.

1. Lwasi studied the rain patterns in his home town. He made boxplots to show the number of days with rain in September and October from data collected over 20 years.



a. Describe the overlap of the two distributions.

b. Based on the medians, overall which month has more days with rain?

Which month has the greater variability in the number of days with rain?

c. Estimate from the plot the medians for October and September and their difference.

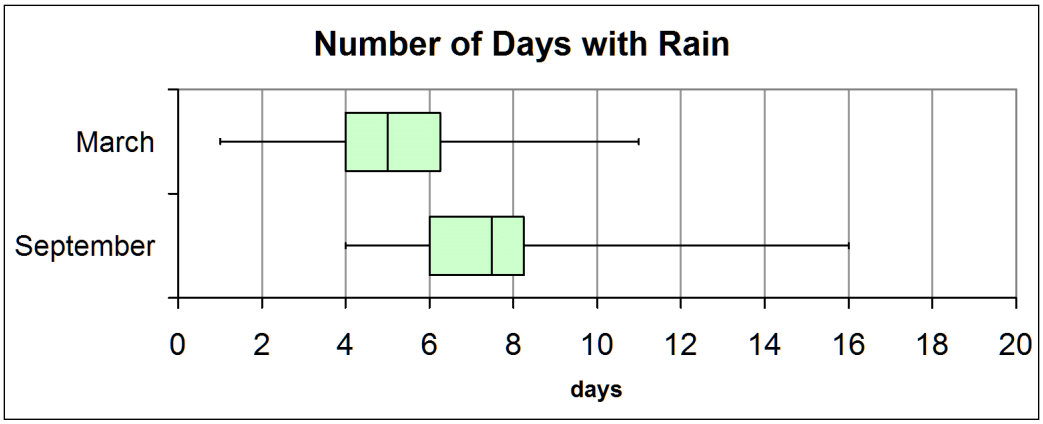
median (September)_____ median (October)_____ difference _____

d. Estimate from the plot the interquartile ranges for October and September.

IQR (September) _____ IQR (October) _____

e. Based on your answers to (c) and (d), is the difference in the medians significant?

2. Lwazi also made boxplots to compare the number of days with rain in March and September.



a. We can see that these two distributions overlap each other quite a bit, but not completely.
Let's say that a certain month had 4 rainy days.
Which is more likely, that the month was March or September?

b. Let's say that a certain month had 8 rainy days.
Which is more likely, that the month was March or September?

c. Based on the plots, overall which month has more days with rain?

Which month has the greater variability in the number of days with rain?

d. Find the difference in the medians and the interquartile ranges.

e. Compare the difference in the medians to the variability of the data.

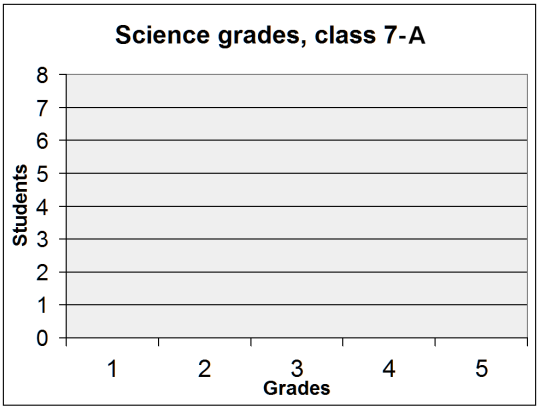
The difference in the medians is about _____ times the interquartile range.

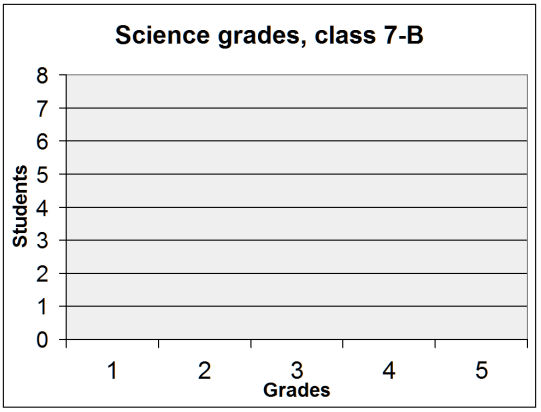
Are there significantly more days with rain in September than in March?

3. The following are the science grades of two 7th grade classes. This school grades on a five-point system where 5 = A, 4 = B, 3 = C, 2 = D, and 1 = F. Make bar graphs from the data.

Class 7-A	
Grades	**Students**
1	5
2	8
3	7
4	5
5	2

Class 7-B	
Grades	**Students**
1	3
2	6
3	7
4	7
5	4





a. Compare the bar graphs visually.
Based on the graphs, did either class do better overall? If so, which one?

Did either class have more variability in the grades? If so, which one?

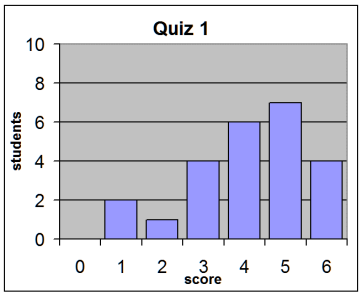
b. Now, calculate the mean of the grades for each class and the difference in the means.

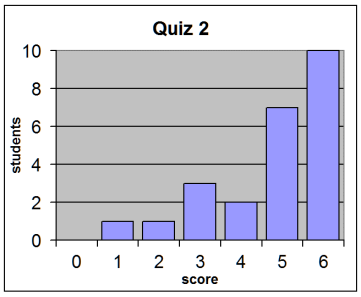
Class A, mean: _____ Class B, mean: _____ Difference: _____

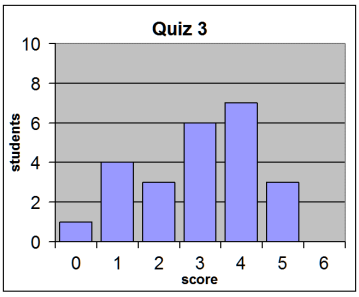
c. The mean absolute deviations of the data are 1,01 (class A) and 1,02 (class B).
The numbers are quite close. This means the variability is similar in both sets of data.

Compare the difference in the mean scores to the variability of the data, and use that to explain whether one of the classes did *significantly* better than the other.

4. Mrs. Mda gave her calculus class three quizzes. The bar graphs for the scores are below.







mean _____ mean _____ mean _____

MAD 1,14 MAD 1,13 MAD 1,22

a. Look at the graphs. Mrs. Mda felt one of the quizzes turned out too easy (the students didn't!). Which one?

b. In which quiz did the students fare the worst?

c. The mean scores for the three quizzes were: 2,96; 4,13; and 4,79. Match each mean with the correct graph.

d. Compare quiz 2 and quiz 3 now. What is the difference in the means for quiz 2 and quiz 3? _____

This difference is about _____ times the mean absolute deviation of the data (1,13).

Is the difference in the means significant?

5. The data below give you the heights in inches, of the players in a women's field hockey team and in a women's basketball team.

a. Determine the five-number summaries and draw side-by-side boxplots for the data.

Field Hockey Team Heights (inches)	
60	66
62	66
62	66
62	67
62	67
63	68
63	68
64	68
64	68
64	70
65	

Five-number summary

Minimum: _____

1st quartile: _____

Median: _____

3rd quartile: _____

Maximum: _____

Interquartile range:

_____

Basketball Team Heights (inches)	
67	72
68	72
69	72
69	74
69	75
70	76
71	

Five-number summary

Minimum: _____

1st quartile: _____

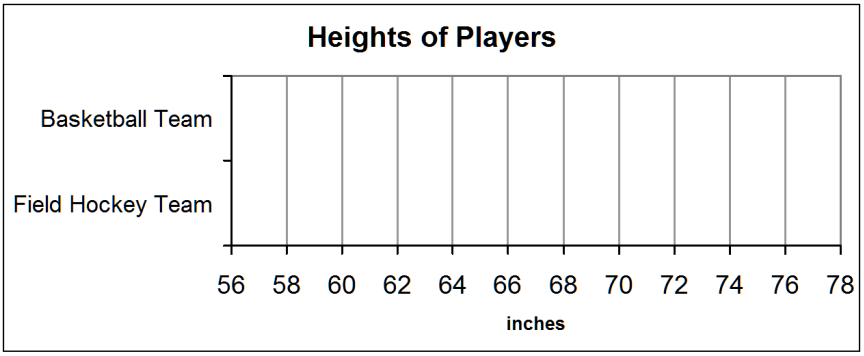
Median: _____

3rd quartile: _____

Maximum: _____

Interquartile range:

_____



b. Describe the overlap of the distributions.

c. Let's say you met a female athlete who was 70 inches tall. Which is more likely, that she plays field hockey or that she plays basketball?

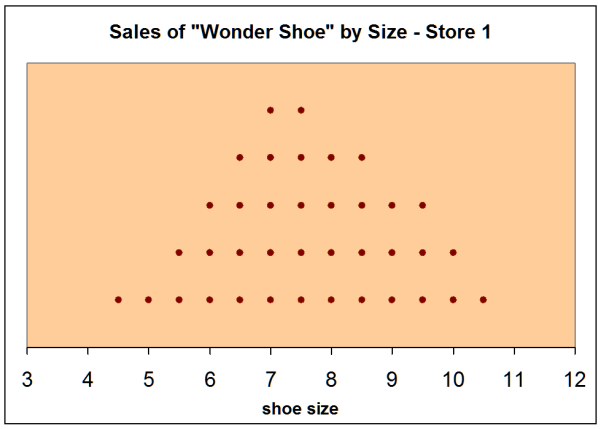
d. Based on the boxplots, overall which team appears to be taller?

Which team appears to have greater variability in the heights?

e. Is the difference in the medians significant?

Justify your reasoning.

6. Over the course of a month, a shoe store chain studied the sales of a "Wonder Shoe" by shoe size in two different stores. They did that for two purposes: (1) to check if there was any difference in the sales in the two stores, and (2) to obtain data to know how many of each size to order from their distributor.

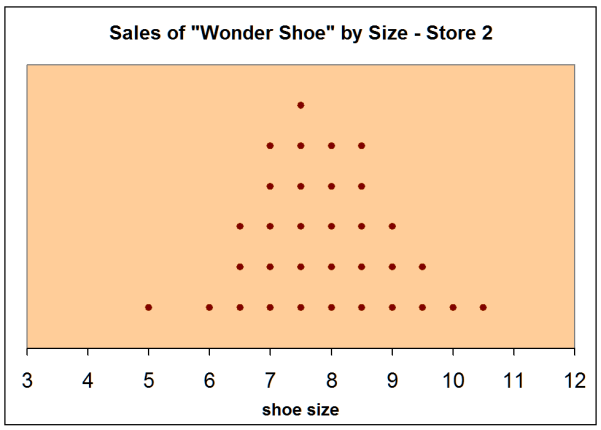


mean 7,61 MAD 1,20

a. Underline the correct choice.
There is (not much / some / a lot) of overlap in these distributions, and overall, they appear (different / similar).

b. Calculate the difference in the mean shoe size sold in these two stores

difference _____



mean 7,86 MAD 0,93

c. Compare the difference in the mean scores to the variability of the data, and use that to explain whether the difference in the mean scores is significant or not.

d. The shoe store is planning to order 500 pairs of Wonder Shoes. Based on the sales in these two stores, suggest how many of each size they should order. Keep in mind that the total has to be 500.