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Foreword

Math Mammoth Grade 6, comprises a complete maths curriculum for the sixth grade mathematics studies. This curriculum is essentially the same as the *Math Mammoth Grade 6* sold in the United States, only customised for South Africa use in a few aspects (listed below). The curriculum meets the Common Core Standards in the United States, but it may not properly align to the sixth grade standards in your country. However, you can probably find material for any missing topics in the neighbouring grades of Math Mammoth.

The South Africa version of Math Mammoth differs from the US version in these aspects:

- The curriculum uses metric measurement units, not both metric and customary (imperial) units.
- Spelling is British English instead of American English
- The pages are formatted for A4 paper size.
- Large numbers are formatted with a space as the thousands separator (such as 12 394).
  (The decimals are formatted with a decimal comma.)

This year starts out, in chapter 1 of part 6-A, with a revision of the four operations with whole numbers (including long division), place value and rounding. Students are also introduced to exponents and do some problem solving.

Chapter 2 starts the study of algebra topics, delving first into expressions and equations. Students practise writing expressions in different ways, and use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple equations. We also briefly study inequalities and using two variables.

Chapter 3 has to do with decimals. This is a long chapter, as we revise all of decimal arithmetic, just using more decimal digits than in 5th year. Students also convert measuring units in this chapter.

Ratios (chapter 4) is a new topic. Students are already familiar with finding fractional parts, and now it is time to advance that knowledge into the study of ratios, which arise naturally from dividing a quantity into many equal parts. We study such topics as rates, unit rates, equivalent ratios and problem solving using bar models.

Percent (chapter 5) is an important topic, because of its many applications in real life. The goal of this chapter is to develop a basic understanding of percent, to see percentages as decimals and to learn to calculate discounts.

In part 6-B, students study number theory, fractions, integers, geometry and statistics.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching maths!

Maria Miller, the author

Chapter 1: Revision of the Basic Operations
Introduction

The goal of the first chapter in year 6 is to revise the four basic operations with whole numbers, place value and rounding, as well as to learn about exponents and problem solving.

A lot of this chapter is revision, and I hope this provides a gentle start for 6th year maths. In the next chapter, we will delve into some beginning algebra topics.

Special notes for this chapter: problem solving

This chapter doesn't have many new concepts – only the concept of exponents and powers. Besides revising how to perform the four basic operations with pencil and paper, students also get some practice for problem solving.

Solving (word) problems in maths works much the same way as solving problems in real life. You may start out one way, come to a "dead end", and have to take an entirely different approach. Good problem solvers monitor their progress as they work, and change course if necessary.

Here is a list of general tips and strategies for solving mathematical problems that you can share with your student(s).

- If you cannot solve the original problem, try to **solve an easier, related problem first**. This may help you find a way to solve the original. For example, if the numbers in the problem seem intimidating, change them temporarily to easier numbers and see if you can then solve the problem. Or reduce the details mentioned in the problem to make it simpler, solve the simpler problem, then go back to the original. You can also try special cases of the problem at hand, at first.
- Drawing a sketch, a diagram (e.g. a bar model), or making a table can be very helpful.
- **Check your final answer** if at all possible, using a different method. For example, division problems can be checked by multiplication and subtractions by addition. Multi-step problems can often be solved in different ways or in a different order.

  At the very least, **check that your answer is reasonable** and actually makes sense. If the problem is asking how many days of vacation someone might get in a year, and you get an answer in the thousands, you can tell something went really wrong.

  Once you find your answer is wrong – maybe it doesn't make sense – it is NOT time to cry and give up. Do you know how many times Thomas Alva Edison tried and failed, until he finally found a way to make a commercially viable electric light bulb? Thousands of times.

  Perseverance is something that is very necessary when you encounter problems in real life, and I don't mean maths problems. Everyone fails, but it is those who keep trying who will ultimately succeed. Every successful entrepreneur can tell you that. Failing is *not* a sign of being stupid. It is a sign of being a human. ALL of us make mistakes and fail, and ALL of us improve as we keep trying.
- Often, it is easier and neater to perform paper-and-pen calculations (long addition, subtraction, multiplication, division) on a grid paper.
- The space in the worktext may not be enough. Use as much scrap paper (extra paper) as necessary.
- Remember to include a unit (if applicable) in the answers to word problems.

General principles in using the curriculum

Please note that it is not recommended to assign all the exercises by default. Use your judgement, and try to vary the number of assigned exercises according to the student's needs.

The specific lessons in the chapter can take several days to finish. They are not "daily lessons." Instead, use the general guideline that sixth graders should finish about two pages daily or 10 pages a week in order to finish the curriculum in about 36 weeks.

See the user guide at in the beginning of this book or online at **https://www.mathmammoth.com/userguides/** for more guidance on using and pacing the curriculum.

The Lessons in Chapter 1

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch1

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Powers and Exponents

Exponents are a "shorthand" for writing repeated multiplications by the same number.

For example, $2 \times 2 \times 2 \times 2 \times 2$ is written 2^5.
$5 \times 5 \times 5 \times 5 \times 5 \times 5$ is written 5^6.

The tiny raised number is called the **exponent**. It tells us how many times the *base* number is multiplied by itself.

exponent

$$12^{4} = 12 \times 12 \times 12 \times 12$$
$$= 20\ 736$$

base

The expression 2^5 is read as "two to the fifth power," "two to the fifth," or "two raised to the fifth power."

Similarly, 7^9 is read as "seven to the ninth power," "seven to the ninth," or "seven raised to the ninth power."

The "powers of 6" are simply expressions where 6 is raised to some power: For example, 6^3, 6^4, 6^{45} and 6^{99} are powers of 6. What would powers of 10 be?

Expressions with the exponent 2 are usually read as something "**squared.**" For example, 11^2 is read as "**eleven squared.**" That is because it gives us *the area of a square* with the side length of 11 units.

Similarly, if the exponent is 3, the expression is usually read using the word "**cubed.**" For example, 31^3 is read as "**thirty-one cubed**" because it gives the *volume of a cube* with the edge length of 31 units.

1. Write the expressions as multiplications, and then solve them in your head.

a. $3^2 = \underline{\ 3 \times 3 = 9\ }$ **b.** 1^6

c. 4^3 **d.** 10^4

e. 5^3 **f.** 10^2

g. 2^3 **h.** 8^2

i. 0^5 **j.** 10^5

k. 50^2 **l.** 100^3

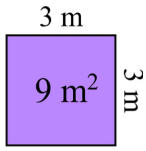
2. Rewrite the expressions using an exponent, then solve them. You may use a calculator.

a. $2 \times 2 \times 2 \times 2 \times 2 \times 2$ **b.** $8 \times 8 \times 8 \times 8 \times 8$

c. 40 squared   **d.** $10 \times 10 \times 10 \times 10$

e. nine to the eighth power   **f.** eleven cubed

You just learned that the expression 7^2 is read "seven *squared*" because it tells us the area of a *square* with a side length of 7 units. Let's compare that to square metres and other units of area.



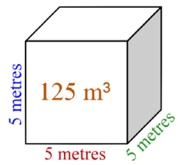
If the sides of a square are 3 m long, then its area is 3 m × 3 m = 9 m^2 or nine square metres.

Notice that the symbol for square metres is $\mathbf{m^2}$. This means "**metre** × **metre**." We are, in effect, squaring the unit *metre* (multiplying the unit of length *metre* by itself)!

The expression $(9\text{ cm})^2$ means 9 cm × 9 cm. We multiply 9 by itself, but we also multiply the unit *cm* by itself. That is why the result is **81 $\mathbf{cm^2}$**. The square centimetre (cm^2) comes from multiplying "**centimetre** × **centimetre**."

We do the same thing with any other unit of length to form the corresponding unit for area, such as square kilometres or square millimetres.

In a similar way, to calculate the volume of this cube, we multiply 5 m × 5 m × 5 m = 125 m^3. We not only multiply 5 by itself three times, but also multiply the unit *metre* by itself three times (metre × metre × metre) to get the unit of volume "cubic metre" or m^3.



3. Express the area (A) as a multiplication, and solve.

a. A square with a side of 12 kilometres: A = 12 km × 12 km = ____________	**b.** A square with sides 6 m long: A = ____________________
c. A square with a side length of 6 centimetres: A = ____________________	**d.** A square with a side with a length of 12 cm: A = ____________________

4. Express the volume (V) as a multiplication, and solve.

a. A cube with an edge of 2 cm: V = 2 cm × 2 cm × 2 cm = ____________	**b.** A cube with edges 10 cm long each: V = ____________________
c. A cube with edges 1 m in length: V = ____________________	**d.** A cube with edges that are all 5 m long: V = ____________________

5. **a.** The perimeter of a square is 40 centimetres. What is its area?

b. The volume of a cube is 64 cubic centimetres. How long is its edge?

c. The area of a square is 121 m^2. What is its perimeter?

d. The volume of a cube is 27 cm^3. What is the length of one edge?

The powers of 10 are very special —and very easy! Notice that the exponent tells us *how many zeros* there are in the answer.	$10^1 = 10$ $10^2 = 10 \times 10 = 100$ $10^3 = 10 \times 10 \times 10 = 1000$	$10^4 = 10\,000$ $10^5 = 100\,000$ $10^6 = 1\,000\,000$

6. Fill in the patterns. In part (d), choose your own number to be the base.
Use a calculator in parts (c) and (d).

a.	b.	c.	d.
$2^1 =$	$3^1 =$	$5^1 =$	
$2^2 =$	$3^2 =$	$5^2 =$	
$2^3 =$	$3^3 =$	$5^3 =$	
$2^4 =$	$3^4 =$	$5^4 =$	
$2^5 =$	$3^5 =$	$5^5 =$	
$2^6 =$	$3^6 =$	$5^6 =$	

7. Look at the patterns above. Think carefully how each step comes from the previous one. Then answer.

a. If $3^7 = 2187$, how can you use that result to find 3^8?

b. Now find 3^8 without a calculator.

c. If $2^{45} = 35\,184\,372\,088\,832$, use that to find 2^{46} without a calculator.

8. Fill in.

a. 17^2 gives us the ______________ of a ________________ with sides ______ units long.

b. 101^3 gives us the ______________ of a ________________ with edges ______ units long.

c. 2×6^2 gives us the _____________ of two ________________ with sides ______ units long.

d. 4×10^3 gives us the _____________ of ____ ________________ with edges ______ units long.

Puzzle Corner

Make a pattern, called a **sequence**, with the powers of 2, starting with 2^6 and going *backwards* to 2^0. At each step, *divide* by 2. What is the logical (though surprising) value for 2^0 from this method?

Make another, similar, sequence for the powers of 10. Start with 10^6 and divide by 10 until you reach 10^0. What value do you calculate for 10^0?

Try this same pattern for at least one other base number, *n*. What value do you calculate for n^0? Do you think it will come out this way for every base number?

Why or why not?

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Chapter 2: Expressions and Equations
Introduction

In this chapter students start learning *algebra* – in a nutshell, the way to "do arithmetic with variables". Algebra enables us to solve real-life problems abstractly, in terms of variable(s) instead of numbers, and it is a very powerful tool.

Special notes for this chapter: algebra

The chapter focuses on two important basic concepts: **expressions** and **equations**. We also touch on inequalities and graphing on a very introductory level. In order to make the learning of these concepts easier, the expressions and equations in this chapter do not involve negative numbers (as they typically do when studied in pre-algebra and algebra). Integers are introduced in part 6-B, and then Math Mammoth grade 7 deals with algebraic concepts including with negative numbers.

We start out by revising the order of operations. Then the lessons focus on algebraic expressions. Students encounter the exact definition of an expression, a variable, and a formula, and practise writing expressions in many different ways. They study equivalent expressions and simplifying expressions. Length and area are two simple contexts I have used extensively for students to learn to write and simplify expressions.

In these lessons, students have opportunities to **write real-world scenarios in terms of variables**. In other words, they *decontextualise* – they abstract a given situation and represent it symbolically. Then, as they learn algebra, they learn to manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents, and to reason abstractly about those quantities represented by the variables.

The other major topic of the chapter is equations. Students learn some basics, such as, the solutions of an equation are the values of the variables that make the equation true. They use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. I have also included a few easy two-step equations.

Next, students solve and graph simple inequalities, again practising the usage of variables to represent quantities. Lastly, they are introduced to the usage of *two* variables in algebra, including how to graph that relationship on a coordinate plane. This is an important topic, as so many real-life situations involve a relationship between two quantities, and graphing that relationship is an important tool in mathematical modelling.

You will find free videos covering many topics of this chapter of the curriculum at https://www.mathmammoth.com/videos/ (choose 6th grade).

The Lessons in Chapter 2

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
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- **animations** and interactive **illustrations** of maths concepts;
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https://l.mathmammoth.com/gr6ch2

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The Distributive Property

The **distributive property** states that $a(b + c) = ab + ac$

It may look like a meaningless or difficult equation to you now, but don't worry, it will become clearer!

The equation $\boldsymbol{a(b + c) = ab + ac}$ means that you can *distribute* the multiplication (by a) over the sum $b + c$ so that you multiply the numbers b and c separately by a, and add last.

You have already used the distributive property! When you separated $3 \cdot 84$ into $3 \cdot (80 + 4)$, you then multiplied 80 and 4 *separately* by 3, and added last: $3 \cdot 80 + 3 \cdot 4 = 240 + 12 = 252$.
We called this using "partial products" or "multiplying in parts."

Example 1. Using the distributive property, we can write the product $2(x + 1)$ as $2x + 2 \cdot 1$, which simplifies to $2x + 2$.

Notice what happens: Each term in the sum $(x + 1)$ gets multiplied by the factor 2! Graphically:

$$2(x + 1) = \underline{2x} + \underline{2 \cdot 1}$$

Example 2. To multiply $s \cdot (3 + t)$ using the distributive property, we need to multiply *both* 3 and t by s:

$$s \cdot (3 + t) = s \cdot 3 + s \cdot t, \text{ which simplifies to } 3s + st.$$

1. Multiply using the distributive property.

a. $3(90 + 5) = 3 \cdot$ ___ $+ 3 \cdot$ ___ $=$	**b.** $7(50 + 6) = 7 \cdot$ ___ $+ 7 \cdot$ ___ $=$
c. $4(a + b) = 4 \cdot$ ___ $+ 4 \cdot$ ___ $=$	**d.** $2(x + 6) = 2 \cdot$ ___ $+ 2 \cdot$ ___ $=$
e. $7(y + 3) =$	**f.** $10(s + 4) =$
g. $s(6 + x) =$	**h.** $x(y + 3) =$
i. $8(5 + b) =$	**j.** $9(5 + c) =$

Example 3. We can use the distributive property also when the sum has three or more terms. Simply multiply *each term* in the sum by the factor in front of the brackets:

$$5(x + y + 6) = 5 \cdot x + 5 \cdot y + 5 \cdot 6, \text{ which simplifies to } 5x + 5y + 30$$

2. Multiply using the distributive property.

a. $3(a + b + 5) =$	**b.** $8(5 + y + r) =$
c. $4(s + 5 + 8) =$	**d.** $3(10 + c + d + 2) =$

Example 4. Now one of the terms in the sum has a coefficient (the 2 in $2x$):

$$6(2x + 3) \ = \ 6 \cdot 2x \ + \ 6 \cdot 3 \ = \ 12x + 18$$

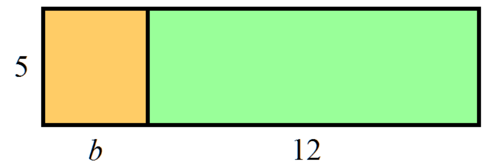
3. Multiply using the distributive property.

a. $2(3x + 5) \ =$	**b.** $7(7a + 6) \ =$
c. $5(4a + 8b) \ =$	**d.** $2(4x + 3y) \ =$
e. $3(9 + 10z) \ =$	**f.** $6(3x + 4 + 2y) \ =$
g. $11(2c + 7a) \ =$	**h.** $8(5 + 2a + 3b) \ =$

To understand even better why the the distributive property works, let's look at an area model (this, too, you have seen before!).

The area of the whole rectangle is 5 times (b + 12).

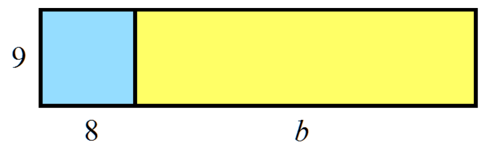
But if we think of it as *two* rectangles, the area of the first rectangle is $5b$, and of the second, $5 \cdot 12$.



Of course, these two expressions have to be equal:

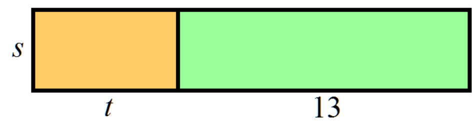
$$5 \cdot (b + 12) \ = \ 5b \ + \ 5 \cdot 12 \ = \ 5b + 60$$

4. Write an expression for the area in two ways, thinking of one rectangle or two.



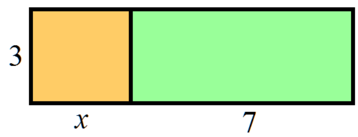
a. 9(_____ + _____) and

$9 \cdot$ _____ $+ 9 \cdot$ _____ =

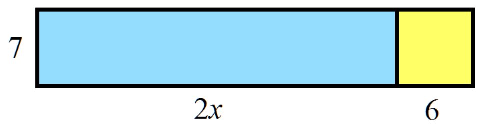


b. s(_____ + _____) and

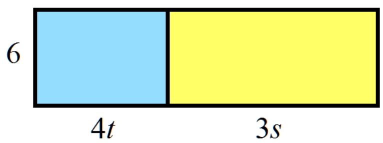
$s \cdot$ _____ $+ s \cdot$ _____ =



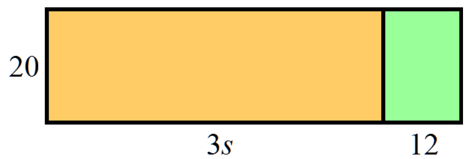
c. ____ (_____ + _____) and



d.

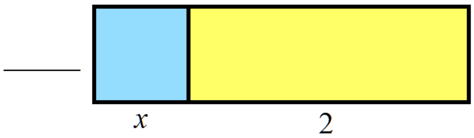


e.

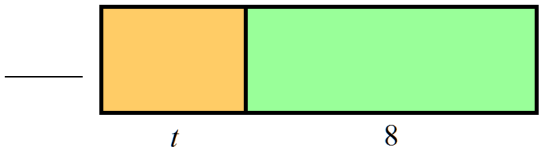


f.

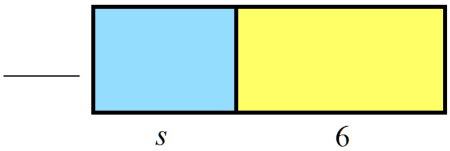
5. Find the missing number or variable in these area models.



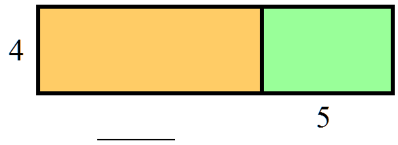
a. ____ $(x + 2) = 3x + 6$



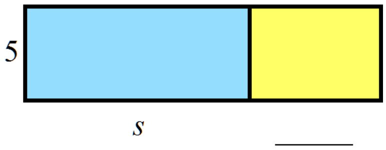
b. _____ $(t + 8) = 7t + 56$



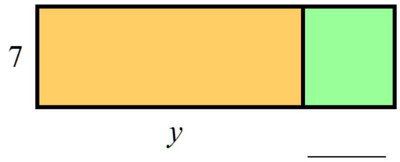
c. The total area is $9s + 54$.



d. $4($ ____ $+ 5) = 4z + 20$



e. $5(s +$ ____ $) = 5s + 30$



f. The total area is $7y + 42$.

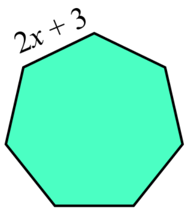
6. Find the missing number in the equations.

a. ____ $(x + 5) = 6x + 30$	**b.** $10(y +$ ____ $) = 10y + 30$
c. $6($ ____ $+ z) = 12 + 6z$	**d.** $8(r +$ ____ $) = 8r + 24$

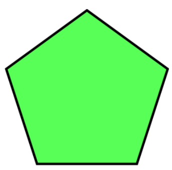
7. Find the missing number in the equations. These are just a little bit trickier!

a. ____ $(2x + 5) = 6x + 15$	**b.** ____ $(3w + 5) = 21w + 35$
c. ____ $(6y + 4) = 12y + 8$	**d.** ____ $(10s + 3) = 50s + 15$
e. $2($ _____ $+ 9) = 4x + 18$	**f.** $4($ ______ $+ 3) = 12x + 12$
g. $5($ _____ $+ 3) = 20y + 15$	**h.** $8($ ____ $+$ ____ $+ 7) = 40t + 8s + 56$

8. Write an expression for the perimeter of this regular heptagon as a *product*.
Then multiply the expression using the distributive property



9. The perimeter of a regular pentagon is $15x + 5$. How long is one of its sides?



When we use the distributive property "backwards," and write a sum as a product, it is called **factoring**.

Example 5. The sum $5x + 5$ can be written as $5(x + 1)$. We took the SUM $5x + 5$ and wrote it as a PRODUCT— something times something, in this case 5 times the quantity $(x + 1)$.

Example 6. The sum $24x + 16$ can be written as the product $8(3x + 2)$.

Notice that the numbers 24 and 16 are *both divisible by 8*! That is why we write 8 as one of the factors.

10. Think of the distributive property "backwards," and factor these sums. Think of divisibility!

a. $6x + 6$ = ____ $(x + 1)$	**b.** $8y + 16$ = 8(____ + ____)
c. $15x + 45$ = ____ (x + ____)	**d.** $4w + 40$ = ____ (w + ____)
e. $6x + 30$ = ____ (____ + ____)	**f.** $8x + 16y + 48$ = ____ (____ + ____ + ____)

11. Factor these sums (writing them as products). Think of divisibility!

a. $8x + 4$ = ____ ($2x$ + ____)	**b.** $15x + 10$ = ____ ($3x$ + ____)
c. $24y + 8$ = ____ (____ + ____)	**d.** $6x + 3$ = ____ (____ + ____)
e. $42y + 14$ = ____ (____ + ____)	**f.** $32x + 24$ = ____ (____ + ____)
g. $27y + 9$ = ____ (____ + ____)	**h.** $55x + 22$ = ____ (____ + ____)
i. $36y + 12$ = ____ (____ + ____)	**j.** $36x + 9z + 27$ = ____ (____ + ____ + ____)

12. The perimeter of a square is $48x + 16$. How long is its side?

Puzzle Corner

As a shopkeeper, you need to purchase 1000 items to get a wholesale (cheaper) price of R8 per item, so you do. You figure you might sell 600 of them. You also want to advertise a R3 discount to your customers. What should the non-discounted selling price be for you to actually earn a R500 profit from the sale of these items?

Epilogue: It may be hard to see now where distributive property or factoring might be useful, but it IS extremely necessary later in algebra when solving equations.

To solve the problem above, you *can* figure it out without algebra, but it becomes fairly straightforward if we write an equation for it. Let p be the non-discounted price. Then p − R3 is the price with the discount. We get:

$$\begin{aligned} \text{What we need to take in} &= \text{pay to supplier + profit} \\ 600(p - \text{R3}) &= 1000 \cdot \text{R8} + \text{R500} \end{aligned}$$

To solve this equation, one needs to use the distributive property in the very first step:

$$\begin{aligned} 600p - \text{R1800} &= \text{R8500} \\ 600p &= \text{R10 300} \end{aligned}$$

(Can you solve this last step yourself?)

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Chapter 3: Decimals
Introduction

In this chapter we study all four operations of decimals, the metric system and using decimals with measuring units. Most of these topics have already been studied in 5th grade, but in 5th grade we were using numbers with a maximum of three decimal digits. This time there is no such restriction, and the decimals used can have many more decimal digits than that.

However, since the topics are the same, consider assigning only one-fourth to one-half of the exercises initially. Monitor the student's progress and assign more if needed. The skipped problems can be used for revision later.

We start out by studying place value with decimals and comparing decimals up to six decimal digits. The next several lessons contain a lot of revision, just using longer decimals than in 5th grade: adding and subtracting decimals, rounding decimals, multiplying and dividing decimals, fractions and decimals, and multiplying and dividing decimals by the powers of ten.

Since the chapter focuses on restudying the mechanics of decimal arithmetic, it is a good time to stress to your student(s) the need for **accurate calculations** and for **checking one's final answer**. Notice how the lessons often ask students to estimate the answer before calculating the exact answer. Estimation can be used as a type of check for the final answer: if the final answer is far from the estimation, there is probably an error in the calculation. It can also be used to check if an answer calculated with a calculator is likely correct.

In the lessons about multiplication and division of decimals, students work both with mental maths and with standard algorithms. The lessons that focus on mental maths point out various patterns and shortcuts for students, helping them to see the structure and logic in maths. I have also explained why the common rules (or shortcuts) for decimal multiplication and decimal division actually work, essentially providing a mathematical proof on a level that 6th graders can hopefully understand.

The last lessons deal with measuring units and the metric system, rounding out our study of decimals.

Consider mixing the lessons here with lessons from some other chapter. For example, the student could study decimals and some other topic on alternate days, or study a little of each topic each day. Such, somewhat spiral, usage of the curriculum can help prevent boredom, and also to help students retain the concepts better.

You will find free videos covering many topics of this chapter at **https://www.mathmammoth.com/videos/**.

As a reminder, check out this list of resources for challenging problems:
https://l.mathmammoth.com/challengingproblems

I recommend that you at least use the first resource listed, Math Stars Newsletters.

The Lessons in Chapter 3

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch3

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Fractions and Decimals

You already know how to change decimals to fractions. The number of decimal digits tells you the denominator —it is always a power of ten with as many zeros as you have decimal digits. For the numerator, just copy all the digits from the number. **Example:** $3{,}0928 = \dfrac{30\,928}{10\,000}$	You can also write this as a mixed number, in which case you take the whole number part from the decimal, and the actual decimal digits form the numerator: $15{,}30599 = \dfrac{1\,530\,599}{100\,000} = 15\,\dfrac{30\,599}{100\,000}$

1. Write as fractions.

a. 0,09	**b.** 0,005	**c.** 0,045
d. 0,00371	**e.** 0,02381	**f.** 0,0000031

2. Write as fractions and also as mixed numbers.

a. 2,9302	**b.** 2,003814
c. 5,3925012	**d.** 3,0078
e. 3,294819	**f.** 45,00032

When changing a **fraction into a decimal**, we have several tools in our "toolbox." <u>**Tool 1.**</u> If the denominator of a fraction is already a power of ten, there is not much to do but to write it as a decimal. The number of zeros in the power of ten tells you the number of decimal digits you need.	$\dfrac{3}{10} = 0{,}3$	$\dfrac{451\,593}{10\,000} = 45{,}1593$

3. Write as decimals.

a. $\dfrac{36}{100}$	**b.** $\dfrac{5009}{1000}$	**c.** $1\,\dfrac{45}{1000}$
d. $\dfrac{3908}{10\,000}$	**e.** $2\,\dfrac{593}{100\,000}$	**f.** $\dfrac{5903}{1\,000\,000}$
g. $\dfrac{45\,039\,034}{1\,000\,000}$	**h.** $\dfrac{435\,112}{10\,000}$	**i.** $\dfrac{450\,683}{100\,000}$

<u>Tool 2.</u> With some fractions, you can find an equivalent fraction with a denominator of 10, 100, 1000, *etc*. and then write the fraction as a decimal.	$\frac{27}{30} \overset{\div 3}{=} \frac{9}{10} = 0{,}9$	$\frac{66}{200} \overset{\div 2}{=} \frac{33}{100} = 0{,}33$	$\frac{3}{8} \overset{\cdot 125}{=} \frac{375}{1000} = 0{,}375$

4. Write as decimals. Think of the equivalent fraction that has a denominator of 10, 100, or 1 000.

a. $\frac{1}{5}$	**b.** $\frac{1}{8}$	**c.** $1\frac{1}{20}$
d. $3\frac{9}{25}$	**e.** $\frac{12}{200}$	**f.** $8\frac{3}{4}$
g. $4\frac{3}{5}$	**h.** $\frac{13}{20}$	**i.** $\frac{7}{8}$
j. $\frac{11}{125}$	**k.** $\frac{24}{400}$	**l.** $\frac{95}{500}$

5. In these problems, you see both fractions and decimals. Either change the decimal into a fraction, or vice versa. You decide which way is easier! Then, calculate in your head.

a. $0{,}2 + \frac{1}{4}$	**b.** $0{,}34 + 1\frac{1}{5}$	**c.** $2\frac{3}{5} + 1{,}3$	**d.** $\frac{5}{8} - 0{,}09$
e. $0{,}02 + \frac{3}{4}$	**f.** $1{,}9 + 3\frac{1}{8}$	**g.** $\frac{14}{20} - 0{,}23$	**h.** $\frac{18}{25} + 0{,}07$

<u>Tool 3.</u> Most of the time, in order to change a fraction to a decimal, you simply treat the fraction as a division problem and divide (with a calculator or long division).	$\frac{5}{6} = 5 \div 6 = 0{,}83333... \approx 0{,}83$

6. Write the fractions as decimals. Use long division on blank paper. Give your answers to three decimal digits.

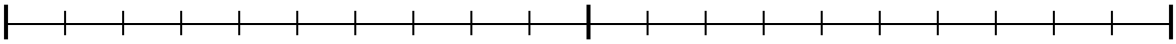
a. $\frac{2}{9} =$	**b.** $\frac{3}{7} =$	**c.** $\frac{7}{16} =$

7. Use a calculator to write these fractions as decimals. Give your answers to three decimal digits.

a. $\frac{1}{11} =$	**b.** $\frac{3}{23} =$	**c.** $\frac{47}{56} =$

8. Label the bold tick marks on the number line as “0,” “1” and “2.” Then mark the following numbers on it where they belong.

$$0{,}2, \quad \frac{1}{4}, \quad 0{,}65, \quad 1\frac{1}{3}, \quad 0{,}04, \quad \frac{2}{5}, \quad 1{,}22, \quad 1\frac{3}{4}, \quad 1{,}95, \quad 1\frac{4}{5}$$



9. One bag of powdered milk contains 900 g. Another contains 3/4 kg.
What is the combined weight of the two?

10. Flax seed costs R180 per kilogram. Sally bought 1,75 kg of it.
Calculate the total price of Sally’s purchase (in rand).

11. Explain two different ways to calculate the price of 3/8 of a kilogram of bird seed, if one kilogram costs R12,95. You do not have to calculate the price; just explain or show two ways of *how* to do it.

12. A foundation for a building measures 14 ¾ m by 20 ⅖ m.
Find its area in square metres, as a decimal.

13. Anna lives in Australia, and her friend Cindy lives in the USA. Cindy said that she used 1 ¾ pounds of beef for a certain dish. Anna needs to know this amount in grams, but also, she wants to make only ⅔ of the recipe since her family is smaller. One pound is 454 grams. Find how much beef, in grams, Anna needs to make the dish for her family.

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Chapter 4: Ratios
Introduction

In this chapter we concentrate on the concept of ratio and various applications involving ratios and rates.

The chapter starts out with the basic concepts of ratio, rate and unit rate. We also connect the concept of rates (specifically, tables of equivalent rates) with ordered pairs, use equations (such as $y = 3x$) to describe these tables, and plot the ordered pairs in the coordinate plane.

Next, we study various kinds of word problems involving ratios and use a bar model to solve these problems in two separate lessons. These lessons tie ratios in with the student's previous knowledge of bar models as a tool for problem solving.

Lastly, students encounter the concept of aspect ratio, which is simply the ratio of a rectangle's width to its height or length, and they solve a variety of problems involving aspect ratio.

This chapter contains lots of opportunities for problem solving, once again. In the lessons that use bar models, encourage your students to communicate their thinking and explain (justify) how they solved the problems. It doesn't have to be fancy. All we are looking for is some explanation of what the student did and why. The bar models provide an excellent way for the students to demonstrate their reasoning here. Essentially, they are practising constructing a **mathematical argument (parameter)**.

Once again, there are some free videos for the topics of this chapter at **https://www.mathmammoth.com/videos/** (choose 6th grade).

The Lessons in Chapter 4

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch4

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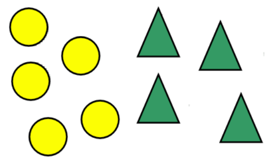
Ratios and Rates

A **ratio** is simply a *comparison* of two numbers or other quantities.

To compare the circles to the triangles in the picture, we say that the *ratio of circles to triangles* is 5:4 (read "five to four").

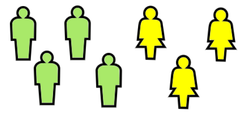


We can write this ratio (in text) in many different ways:

- The ratio of circles to triangles is 5:4 (read "5 to 4").
- The ratio of circles to triangles is 5 to 4.
- The ratio of circles to triangles is $\frac{5}{4}$.
- For each five circles, there are four triangles.

The two numbers in the ratio are called the **first term** and the **second term** of the ratio.

In this picture, the ratio of males to females is 4:3. However, the ratio of *females to males* is **3:4**. The order in which the terms are mentioned does matter!

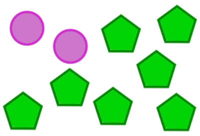


We can also compare a part to the whole. The ratio of males to everyone is 4:7.

Also, we can use fractions to describe the same image: $\frac{4}{7}$ of the people are males, and $\frac{3}{7}$ are females.

1\. Describe the images using ratios and fractions.

a.



The ratio of circles to pentagons is ____ : ____

The ratio of pentagons to all shapes is ____ : ____

$\frac{}{}$ of the shapes are pentagons.

b.

The ratio of hearts to stars is ____ : ____

The ratio of stars to all shapes is ____ : ____

$\frac{}{}$ of the shapes are stars.

2\. **a.** Draw a picture: There are hearts and circles, and the ratio of hearts to all the shapes is 1:3.

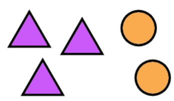
b. What is the ratio of hearts to circles?

3\. Look at the picture of the triangles and circles. If we drew more triangles and circles in the same ratio, how many circles would there be …

a. … for 9 triangles?

b. … for 15 triangles?

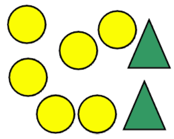
c. … for 300 triangles?



We can **simplify** ratios in exactly the same way we simplify fractions (using division).

The ratio of circles to triangles is $\frac{6}{2} = \frac{3}{1}$. We say that 6:2 and 3:1 are **equivalent ratios**.

The simplified ratio 3:1 means that for each three circles, there is one triangle.



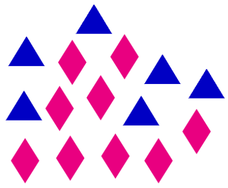
Example 1. When we simplify the ratio of hearts to stars to the *lowest terms*, we get $\frac{12}{16} = \frac{3}{4}$, or 12:16 = 3:4. This means that for each three hearts, there are four stars.

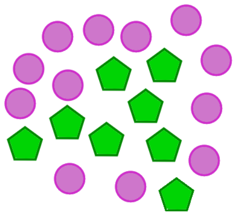
We could also simplify like this: 12:16 = 6:8. These two ratios are equivalent, but neither is simplified to the lowest terms.

4. Write the ratios, and then simplify them to lowest terms.

a. The ratio of diamonds to triangles is _____ : _____ or _____ : _____ .

There are ______ diamonds to every _______ triangles.





b. The ratio of pentagons to circles is _____ : _____

or _____ : _____ .

There are ______ circles to every _______ pentagons.

5. **a.** Draw a picture in which (1) there are five squares for each two hearts, and (2) there is a total of six hearts.

b. Write the ratio of all hearts to all squares, and simplify this ratio to lowest terms.

c. Write the ratio of all shapes to hearts, and simplify this to lowest terms.

6. Write the ratios using a colon. Simplify the ratios if possible.

a. 15 to 20 **b.** 16 to 4

c. 25 to 10 **d.** 13 to 30

7. Write the equivalent ratios. Think about equivalent fractions.

a. $\frac{5}{2} = \frac{20}{\square}$	**b.** 3 : 4 = 9 : _____	**c.** 16 : 18 = ____ : 9	**d.** $\frac{5}{1} = \frac{\square}{4}$
e. 2 to 100 = 1 to ________	**f.** _____ to 40 = 3 to 5	**g.** 5 : ____ = 1 to 20	

We can also form ratios using quantities with units. For example, in the ratio 5 km : 8 km, both terms contain the unit "km". We can then simplify the ratio, cancelling the units "km": $\frac{5 \text{ km}}{8 \text{ km}} = \frac{5}{8}$.

8. Write ratios of the given quantities. Use the fraction line to write the ratios. Then, simplify the ratios. In most of the problems, you will need to *convert* one quantity so it has the same measuring unit as the other.

a. 2 kg and 400 g $\frac{2 \text{ kg}}{400 \text{ g}} = \frac{2000 \text{ g}}{400 \text{ g}} = \frac{2000}{400} = \frac{5}{1}$	**b.** 200 ml and 2 L
c. 400 ml and 5 L	**d.** 800 m and 1,4 km
e. 120 cm and 1,8 m	**f.** 3 cm 4 mm and 1 cm 4 mm

If the two terms in the ratio have *different* units, then the ratio is also called a **rate.**

Example 2. "5 kilometres to 40 minutes" is a rate. It is comparing the quantities "5 kilometres" and "40 minutes," perhaps for the purpose of giving us the speed at which a person is walking.

We can write this rate as 5 kilometres : 40 minutes or $\frac{5 \text{ kilometres}}{40 \text{ minutes}}$ or 5 kilometres *per* 40 minutes.

The word "per" in a rate signifies the same thing as a colon or a fraction line.

This rate can be simplified: $\frac{5 \text{ kilometres}}{40 \text{ minutes}} = \frac{1 \text{ kilometre}}{8 \text{ minutes}}$. The person walks 1 kilometre in 8 minutes.

Example 3. Simplify the rate "15 pencils per 100c." Solution: $\frac{15 \text{ pencils}}{100\text{c}} = \frac{3 \text{ pencils}}{20\text{c}}$.

9. Write each rate using a colon, the word "per," or a fraction line. Then simplify it.

a. Annie walks at a constant speed of 3 kilometres in half an hour.

b. In this county, there are five teachers for every 60 pupils.

c. Each three kilograms of rice costs R48,00.

10. Fill in the missing numbers to form equivalent rates.

a. $\frac{2\text{ cm}}{30\text{ min}} = \frac{\_\_\_}{15\text{ min}} = \frac{\_\_\_}{45\text{ min}}$	**b.** $\frac{\text{R}72}{8\text{ g}} = \frac{\_\_\_}{1\text{ g}} = \frac{\_\_\_}{10\text{ g}}$
c. $\frac{1/4\text{ km}}{10\text{ min}} = \frac{\_\_\_}{1\text{ hr}} = \frac{\_\_\_}{5\text{ hr}}$	**d.** $\frac{\text{R}84{,}40}{8\text{ hr}} = \frac{\_\_\_}{2\text{ hr}} = \frac{\_\_\_}{10\text{ hr}}$

11. Express these rates in lowest terms.

a. R44 : 4 L	**b.** R630 : 8 kg
c. 420 km : 8 hr	**d.** 16 apples for R75

12. The rate of pencils to rand is constant. Fill in the table.

Pencils	**Rand**
1	
2	
3	15
6	
7	
8	

13. The rate of kilometres to litres remains constant. Fill in the table.

Kilometres							150	
Litres	1	2	3	4	5	10	15	50

14. An automobile travels at a constant speed of 80 km/hour. This means the *rate* of kilometres to hours remains the same. Fill in the table.

Km	10	20	80	100	150	200	500
Minutes							

15. You can use a table like in the previous problems to solve this problem. Six pairs of scissors cost R108. How much would five pairs cost?

16. You can use a table like in the previous problems to solve this problem. Mark can type at a constant rate 225 words in five minutes. How many words can he type in 12 minutes?

Unit Rates

In a **unit rate**, the second term of the rate is *one* of something, or a unit.

For example, 5 rand per 1 kilogram is a unit rate. It is commonly said as "5 rand per kilogram," but the "per kilogram" means "per one kilogram". See more examples of unit rates:

35 words per (one) minute	R3,70/kg	2/3 cup of sugar per 1 cup of flour
45 kilometres per hour	each student gets 3 pencils	R10,70 per marker

To change a rate that is not a unit rate into a unit rate, simply divide.

Example 1. To change the rate R696 for 6 cups into a unit rate, divide the numbers (696 divided by 6):

We get: $\frac{\text{R696}}{\text{6 cups}} = \frac{\text{R116}}{\text{1 cup}}$, which is more commonly written as R116 per cup.

Example 2. Two tablespoons of salt in 5 decilitres of water is the unit rate 2/5 tablespoons of salt per 1 decilitre of water.

To see that, write it using the fraction line: $\frac{\text{2 tbsp}}{\text{5 dl}}$ = 2/5 tbsp per dl.

1. Give two examples of unit rates (for example, a unit price and a speed).

2. Change to unit rates.

a. R715 for five cups	**b.** 180 kilometres in six hours	**c.** 10 000 people and five doctors

3. Change to unit rates. Give the rate using the word "per" or the slash /.

a. To paint 130 square metres, you need to use 15 litres of paint.

b. Joanne's Internet speed is 100 megabits in 25 seconds.

c. Each group of five students gets two calculators.

d. 7 teaspoons of vanilla for each 4 cups of batter.

e. We paid R75 for fifteen flowers.

f. R111 for three kilograms of beans.
Hint: use long division.

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Chapter 5: Percent
Introduction

This chapter is all about the basics of the concept of percent—a very important topic in regards to real life. We focus on how to calculate percentages (e.g. what percentage is R20 of R50) and how to find a certain percentage of a given number or quantity (e.g. what is 20% of 80 km). In 7th grade, students learn about percent of change and how to make comparisons with percent.

The lessons emphasise the connection between percentages and fractions and decimals in various ways. After all, percentages *are* fractions: the word percent simply means "a hundredth part," and the concept of percent builds on the student's previous understanding of fractions and decimals.

Specifically, the student should be very familiar with the idea of finding a fractional part of a whole (such as finding 3/4 of $240). Students using Math Mammoth have been practising that concept since 4th grade, and one reason why I have emphasised finding a fractional part of a whole in the earlier grades is specifically to lay a groundwork for the concept of percent. Assuming the student has mastered that, and can easily convert fractions to decimals, then studying the concept of percent should not be difficult.

In this context of thinking of percentages as fractions, students learn how to find a percentage of a given number or quantity using **mental maths techniques**. For example, students find 10% of R400 by thinking of it as 1/10 of R400, and thus dividing R400 by 10. They also learn to find a percentage of a quantity using *decimal* multiplication, both manually and with a calculator. For example, students find 17% of 45 km by multiplying 0,17 × 45 km.

In fact, in cases where mental maths is not a good option, I prefer teaching students to calculate percentages of quantities using decimals, instead of using percent proportion or fractions. That is because using decimals is simpler and quicker. Also, this method is often superior later on in algebra courses, when students need to write equations from verbal descriptions, and symbolically represent situations that involve percentages.

The last lesson of the chapter teaches students how to find the total when the percentage and the partial amount are known. For example: "Three-hundred twenty students, which is 40% of all students, take PE. How many students are there in total?" Students solve these with the help of the visual bar models, which they are already familiar with.

As the lessons constantly refer back to fractions and decimals, students can relate calculations with percentages to their earlier knowledge, and thus see **the logical structure of mathematics**. It will also prevent students from memorising calculations with percentages without understanding what is going on.

As a reminder, it is not recommended that you assign all the exercises by default. Use your judgment, and strive to vary the number of assigned exercises according to the student's needs. Some students might only need half or even less of the available exercises, in order to understand the concepts.

You will find free videos covering many topics of this chapter at **https://www.mathmammoth.com/videos/**.

The Lessons in Chapter 5

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch5

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Percentage of a Number: Using Decimals

You have learned that finding 1% of a number means finding 1/100 of it. Similarly, finding 60% of a number means finding 60/100 (or 6/10) of it.

In these types of expressions, the word "of" translates into **multiplication**:

1% of 90		60% of R700
↓	OR	↓
1% · 90		60% · R700

Next, let's write those percentages as *decimals*. We get:

1% of 90		60% of R700
↓	OR	↓
0,01 · 90		0,6 · R700

This gives us another way to calculate a certain percentage of a number (or a percentage of some quantity):

To calculate a percentage of a number, you need to make TWO simple changes:

1. **Change the percentage into a decimal.**
2. **Change the word "of" into multiplication.**

Example 1. Find 70% of 80.

Making the two changes, we write this as 0,7 · 80.

(Remember, in decimal multiplication, you multiply just as if there were no decimal comma, and the answer will have as many decimal digits as the total number of decimal digits in all of the factors.)

So, when you multiply 0,7 · 80, think of multiplying 7 · 80 = 560. Since 0,7 has one decimal digit, and 80 has none, the answer has one decimal digit. Thus, 0,7 · 80 = 56,0 or just 56.

You can also use common sense and estimation: 0.7 · 80 must be less than 80, yet more than 1/2 of 80, which is 40. Since 7 · 8 = 56, you know that the answer must be 56—not 5.6 or 560.

Example 2. Find 3% of R4000.

First, write this as 0,03 · R4000. Next, multiply without decimal commas: 3 · R4000 = R12 000. Lastly, put the decimal comma so that the answer will have <u>two</u> decimal digits: R120,00.

Example 3. Find 23% of 5500 km.

Write this as 0.23 · 5500 km and use a calculator. The answer is 1265 km. This makes sense, because 10% of 5500 km is 550 km, and 20% of it is 1100 km. Therefore, 1265 km as 23% of 5500 km is reasonable.

1. "Translate" the expressions into multiplications by a decimal. Solve, using mental maths.

a. 20% of 70 _______ · _______ = _______	**b.** 90% of 50 _______ · _______ = _______	**c.** 80% of 400 _______ · _______ = _______
d. 60% of R8 _______ · _______ = _______	**e.** 9% of 3000 _______ · _______ = _______	**f.** 7% of 40 L _______ · _______ = _______
g. 150% of 44 kg _______ · _______ = _______	**h.** 200% of 56 students _______ · _______ = _______	**i.** 2% of 1500 km _______ · _______ = _______

2. "Translate" the other way: Write the multiplications as expressions of a "percentage of the number".

a. 0,6 · 50 ______% of _______ = _______	**b.** 0,03 · R400 ______% of _______ = _______	**c.** 0,8 · 400 km ______% of _______ = _______
d. 0,08 · 6 ______% of _______ = _______	**e.** 0,11 · R300 ______% of _______ = _______	**f.** 0,2 · 70 kg ______% of _______ = _______

3. Use a calculator to find percentages of these quantities.

a. 17% of R4500 **b.** 67% of 27 m **c.** 48% of 7.8 kg

4. Use mental maths to find percentages of these quantities.

a. 25% of 240 m **b.** 80% of 30 000 km **c.** 75% of 3,2 kg

5. **a.** A lake has a 30-km long shoreline. Six percent of it is sandy beach.
What *percentage* of the shoreline is *not* sandy beach?

b. Find the length (in km) of the shoreline that *is* sandy beach.

6. Twenty percent of a university's 4000 students have a scholarship.

a. What *percentage* of the students do *not* have a scholarship?

b. How many students have a scholarship?

c. How many students do *not* have a scholarship?

7. A farmer had 1200 hectares of land. He planted 30% of it with wheat, 45% with corn and the rest with oats. Find how many hectares he planted with each kind of grain.

8. Identify the errors that these children made. Then find the correct answers.

a. Find 80% of 50. Gladys's solution: $80 \cdot 50 = 4000$	**b.** Find 75% of 84 000. Glenn's solution: This is the same as $84\,000 \div 4 = 21\,000$.

9. Circle the expressions with the same value as 20% of R620.

0,02 · R620	R620 ÷ 5	R620 ÷ 10 · 2	2 · R62
$\frac{1}{5}$ · R620	0,2 · R620	20 · R620	R620 ÷ 4

10. About 92% of Argentina's population of 41 million live in cities.
About 25% of Tanzania's population of 41 million live in cities.

How many more Argentineans than Tanzanians live in cities?

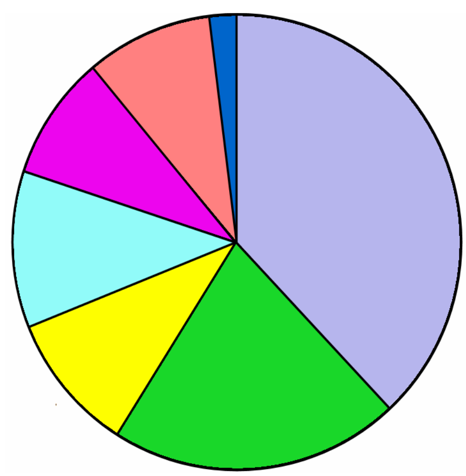
11. The table below shows Andy's usage of time in one day.

a. Calculate the time he spent doing each activity. Round the minutes to the nearest minute.

b. Label the sections in the circle graph with the name of each activity.

Activity	Percentage	Minutes	Hours/minutes
Sleep	38%		
School	21%		
Soccer	10%	144	2 h 24 min
Play	11%		
Eating	9%		
Chores	9%		
Hygiene	2%		
TOTAL	100%	1440	24 hours

Andy's Usage of Time



Discounts

Other than figuring sales tax, the area of life in which you will probably most often need to use percentages is in calculating discounts.

A laptop that costs R6000 is 20% off. What is the sale price?

Method 1. We calculate 20% of R6000. That is the discounted amount in *rand*.
Then we subtract that from the original price, R6000.

20% of R6000 is R1200. And R6000 − R1200 = R4800. So the sale price is R4800.

Method 2. Since 20% of the price has been removed, 80% of the price is *left*.
By calculating 80% of the original price, you will get the new discounted price: $0{,}8 \cdot$ R6000 = R4800

Two methods for calculating the discounted price:

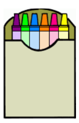
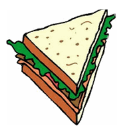
1. Calculate the discount amount as a percentage of the original price. Then subtract.
2. Find what percentage of the price is left. Then calculate that percentage of the normal price.

1. All of these items are on sale. Calculate the discount in rand and the resulting sale price.

a.  Price: R900 20% off Discount amount: R 180 Sale price: R______________	**b.**  Price: R50 40% off Discount amount: R__________ Sale price: R______________	**c.**  Price: R150 30% off Discount amount: R__________ Sale price: R____________

2. A swimsuit that cost R250 was on sale for 20% off.
Monica tried to calculate the discounted price this way: R250 − R20 = R230.
What did she do wrong? Find the correct discounted price.

3. All these items are on sale. Find the discounted price.

a. Price: R11,20 25% off  Discount amount: R__________ Discounted price: R_________	**b.** Price: R180 25% off  Discount amount: R__________ Discounted price: R_________	**c.** Price: R950 30% off  Discount amount: R__________ Discounted price: R_________
d. Price: R55 40% off  Discount amount: R__________ Discounted price: R__________	**e.** Price: R40,00 10% off  Discount amount: R__________ Discounted price: R_________	**f.** Price: R13 50% off  Discount amount: R__________ Discounted price: R__________

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Revision: Percent

1. Find a percentage of a number	2. A fractional part as a percentage
What is 60% of 300 kilometres? Calculate 0,6 · 300 kilometres = 180 kilometres. Or, using mental maths, first calculate 10% of 300 kilometres, which is 1/10 of it, or 30 kilometres. Then multiply 6 · 30 kilometres = 180 kilometres. *Of the 15 400 workers in a city, 22% work in a steel factory. How many workers is that?* Calculate: 0,22 · 15 400 = 3388 workers.	*What percent is 600 g of 2 kg?* Write the fraction $\frac{600\text{ g}}{2000\text{ g}} = \frac{6}{20} = \frac{30}{100} = 30\%$. *One backpack costs R180 and another costs R169. What percentage is the price of the cheaper backpack of the price of the more expensive one?* Write the fraction $\frac{\text{R169}}{\text{R180}} = 0{,}9388... \approx 94\%$.
1. Change the percentage into a decimal. 2. Then multiply the number by that decimal. Alternatively, use mental maths shortcuts for finding 5%, 10%, 20%, 25%, 50%, *etc.* of a number.	1. First write the fraction. Note that the two quantities in the fraction must both be in the same units: both grams, both metres, both rand, *etc.* 2. Then convert the fraction into a decimal and finally a percentage.

1. Write as percentages, fractions and decimals.

a. ______% $= \frac{68}{100} =$ _______	**b.** 7% $= \frac{}{} =$ _______	**c.** _______% $= \frac{}{} = 0{,}15$
d. 120% $= \frac{}{} =$ _______	**e.** ______% $= \frac{224}{100} =$ ______	**f.** _______% $= \frac{}{} = 0{,}06$

2. Fill in the table. Use mental maths.

percentage ↓ number →	**6 100**	**90**	**57**	**6**
1% of the number				
4% of the number				
10% of the number				
30% of the number				

3. A skating group has 15 girls and 5 boys.
What percentage of the skaters are girls?

4. Write as percentages. You may need long division in some problems.
   If necessary, round your answers to the nearest percent.

   a. 3/4

   b. 2/25

   c. 1 5/8

5. Emma is 1 m 63 cm tall and Madison is 1 m 22 cm tall. How many percent is Emma's height of Madison's height?

6. A cheap chair costs R250. The price of another chair is 140% of that. How much does the other chair cost?

7. A bag has 25 green marbles and some white ones, too. The green marbles are 20% of the total. How many marbles are there in total? How many white marbles are there?

8. Andrew earns R2000 monthly. He pays R540 of his salary in taxes. What percentage of his income does Andrew pay in taxes?

9. Which is cheaper, a shirt that was R180 discounted by 20%, or shirt that was R160 discounted by 10%?

10. (*Challenge*) One square has sides 2 cm long, and another has sides 4 cm long. How many percent is the area of the smaller square of the area of the larger square?

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Contents

Chapter 9: Geometry

Chapter 10: Statistics

Foreword

Math Mammoth Grade 6, South Africa Version, comprises a complete maths curriculum for the sixth grade mathematics studies. This curriculum is essentially the same as the *Math Mammoth Grade 6* sold in the United States, only customised for South Africa use in a few aspects (listed below). The curriculum meets the Common Core Standards in the United States, but it may not properly align to the sixth grade standards in your country. However, you can probably find material for any missing topics in the neighbouring grades of Math Mammoth.

The South Africa version of Math Mammoth differs from the US version in these aspects:

- The curriculum uses metric measurement units, not both metric and customary (imperial) units.
- Spelling is British English instead of American English
- The pages are formatted for A4 paper size.
- Large numbers are formatted with a space as the thousands separator (such as 12 394).
  (The decimals are formatted with a decimal comma.)

The worktext 6-A covered the first half of the topics for 6th year: a revision of the four basic operations, an introduction to algebra, and decimals, ratios and percent.

This part B covers the remainder of the topics for 6th year: prime factorisation, the greatest common factor, least common multiple; fractions; integers and graphing in the coordinate plane; geometry and statistics.

Chapter 6 first revises prime factorisation and then applies those principles to using the greatest common factor to simplify fractions and the least common multiple to find common denominators. Chapter 7 provides a thorough revision of the fraction operations from fifth year and includes ample practice in solving problems with fractions.

Chapter 8 introduces students to integers (signed numbers). Students plot points in all four quadrants of the coordinate plane, reflect and translate simple figures, and learn to add and subtract with negative numbers. (The multiplication and division of integers will be studied in 7th year.)

The next chapter, Geometry, focuses on calculating the area of polygons. The final chapter is about statistics. Beginning with the concept of a statistical distribution, students learn about measures of centre and measures of variability. They also learn how to make dot plots, histograms, boxplots, and stem-and-leaf plots as ways to summarise and analyse distributions.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching maths!

Maria Miller, the author

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Chapter 6: Prime Factorisation, GCF and LCM
Introduction

The topics of this chapter belong to a branch of mathematics known as *number theory*. Number theory has to do with the study of whole numbers and their special properties. In this chapter, we revise prime factorisation and study the greatest common factor (GCF) and the least common multiple (LCM).

The main application of factoring and the greatest common factor in arithmetic is in simplifying fractions, so that is why I have included a lesson on that topic. However, it is not absolutely necessary to use the GCF when simplifying fractions, and the lesson emphasises that fact.

The concepts of factoring and the GCF are important to understand because they will be carried over into algebra, where students will factor polynomials. In this chapter, we lay the groundwork for that by using the GCF to factor simple sums, such as 27 + 45. For example, a sum like 27 + 45 factors into 9(3 + 5).

Similarly, the main use for the least common multiple in arithmetic is in finding the smallest common denominator for adding fractions, and we study that topic in this chapter in connection with the LCM.

Primes are fascinating "creatures," and you can let students read more about them by accessing the Internet resources mentioned below. The really important, but far more advanced, application of prime numbers is in cryptography. Some students might be interested in reading additional material on that subject—please see the list for Internet resources.

Keep in mind that the specific lessons in the chapter can take several days to finish. They are not "daily lessons." Instead, use the general guideline that sixth graders should finish about 2 pages daily or 9-10 pages a week in order to finish the curriculum in about 40 weeks. Also, I recommend not assigning all the exercises by default, but that you use your judgement, and strive to vary the number of assigned exercises according to the student's needs. Please see the user guide at **https://www.mathmammoth.com/userguides/** for more guidance on using and pacing the curriculum.

You can find some free videos for the topics of this chapter at **https://www.mathmammoth.com/videos/** (choose 6th grade).

The Lessons in Chapter 6

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better, and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch6

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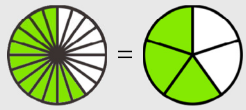
Using Factoring When Simplifying Fractions

You have seen the process of **simplifying fractions** before.

In simplifying fractions, we divide both the numerator and the denominator by the same number. The fraction becomes *simpler*, which means that the numerator and the denominator are now *smaller* numbers than they were before.

However, this does NOT change the actual value of the fraction. It is the "same amount of pie" as it was before. It is just cut differently.



Every four slices have been joined together.

$$\frac{12}{20} \overset{\div 4}{=} \frac{3}{5}$$

Why does this work?

It is based on finding common factors and on how fraction multiplication works. In our example above, the fraction $\frac{12}{20}$ can be written as $\frac{4 \cdot 3}{4 \cdot 5}$. Then we can **cancel out** those fours: $\frac{\cancel{4} \cdot 3}{\cancel{4} \cdot 5} = \frac{3}{5}$.

The reason this works is because $\frac{4 \cdot 3}{4 \cdot 5}$ is equal to the fraction multiplication $\frac{4}{4} \cdot \frac{3}{5}$. And in that, 4/4 is equal to 1, which means we are only left with 3/5.

Example 1. Often, the simplification is simply written or indicated this way →
Notice that here, the 4's that were cancelled out do ***not*** get indicated in any way! You only think it: "I divide 12 by 4, and get 3. I divide 20 by 4, and get 5."

$$\frac{\overset{3}{\cancel{12}}}{\underset{5}{\cancel{20}}} = \frac{3}{5}$$

Example 2. Here, 35 and 55 are both divisible by 5. This means we can cancel out those 5's, but notice this is not shown in any way. We simply cross out 35 and 55, think of dividing them by 5, and write the division result above and below.

$$\frac{\overset{7}{\cancel{35}}}{\underset{11}{\cancel{55}}} = \frac{7}{11}$$

1. Simplify the fractions, if possible.

a. $\frac{12}{36}$	**b.** $\frac{45}{55}$	**c.** $\frac{15}{23}$	**d.** $\frac{13}{6}$
e. $\frac{15}{21}$	**f.** $\frac{19}{15}$	**g.** $\frac{17}{24}$	**h.** $\frac{24}{30}$

2. Leah simplified various fractions like you see below. She did not get them right though. Explain to her what she is doing wrong.

$\frac{24}{84} = \frac{20}{80} = \frac{1}{4}$ $\qquad$ $\frac{27}{60} = \frac{7}{40}$ $\qquad$ $\frac{14}{16} = \frac{10}{12} = \frac{6}{8} = \frac{3}{4}$

Using factoring when simplifying

Carefully study the example on the right where we simplify the fraction 144/96 to **lowest terms —** in other words, where the numerator and the denominator have no common factors.

- First we factor (write) 144 as 12 · 12 and 96 as 8 · 12.
- Then we simplify in two steps:
    1. 12 and 8 are both divisible by 4, so they simplify into 3 and 2.
    2. 12 and 12 are divisible by 12, so they simplify into 1 and 1. Essentially, they cancel each other out.
- Lastly we write the improper fraction 3/2 as a mixed number.

$$\frac{144}{96} = \frac{\overset{3}{\cancel{12}} \cdot \overset{1}{\cancel{12}}}{\underset{2}{\cancel{8}} \cdot \underset{1}{\cancel{12}}} = \frac{3}{2} = 1\frac{1}{2}$$

For a comparison, here is another way to write the simplification in several steps, and that you've seen in earlier grades in Math Mammoth:

$$\frac{144}{96} \;\underset{\div 12}{\overset{\div 12}{=}}\; \frac{12}{8} \;\underset{\div 4}{\overset{\div 4}{=}}\; \frac{3}{2} = 1\frac{1}{2}$$

Let's study some more examples. (Remember that they don't show the number that you divide by.)

$$\frac{42}{105} = \frac{\overset{1}{\cancel{7}} \cdot \overset{2}{\cancel{6}}}{\underset{5}{\cancel{35}} \cdot \underset{1}{\cancel{3}}} = \frac{2}{5}$$

$$\frac{45}{150} = \frac{\overset{3}{\cancel{9}} \cdot \overset{1}{\cancel{5}}}{\underset{10}{\cancel{30}} \cdot \underset{1}{\cancel{5}}} = \frac{3}{10}$$

3. Simplify. Write the simplified numerator above and the simplified denominator below the old ones.

a. $\frac{14}{16}$	**b.** $\frac{33}{27}$	**c.** $\frac{12}{26}$	**d.** $\frac{9}{33}$	**e.** $\frac{42}{28}$

4. The numerator and the denominator have already been factored in some problems. Your task is to simplify. Also, give your final answer as a mixed number, if applicable.

a. $\frac{56}{84} = \frac{7 \cdot 8}{21 \cdot 4} =$	**b.** $\frac{54}{144} = \frac{6 \cdot 9}{12 \cdot 12} =$	**c.** $\frac{120}{72} = \frac{10 \cdot \boxed{}}{\boxed{} \cdot 9} =$
d. $\frac{80}{48} = \frac{\boxed{} \cdot 8}{\boxed{} \cdot 8} =$	**e.** $\frac{36}{90} =$ ________ $=$	**f.** $\frac{28}{140} =$ ________ $=$

5. Simplify the fractions. Use your knowledge of divisibility.

a. $\frac{95}{100}$	**b.** $\frac{66}{82}$	**c.** $\frac{69}{99}$
d. $\frac{120}{600}$	**e.** $\frac{38}{52}$	**f.** $\frac{72}{84}$

Simplify “criss-cross”

These examples are from the previous page. This time the 45 in the numerator has been written as 5 · 9 instead of 9 · 5. We can cancel out the 5 from the numerator with the 5 from the denominator (we simplify criss-cross).

Also, we can simplify the 9 in the numerator and the 30 in the denominator criss-cross. The other example (simplifying 42/105) is similar.

This same concept can be applied to make multiplying fractions easier.

$$\frac{45}{150} = \frac{\overset{1}{\cancel{5}} \cdot \overset{3}{\cancel{9}}}{\underset{10}{\cancel{30}} \cdot \underset{1}{\cancel{5}}} = \frac{3}{10}$$

$$\frac{42}{105} = \frac{\overset{1}{\cancel{7}} \cdot \overset{2}{\cancel{6}}}{\underset{1}{\cancel{3}} \cdot \underset{5}{\cancel{35}}} = \frac{2}{5}$$

6. Simplify. Give your final answer as a mixed number, if applicable.

a. $\frac{14}{84} = \frac{2 \cdot 7}{21 \cdot 4} =$	**b.** $\frac{54}{150} = \frac{9 \cdot \square}{10 \cdot \square} =$	**c.** $\frac{138}{36} = \frac{2 \cdot \square}{\square \cdot 4} =$
d. $\frac{27}{20} \cdot \frac{10}{21} =$	**e.** $\frac{75}{90} = \underline{\qquad\qquad} =$	**f.** $\frac{48}{45} \cdot \frac{55}{64} =$

Example 3. The simplification is done in two steps. In the first step, 12 and 2 are divided by 2, leaving 6 and 1. In the second step, 6 and 69 are divided by 3, leaving 2 and 23.

$$\frac{48}{138} = \frac{\overset{6}{\cancel{12}} \cdot 4}{\underset{1}{\cancel{2}} \cdot 69} = \frac{\overset{2}{\cancel{6}} \cdot 4}{1 \cdot \underset{23}{\cancel{69}}} = \frac{8}{23}$$

These two steps can also be done without rewriting the expression. The 6 and 69 are divided by 3 as before. This time we simply did not rewrite the expression in between but just continued on with the numbers 6 and 69 that were already written there.

$$\frac{48}{138} = \frac{\overset{\overset{2}{\cancel{6}}}{\cancel{12}} \cdot 4}{\underset{1}{\cancel{2}} \cdot \underset{23}{\cancel{69}}} = \frac{8}{23}$$

If this looks too confusing, you do not have to write it in such a compact manner. You can rewrite the expression before simplifying it some more.

7. Simplify the fractions to lowest terms, or simplify before you multiply the fractions.

a. $\frac{88}{100}$	**b.** $\frac{84}{102}$	**c.** $\frac{85}{105}$
d. $\frac{8}{5} \cdot \frac{8}{20} =$	**e.** $\frac{72}{120}$	**f.** $\frac{104}{240}$
g. $\frac{35}{98}$	**h.** $\frac{5}{7} \cdot \frac{17}{15} =$	**i.** $\frac{72}{112}$

The Greatest Common Factor (GCF)

Let's take two whole numbers. We can then list all the factors of each number, and then find the factors that are common in both lists. Lastly, we can choose the greatest or largest among those "common factors." That is the **greatest common factor** of the two numbers. The term itself really tells you what it means!

Example 1. Find the greatest common factor of 18 and 30.

The factors of 18: 1, 2, 3, 6, 9 and 18.
The factors of 30: 1, 2, 3, 5, 6, 10, 15 and 30.

Their common factors are 1, 2, 3 and 6. The greatest common factor is 6.

Here is a **method to find all the factors of a given number**.

Example 2. Find the factors (divisors) of 36.

We check if 36 is divisible by 1, 2, 3, 4 and so on. Each time we find a divisor, we write down *two* factors.

- 36 is divisible by 1. We write $36 = 1 \cdot 36$, and that equation gives us two factors of 36: both the smallest (**1**) and the largest (**36**).
- 36 is also divisible by 2. We write $36 = 2 \cdot 18$, and that equation gives us two more factors of 36: the second smallest (**2**) and the second largest (**18**).
- Next, 36 is divisible by 3. We write $36 = 3 \cdot 12$, and now we have found the third smallest factor (**3**) and the third largest factor (**12**).
- Next, 36 is divisible by 4. We write $36 = 4 \cdot 9$, and we have found the fourth smallest factor (**4**) and the fourth largest factor (**9**).
- Finally, 36 is divisible by 6. We write $36 = 6 \cdot 6$, and we have found the fifth smallest factor (**6**) which is also the fifth largest factor.

We know that we are done because the list of factors from the "small" end (1, 2, 3, 4, 6) has met the list of factors from the "large" end (36, 18, 12, 9, 6).

Therefore, all of the factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18 and 36.

1. List all of the factors of the given numbers.

a. 48	**b.** 60
c. 42	**d.** 99

2. Find the greatest common factor of the given numbers. Your work above will help!

a. 48 and 60	**b.** 42 and 48	**c.** 42 and 60	**d.** 99 and 60

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Chapter 7: Fractions
Introduction

This chapter begins with a revision of fraction arithmetic from fifth grade—specifically, addition, subtraction, simplification, and multiplication of fractions. Then it focuses on division of fractions.

The introductory lesson on the division of fractions presents the concept of reciprocal numbers and ties the reciprocity relationship to the idea that division is the appropriate operation to solve questions of the form, "How many times does this number fit into that number?" For example, we can write a division from the question, "How many times does 1/3 fit into 1?" The answer is, obviously, 3 times. So we can write the division $1 \div (1/3) = 3$ and the multiplication $3 \cdot (1/3) = 1$. These two numbers, 3/1 and 1/3, are reciprocal numbers because their product is 1.

Students learn to solve questions like that through using visual models and writing division sentences that match them. Thinking of fitting the divisor into the dividend (measurement division) also gives us a tool to check whether the answer to a division problem is reasonable.

Naturally, the lessons also present the shortcut for fraction division—that each division can be changed into a multiplication by taking the reciprocal of the divisor, which is often called the "invert (flip)-and-multiply" rule. However, that "rule" is just a shortcut. It is necessary to memorise it, but memorising a shortcut doesn't help students make sense conceptually out of the division of fractions—they also need to study the concept of division and use visual models to better understand the process involved.

In two lessons that follow, students apply what they have learned to solve problems involving fractions or fractional parts. A lot of the problems in these lessons are revision in the sense that they involve previously learned concepts and are similar to problems students have solved earlier, but many involve the division of fractions, thus incorporating the new concept presented in this chapter.

Consider mixing the lessons from this chapter (or from some other chapter) with the lessons from the geometry chapter (which is a fairly long chapter). For example, the student could study these topics and geometry on alternate days, or study a little from both each day. Such, somewhat spiral, usage of the curriculum can help prevent boredom, and also to help students retain the concepts better.

Also, don't forget to use the resources for challenging problems:
https://l.mathmammoth.com/challengingproblems
I recommend that you at least use the first resource listed, Math Stars Newsletters.

The Lessons in Chapter 7

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better, and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch7

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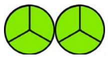
Dividing Fractions: Reciprocal Numbers

One interpretation of division is **measurement division**, where we think: *How many times does one number go into another?* For example, to solve how many times 11 fits into 189, we divide 187 ÷ 11 = 17.

(The other interpretation is equal sharing; we will come to that later.)

Let's apply that to fractions. How many times does  go into  ?

We can solve this just by looking at the pictures: three times. We can write the division: $2 \div \frac{2}{3} = 3$.

To check the division, we multiply: $3 \cdot \frac{2}{3} = \frac{6}{3} = 2$. Since we got the original dividend, it checks.

We can use measurement division to check whether an answer to a division is reasonable.

For example, if I told you that $7 \div 1\frac{2}{3}$ equals $14\frac{1}{3}$, you can immediately see it doesn't make sense:

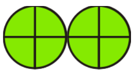
1 2/3 surely does not fit into 7 that many times. Maybe three to four times, but not 14!

You could also multiply to see that: 14-*and-something* times 1-*and-something* is way more than 14, and closer to 28 than to 14, instead of 7.

1. Find the answers that are unreasonable without actually dividing.

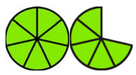
a. $\frac{4}{5} \div 6 = \frac{2}{15}$	**b.** $2\frac{3}{4} \div \frac{1}{4} = \frac{7}{12}$	**c.** $\frac{7}{9} \div 2 = \frac{7}{18}$	**d.** $8 \div 2\frac{1}{3} = 18\frac{1}{3}$	**e.** $5\frac{1}{4} \div 6\frac{1}{2} = 3\frac{1}{8}$

2. Solve with the help of the visual model, checking how many times the given fraction fits into the other number. Then write a division. Lastly, write a multiplication that checks your division.

a. How many times does  go into  ?

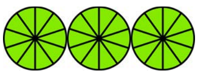
$2 \div \frac{3}{4} =$

Check: _____ $\cdot \frac{3}{4} =$

b. How many times does  go into  ?

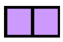
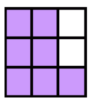
$\square\frac{\square}{\square} \div \frac{\square}{\square} =$

Check:

c. How many times does  go into  ?

$3 \div \frac{\square}{\square} =$

Check:

d. How many times does  go into  ?

$\frac{\square}{\square} \div \frac{\square}{\square} =$

Check:

3. Solve how many times the fraction goes into the whole number. Can you find a *pattern* or a *shortcut*?

a. $3 \div \frac{1}{6} =$	**b.** $4 \div \frac{1}{5} =$	**c.** $3 \div \frac{1}{10} =$	**d.** $5 \div \frac{1}{10} =$
e. $7 \div \frac{1}{4} =$	**f.** $4 \div \frac{1}{8} =$	**g.** $4 \div \frac{1}{10} =$	**h.** $9 \div \frac{1}{8} =$

The shortcut is this:

$$5 \div \frac{1}{4} \quad\downarrow\quad 5 \cdot 4 = 20$$

$$3 \div \frac{1}{8} \quad\downarrow\quad 3 \cdot 8 = 24$$

$$9 \div \frac{1}{7} \quad\downarrow\quad 9 \cdot 7 = 63$$

Notice that 1/4 inverted (upside down) is 4/1 or simply 4. We call 1/4 and 4 reciprocal numbers, or just reciprocals. So the shortcut is: multiply by the reciprocal of the divisor.

Does the shortcut make sense to you? For example, consider the problem 5 ÷ (1/4). Since 1/4 goes into 1 exactly four times, it must go into 5 exactly 5 · 4 = 20 times.

Two numbers are reciprocal numbers (or reciprocals) of each other if, when multiplied, they make 1.

$\frac{3}{4}$ is a reciprocal of $\frac{4}{3}$, because $\frac{3}{4} \cdot \frac{4}{3} = \frac{12}{12} = 1$.

$\frac{1}{7}$ is a reciprocal of 7, because $\frac{1}{7} \cdot 7 = \frac{7}{7} = 1$.

You can find the reciprocal of a fraction $\frac{m}{n}$ by flipping the numerator and denominator: $\frac{n}{m}$.

This works, because $\frac{m}{n} \cdot \frac{n}{m} = \frac{n \cdot m}{m \cdot n} = \frac{m \cdot n}{m \cdot n} = 1$.

To find the reciprocal of a mixed number or a whole number, first <u>write it as a fraction</u>, then "flip" it.

Since $2\frac{3}{4} = \frac{11}{4}$, its reciprocal number is $\frac{4}{11}$. And since $28 = \frac{28}{1}$, its reciprocal number is $\frac{1}{28}$.

4. Find the reciprocal numbers. Then write a multiplication with the given number and its reciprocal.

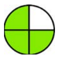
a. $\frac{5}{8}$	**b.** $\frac{1}{9}$	**c.** $1\frac{7}{8}$	**d.** 32	**e.** $2\frac{1}{8}$
$\frac{5}{8} \cdot \frac{\square}{\square} = 1$	$\frac{\square}{\square} \cdot \frac{\square}{\square} = 1$	$\frac{\square}{\square} \cdot \frac{\square}{\square} = 1$	$32 \cdot \frac{\square}{\square} = 1$	$\frac{\square}{\square} \cdot \frac{\square}{\square} = 1$

5. Write a division sentence to match each multiplication above.

a. $1 \div \frac{\square}{\square} = \frac{\square}{\square}$	**b.** $1 \div \frac{\square}{\square} = \frac{\square}{\square}$	**c.** $1 \div \frac{\square}{\square} = \frac{\square}{\square}$	**d.** ___ $\div \frac{\square}{\square} = \frac{\square}{\square}$	**e.** ___ $\div \frac{\square}{\square} = \frac{\square}{\square}$

SHORTCUT: instead of dividing, multiply by the reciprocal of the divisor.

Study the examples to see how this works.

How many times does  go into  ?

$$\frac{3}{4} \div \frac{1}{3}$$

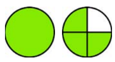
$$\downarrow \; \downarrow$$

$$\frac{3}{4} \cdot 3 = \frac{9}{4} = 2\frac{1}{4}$$

Answer: 2 1/4 times.

Does it make sense?

Yes,  fits into  a little more than two times.

How many times does  go into  ?

$$\frac{7}{4} \div \frac{2}{5}$$

$$\downarrow \; \downarrow$$

$$\frac{7}{4} \cdot \frac{5}{2} = \frac{35}{8} = 4\frac{3}{8}$$

Answer: 4 3/8 times.

Does it make sense?

Yes.  goes into 1 3/4 over four times.

How many times does  go into  ?

$$\frac{2}{9} \div \frac{2}{7} =$$

$$\downarrow \; \downarrow$$

$$\frac{\cancel{2}}{9} \cdot \frac{7}{\cancel{2}} = \frac{7}{9}$$

Answer: 7/9 of a time.

Does it make sense?

Yes, because  does not go into  even one full time!

Remember: There are ***two* changes** in each calculation:

1. **Change the division into multiplication**.
2. **Use the reciprocal of the divisor**.

6. Solve these using the shortcut. Remember to check to make sure your answer makes sense.

a. $\frac{3}{4} \div 5$

$$\downarrow \; \downarrow$$

$$\frac{3}{4} \cdot \frac{1}{5} =$$

b. $\frac{2}{3} \div \frac{6}{7}$

c. $\frac{4}{7} \div \frac{3}{7}$

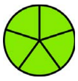
d. $\frac{2}{3} \div \frac{3}{5}$

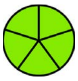
e. $4 \div \frac{2}{5}$

f. $\frac{13}{3} \div \frac{1}{5}$

Now let's try to **make some sense __visually__** out of how reciprocal numbers fit into the division of fractions.

Example 1. We can think of the division **1 ÷ (2/5)** as asking, **"How many times does 2/5 fit into 1?"**

Using pictures: How many times does  go into  ? (From the looks of it, at least two times!)

From the picture we can see that  goes into  two times, and then we have 1/5 left over.

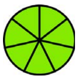
But how many times does $\frac{2}{5}$ fit into the leftover piece, $\frac{1}{5}$? How many times does  go into  ?

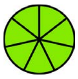
That is like trying to fit a TWO-part piece into a hole that holds just ONE part.
Only 1/2 of the two-part piece fits! So, 2/5 fits into 1/5 exactly half a time.

So we found that, in total, 2/5 fits into 1 exactly **2 ½ times**. We can write the division $1 \div \frac{2}{5} = 2\frac{1}{2}$ or $\frac{5}{2}$.

Notice, we got $1 \div \frac{2}{5} = \frac{5}{2}$. Checking that with multiplication, we get $\frac{5}{2} \cdot \frac{2}{5} = 1$. Reciprocals!

Example 2. We can think of the division **1 ÷ (5/7)** as, **"How many times does 5/7 fit into 1?"**

Using pictures: How many times does  go into  ? (It looks like, a bit over one time.)

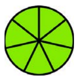
From the picture we can see that  goes into  just once, and then we have 2/7 left over.

But how many times does $\frac{5}{7}$ fit into the leftover piece, $\frac{2}{7}$? How many times does  go into  ?

The five-part piece fits into a hole that is only big enough for two parts just 2/5 of the way.

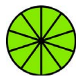
So 5/7 fits into one exactly 1 2/5 times—and this makes sense because, as we noted at first, it looked like 5/7 fit into one a little over one time. The division is $1 \div \frac{5}{7} = 1\frac{2}{5}$ or $1 \div \frac{5}{7} = \frac{7}{5}$. Reciprocals again!

7. Write a division.

a. How many times does  go into  ?

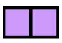
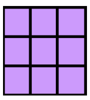
$1 \div \frac{\square}{\square} =$

Check: Does your answer make sense visually?

b. How many times does  go into  ?

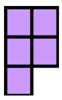
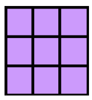
$1 \div \frac{\square}{\square} =$

Check: Does your answer make sense visually?

c. How many times does  go into  ?

$1 \div \frac{\square}{\square} =$

Check: Does your answer make sense visually?

d. How many times does  go into  ?

$1 \div \frac{\square}{\square} =$

Check: Does your answer make sense visually?

8. Fill in the answers and complete the patterns. You'll be able to do a lot of these in your head!

a.	**b.**	**c.**	**d.**
$3 \div \frac{1}{5} =$	$6 \div \frac{1}{4} =$	$1 \div \frac{1}{4} =$	$8 \div \frac{1}{2} =$
$3 \div \frac{2}{5} =$	$6 \div \frac{2}{4} =$	$2 \div \frac{1}{4} =$	$8 \div \frac{2}{2} =$
$3 \div \frac{3}{5} =$	$6 \div \frac{3}{4} =$	$3 \div \frac{1}{4} =$	$8 \div \frac{3}{2} =$
$3 \div \frac{4}{5} =$	$\square \div \frac{\square}{\square} =$	$\square \div \frac{\square}{\square} =$	$\square \div \frac{\square}{\square} =$
$3 \div \frac{5}{5} =$	$\square \div \frac{\square}{\square} =$	$\square \div \frac{\square}{\square} =$	$\square \div \frac{\square}{\square} =$

Epilogue (optional)

The lesson didn't go into full details as to why multiplication by the reciprocal always gives us the answer to a division problem. Let's continue that discussion a bit.

Any division can be turned into a multiplication. Example: from the division $2 \div \frac{3}{4} =$ ______ , we can write the multiplication $\frac{3}{4} \cdot$ ______ $= 2$.

To find what fits on the empty line, let's first of all put there the reciprocal of ¾ : $\frac{3}{4} \cdot \underline{\frac{4}{3} \cdot \quad\quad} = 2$. (This allows us to see better what else needs to go there.)

Notice that the multiplication of the two fractions above equals 1 (since they are reciprocals). To make the left side of the equation equal 2, we place 2 in the empty line: $\frac{3}{4} \cdot \underline{\frac{4}{3} \cdot 2} = 2$ Now the equation is true.

The answer to the original division is what was put on the empty line, which is $\frac{4}{3} \cdot 2$ (or $2 \cdot \frac{4}{3}$)

— or the original dividend times the reciprocal of the divisor.

Let's take the general case: $a \div b =$ ____. From that, we can write the multiplication $b \cdot$ ____ $= a$.

We will first insert the reciprocal of b on the empty line: $b \cdot \underline{\frac{1}{b} \cdot \quad\quad} = a$.

Since $b \cdot (1/b) = 1$, then the number we still need to insert on the empty line must be a, to make a true equation: $b \cdot \underline{\frac{1}{b} \cdot a} = a$.

So, the answer to the original division problem is $\frac{1}{b} \cdot a$, or $a \cdot \frac{1}{b}$ — the original divided times the reciprocal of the divisor.

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Chapter 8: Integers
Introduction

In chapter 8, students are introduced to integers, the coordinate plane in all four quadrants and integer addition and subtraction. The multiplication and division of integers will be studied next year.

Integers are introduced using the number line to relate them to the concepts of temperature, elevation and money. We also study briefly the ideas of absolute value (an integer's distance from zero) and the opposite of a number.

Next, students learn to locate points in all four quadrants and how the coordinates of a figure change when it is reflected across the x or y-axis. Students also move points according to given instructions and find distances between points with the same first coordinate or the same second coordinate.

Adding and subtracting integers is presented through two main models: (1) movements along the number line and (2) positive and negative counters. With the help of these models, students should not only learn the shortcuts, or "rules", for adding and subtracting integers, but also understand *why* these shortcuts work.

A lesson about subtracting integers explains the shortcut for subtracting a negative integer from three different viewpoints (as a manipulation of counters, as movements on a number-line and as a distance or difference). There is also a roundup lesson for addition and subtraction of integers.

The last topic in this chapter is graphing. Students will plot points on the coordinate grid according to a given equation in two variables (such as $y = x + 2$), this time using also negative numbers. They will notice the patterns in the coordinates of the points and the pattern in the points drawn in the grid and also work through some real-life problems.

You will find free videos covering many topics of this chapter at **https://www.mathmammoth.com/videos/** (choose 6th grade).

The Lessons in Chapter 8

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better, and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch8

SCAN ME

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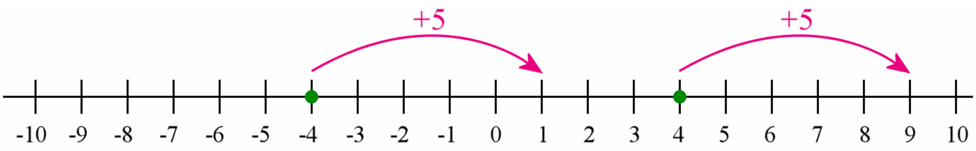
Addition and Subtraction as Movements

Suppose you are at 4. You jump 5 steps *to the right*. You end up at 9. We write <u>*an addition*</u>: 4 + 5 = 9.



Now you are at −4. You jump 5 steps *to the right*. You end up at 1. We write <u>*an addition*</u>*:* −4 + 5 = 1.

Addition can be shown on the number line as a movement to the *right*.

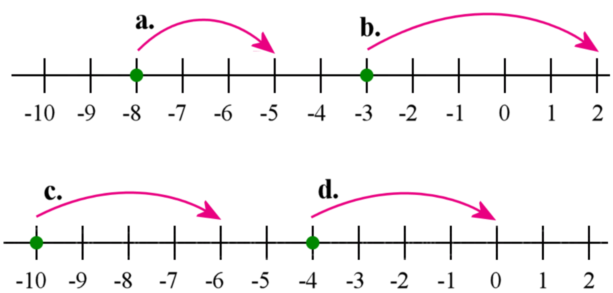
1. Write an addition sentence (an equation) to match each of the number line jumps.

a.

b.

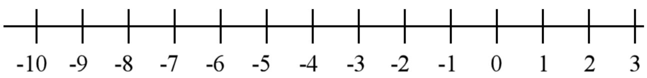
c.

d.

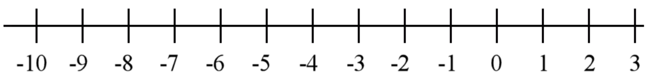


2. Draw a number line jump for each addition sentence.

a. −8 + 2 = ______ **b.** −5 + 4 = ______



c. −7 + 5 = ______ **d.** −10 + 12 = ______



3. Write an addition sentence.

Addition sentence:

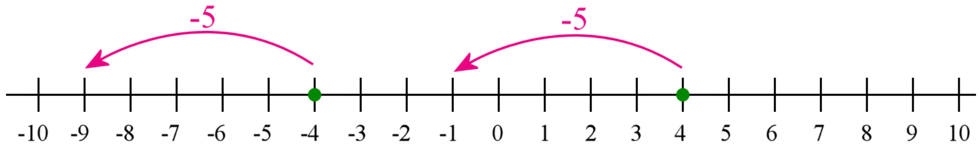
a. You are at −3. You jump 6 to the right. You end up at ______.

b. You are at −8. You jump 8 to the right. You end up at ______.

c. You are at −4. You jump 7 to the right. You end up at ______.

d. You are at −10. You jump 3 to the right. You end up at _____.

You are at 4. You jump 5 steps *to the left*. You end up at −1. We write ***a subtraction***: 4 − 5 = −1.



You are at −4. You jump 5 steps *to the left*. You end up at −9. We write ***a subtraction:*** −4 − 5 = −9.

Subtraction can be shown on the number line as a movement to the *left*.

Note: These three mean the same:

−4 − 5 = −9 (−4) − 5 = (−9) $^{-}4 - 5 = {}^{-}9$	We can use brackets around a negative number if we need to make clear that the minus sign is for "negative", and not for subtraction. We can also use an elevated minus sign for clarity. However, in the above situation, there is no confusion, so the brackets are not necessary and are usually omitted.

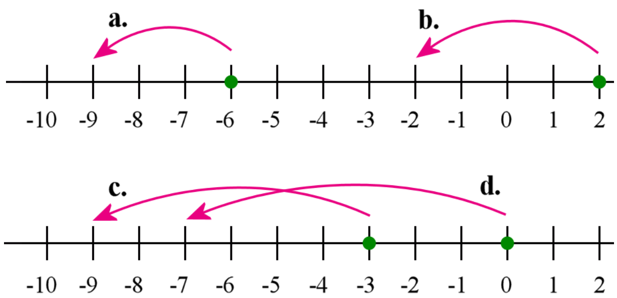
4. Write a subtraction sentence to match the number line jumps.

a.

b.

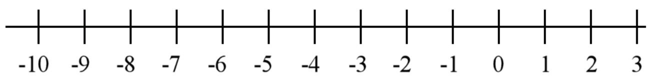
c.

d.

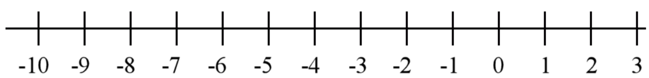


5. Draw a number line jump for each subtraction.

a. 1 − 5 = ______ **b.** 0 − 8 = ______



c. $^{-}2 - 4$ = ______ **d.** $^{-}7 - 3$ = ______



6. Write a subtraction sentence.

Subtraction sentence:

a. You are at $^{-}3$. You jump 5 to the left. You end up at ______.

b. You are at 5. You jump 10 to the left. You end up at ______.

c. You are at $^{-}5$. You jump 5 to the left. You end up at ______.

Number line jumps with mixed addition and subtraction

- The first number tells you where you *start*.
- Next comes the sign: a *plus* sign tells you to jump *right*, and a *minus* sign tells you to jump *left*.
- Next comes the number of *steps* to jump.

Notice that the number of steps that you jump is *not* negative.

$-2 + 6$ means: Start at -2 and move 6 steps to the right. You end up at 4. You started out negative, but you moved towards the positives, and you ended up on the positive side!	$-2 - 6$ means: Start at -2 and move 6 steps to the left. You end up at -8. You started out negative at -2 and ended up even more negative at -8.

7. Add or subtract. Think of the number line jumps.

a. $3 - 4 =$	**b.** $^{-}2 - 1 =$	**c.** $^{-}4 + 4 =$	**d.** $^{-}5 + 6 =$
$2 - 5 =$	$^{-}6 - 4 =$	$^{-}7 + 3 =$	$^{-}8 + 4 =$
$5 - 9 =$	$^{-}7 - 2 =$	$^{-}12 + 5 =$	$^{-}6 + 7 =$

8. Find the number that is missing from the equations. Think of moving on the number line.

a. 1 − ________ = $^{-}4$	**c.** $^{-}7$ + ________ = $^{-}6$	**e.** 2 − _______ = $^{-}5$	**g.** $^{-}3$ + ________ = 0
b. 3 − ________ = $^{-}3$	**d.** $^{-}9$ + ________ = $^{-}1$	**f.** 0 − _______ = $^{-}8$	**h.** $^{-}9$ + ________ = 9

9. The expression $1 - 2 - 3 - 4$ can also be thought of as a person making jumps on the number line. Where does the person end up?

10. James had R50. He bought coffee for R25 and a sandwich for R36. He paid what he could and the shop assistant put the rest of what he owed on his charge account with the shop.

a. Write a maths sentence to show the transaction.

b. How much in debt is James now?

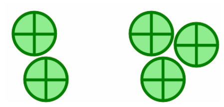
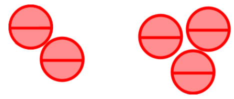
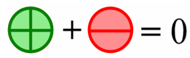
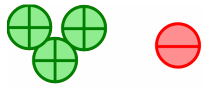
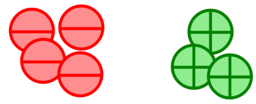
What about adding or subtracting a negative number?

Here is a way to think about $3 - (-2)$. Imagine you are standing at 3 to start with. Because of the subtraction sign, you turn to the left and get ready to take your steps. However, because of the additional minus sign in front of the 2, you have to take those steps BACKWARD—to the right! So, because you ended up taking those 2 steps to the *right*, in effect, you have just performed $3 + 2$.

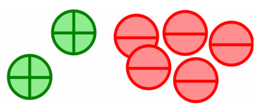
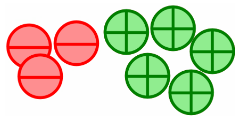
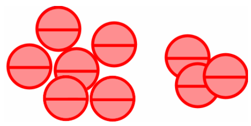
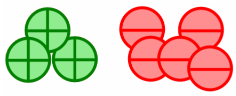
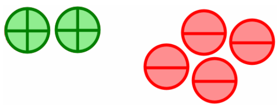
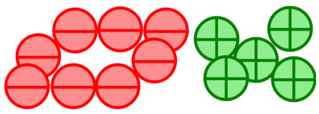
Another example: Here is a way to think about $-4 + (-5)$. Imagine you are standing at -4 to start with. Because of the addition sign, you turn to the right and get ready to move. But because of the additional minus sign in front of the 5, you have to take those 5 steps BACKWARD. So you take those 5 steps to the *left* instead. In essence, you have performed $-4 - 5$. (In the next lesson we will examine *other* ways to think about these situations.)

Adding Integers: Counters

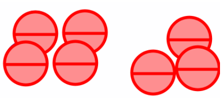
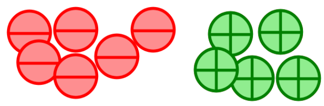
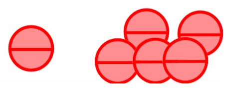
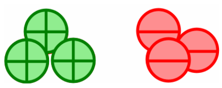
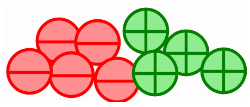
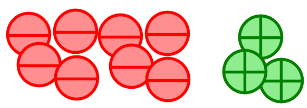
Addition of integers can be modelled using **counters.** We will use green counters with a "+" sign for positives and red counters with a "–" sign for negatives.

 Here we have the sum 2 + 3. There is a group of 2 positives and another of 3 positives.	 This picture shows the sum (−2) + (−3). We *add* negatives and negatives. In total, there are five negatives, so the sum is −5.	 1 + (−1) = 0 One positive counter and one negative counter *cancel* each other. In other words, their sum is zero!
 2 + (−2) = 0 Two negatives and two positives also cancel each other. Their sum is zero.	 3 + (−1) = 2 Here, one "positive-negative" pair is cancelled (you can cross it out!). We are left with 2 positives.	 (−4) + 3 = −1 Now the negatives outweigh the positives. Pair up three negatives with three positives. Those cancel out. There is still one negative left.

1. Refer to the pictures and add. Remember each "positive-negative" pair is cancelled.

 **a.** 2 + (−5) = _____	 **b.** (−3) + 5 = _____	 **c.** (−6) + (−3) = _____
 **d.** 3 + (−5) = _____	 **e.** 2 + (−4) = _____	 **f.** (−8) + 5 = _____

2. Write addition sentences (equations) to match the pictures.

 **a.**	 **b.**	 **c.**
 **d.**	 **e.**	 **f.**

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Chapter 9: Geometry
Introduction

The main topics in this chapter include:

- area of triangles
- area of polygons
- nets and surface area of prisms and pyramids
- volume of rectangular prisms with sides of fractional length

However, the chapter starts out with some revision of topics from earlier grades, as we revise the different types of quadrilaterals and triangles and students do some basic drawing exercises. In these drawing problems, students will need a ruler to measure lengths and a protractor to measure angles.

One focus of the chapter is the area of polygons. To reach this goal, we follow a step-by-step development. First, we study how to find the area of a right triangle, which is very easy, as a right triangle is always half of a rectangle. Next, we build on the idea that the area of a parallelogram is the same as the area of the related rectangle, and from that we develop the usual formula for the area of a parallelogram as the product of its base times its height. This formula then gives us a way to generalise finding the area of any triangle as *half* of the area of the corresponding parallelogram.

Finally, the area of a polygon can be determined by dividing it into triangles and rectangles, finding the areas of those and summing them. Students also practise their new skills in the context of a coordinate grid. They draw polygons in the coordinate plane and find the lengths of their sides, perimeters and areas.

Nets and surface area is another major topic. Students draw nets and determine the surface area of prisms and pyramids using nets. They also learn how to convert between different area units, not using conversion factors or formulas, but using logical reasoning where they learn to determine those conversion factors themselves.

Lastly, we study the volume of rectangular prisms, this time with edges of fractional length. (Students have already studied this topic in fifth grade with edges that are a whole number long.) The basic idea is to prove that the volume of a rectangular prism *can* be calculated by multiplying its edge lengths even when the edges have fractional lengths. To that end, students need to think how many little cubes with edges ½ or ⅓ unit go into a larger prism. Once we have established the formula for volume, students solve some problems concerning the volume of rectangular prisms.

There are quite a few videos available to match the lessons in this chapter at **https://www.mathmammoth.com/videos/** (choose 6th grade).

Also, don't forget to use the resources for challenging problems:
https://l.mathmammoth.com/challengingproblems

I recommend that you at least use the first resource listed, Math Stars Newsletters.

The Lessons in Chapter 9

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better, and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch9

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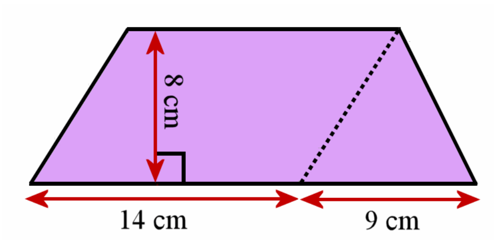
Area of Polygons

In the previous lesson we found the area of a polygon by enclosing it in a rectangle, and by using subtraction.

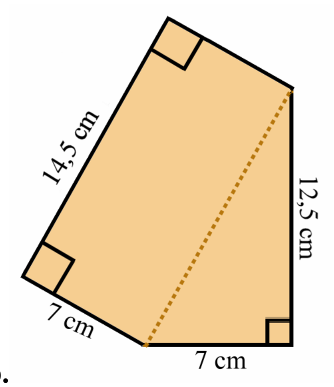
Another, natural way to calculate the area of a polygon is to divide the polygon into easy shapes, such as rectangles, triangles and trapeziums. Calculate the area of each shape separately, and then add them to find the total area.

1. Calculate the total area of the figures.

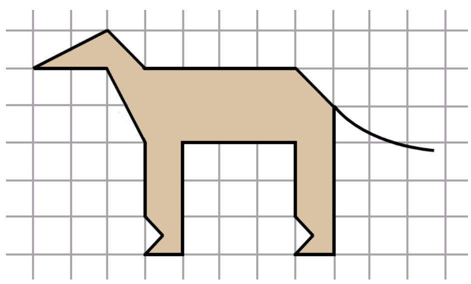
a.



b.

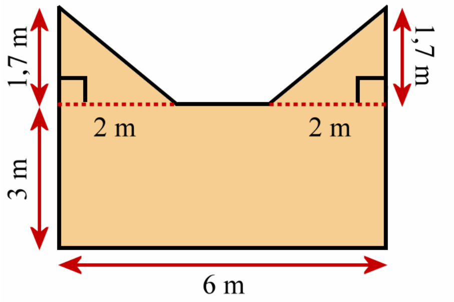


2. **a.** The side of each little square in the drawing on the right is <u>1 centimetre</u>. Find the area of the polygon.



b. Imagine that the side of each little square is <u>2 centimetres</u> instead. What is the area now?

3. Find the area of the polygon.



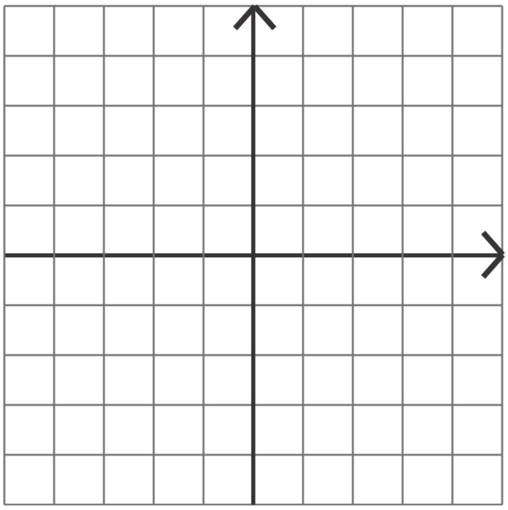
4. The vertices of a right triangle are at (17, 26), (17, −9), and (2, −9). Find its area.

5. The points (1, 2,4), (2,4, 1), (2,4, −1), (1, −2,4) are four vertices of a water fountain in the shape of a regular octagon. The other four points are found by reflecting these four in the *y*-axis.

a. Draw the octagon.

b. Find the length of *one* side of the fountain.

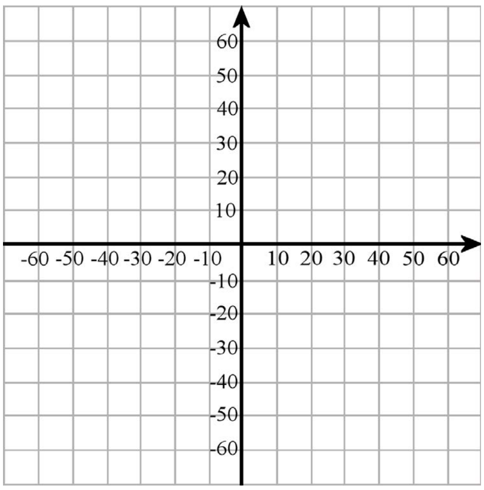
c. Find its perimeter.



Puzzle Corner

Join the following points in order with line segments. Then find the area of the resulting polygon.

(−35, −40), (−35, 40), (−20, 40), (20, −15), (20, 40), (35, 40), (35, −40), (20, −40), (−20, 15), (−20, −40) and (−35, −40)

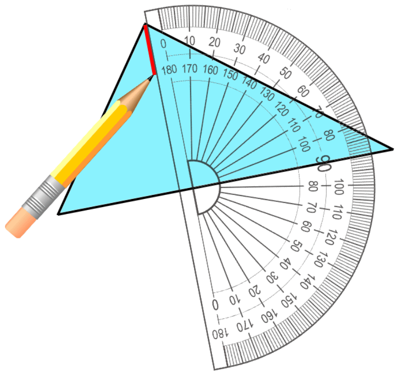


Area of Shapes Not Drawn on Grid

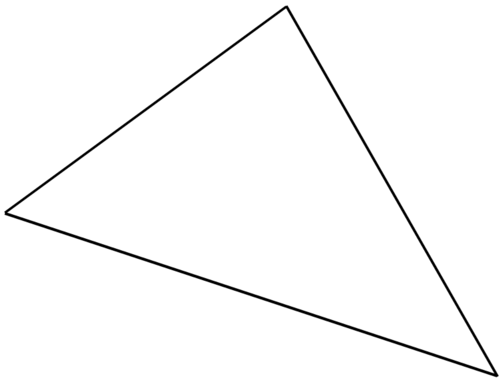
To find the area of a polygon that is not draw on a grid, we often need to draw altitudes and measure the length of various line segments.

1. First, choose one of the sides as the base. It can be any side!
2. Draw the altitude. Use a protractor or a triangular ruler to draw the altitude so that it goes through one vertex and is perpendicular to the base. See the illustration.

   Line up the 90°-mark on the protractor with the base of the triangle and slide it until the line you draw will pass through the vertex.
3. Measure the lengths of the altitude and base as precisely as you can with a ruler.
4. Calculate the area.



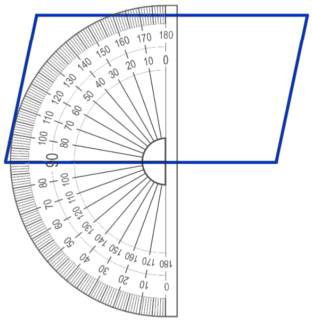
1. Find the area of this triangle in square centimetres. Round your final answer to the nearest whole square centimetre.



2. Draw your own triangle here, and find its area!

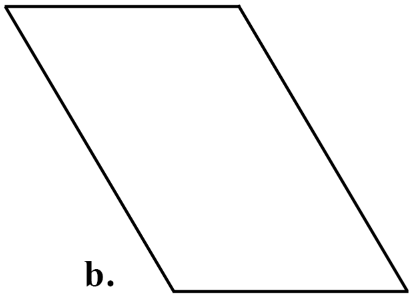
3. Find the of area of each parallelogram below, measuring the necessary parts to the nearest millimetre. Round your final answer to whole square centimetres.

You will need to draw an altitude to each parallelogram. Use a protractor or a triangular ruler—do not “eyeball” it. The picture on the right shows how to position a protractor for drawing the altitude. Notice that the 90° mark is aligned with the base.

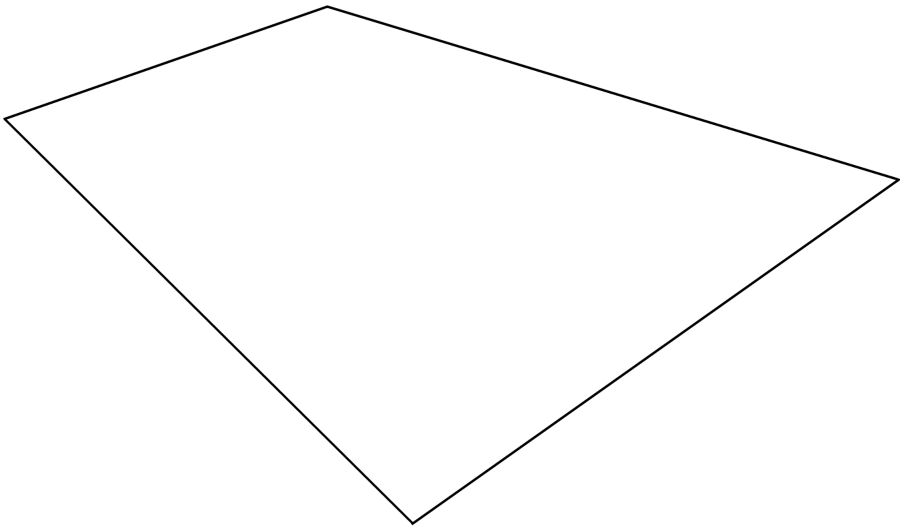


a.

b.



4. Divide this quadrilateral into two triangles, and then find its area in square centimetres. You may use a calculator.



Puzzle Corner

Draw a triangle with an *area* of 18 square centimetres.

Is it only possible to draw just *one* triangle with that area, or is it possible to draw several, with varying shapes/sizes?

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Chapter 10: Statistics
Introduction

The fundamental theme in our study of statistics is the concept of *distribution*. In the first lesson, students learn what a distribution is—basically, it is *how* the data is distributed. The distribution can be described by its centre, spread and overall shape. The shape is read from a graph, such as a dot plot or a bar graph.

Two major concepts when summarising and analysing distributions are its centre and its variability. First we study the centre, in the lessons about mean, median, and mode. Students not only learn to calculate these values, but also relate the choice of measures of centre to the shape of the data distribution and the type of data.

Next, we study measures of variation, starting with range and interquartile range. Students use these measures in the following lesson, as they both read and draw boxplots.

The lesson *Mean Absolute Deviation* introduces students to this measure of variation. It takes many calculations, and the lesson gives instructions on how to calculate it using a spreadsheet program (such as Excel or LibreOffice Calc).

Next, students learn to make histograms. They will also continue summarising distributions by describing their shape, and giving a measure of centre and a measure of variability. The lesson on stem-and-leaf plots is optional.

There are some videos available for these topics at **https://www.mathmammoth.com/videos/** (choose 6th grade).

The Lessons in Chapter 10

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better, and even just for some fun. Enjoy!

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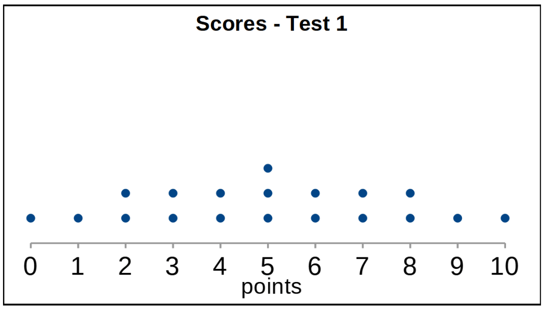
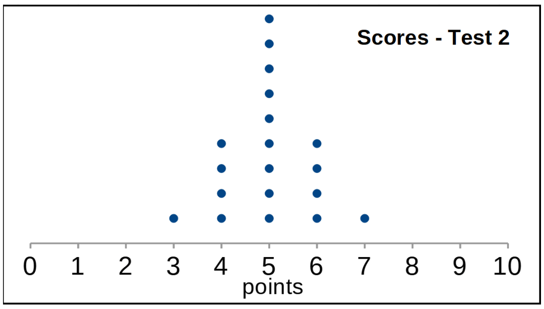
Range and Interquartile Range





Example 1. Look at the two graphs. The first gives the scores for test 1 and the second for test 2. *Both* sets of data have a mean of 5,0 and a median of 5. Yet the distributions are <u>very different</u>.

How? In test 1, the students got a wide range of different scores; the data is very scattered and **varies a lot**. In test 2, nearly all of the students got a score from 4 to 6. The data is concentrated, or *clustered*, around 5.

We have several ways of measuring the variation in a distribution. One way is to use **<u>range</u>**. Simply put, range is **the difference between the largest and smallest data items.**

For test 1, the smallest score is 0 and the largest is 10 so the range is 10. For test 2, the smallest score is 3 and the largest is 7 so the range is 4. Clearly, the range is much smaller for test 2, indicating the data is much more concentrated than in test 1.

Another measure of variation is the **<u>interquartile range</u>**.

To determine this measure, we first identify the **quartiles**, which are the numbers that divide the data into quarters. The **interquartile range is the difference between the first and third quartiles**. Since the quartiles divide the data into quarters, exactly half of it lies between the first and third quartiles—and it is the middlemost half of the data. The smaller this measure is, the more concentrated the data is.

Example 2. The scores for test 1 are: 0, 1, 2, 2, 3, 3, 4, 4, 5, 5, 5, 6, 6, 7, 7, 8, 8, 9, 10. Let's now find the interquartile range. For that, we need to divide the data into quarters.

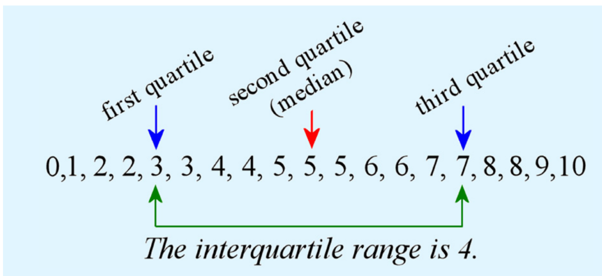
The median naturally divides the data into two halves: 0, 1, 2, 2, 3, 3, 4, 4, 5, **<u>5</u>** , 5, 6, 6, 7, 7, 8, 8, 9, 10.

Now we take the lower half of the data, *excluding the median,* and find *its* median*:* 0, 1, 2, 2, **<u>3</u>**, 3, 4, 4, 5. That is the **first quartile.**

Similarly, the median of the upper half of the data is the **third quartile**: 5, 6, 6, 7, **<u>7</u>**, 8, 8, 9, 10

The median itself is the **second quartile**.

Together, the three quartiles divide the data into quarters. The interquartile range is the difference between the third and first quartile, or in this case 7 − 3 = **<u>4 points</u>**.



So, exactly half of the test scores lie within 4 points (from 3 to 7 points) around the median of 5 points.

The scores for test 2 are: 3, 4, 4, 4, **<u>4</u>**, 5, 5, 5, 5, **<u>5</u>**, 5, 5, 5, 5, **<u>6</u>**, 6, 6, 6, 7. The quartiles are marked in bold.

The interquartile range is 6 − 4 = **<u>2 points</u>**. So, the middlemost half of the data lies within only 2 points of the median (5 points)—very close to the middle peak of the distribution. This is what we also see in the graph. Clearly, the interquartile ranges show us the same story: the data for test 1 varies much more than for test 2.

1. Find the quartiles and the interquartile range of the data sets.

a. 5, 5, 6, 6, 7, 7, 7, 7, 7, 7, 8, 8, 8, 9, 10, 10

first quartile ________ median ________ third quartile ________ interquartile range ________

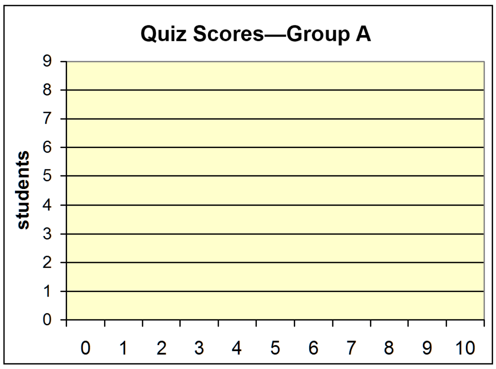
b. 2, 2, 3, 4, 5, 5, 5, 5, 6, 6, 6, 7, 7, 7, 9, 9

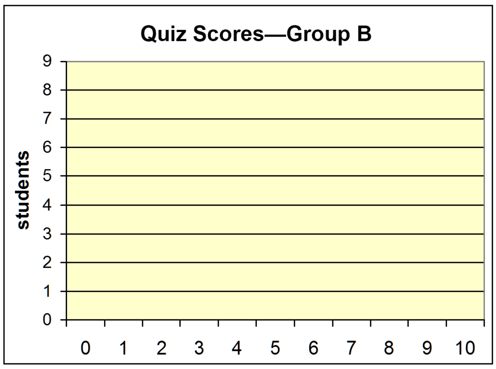
first quartile ________ median ________ third quartile ________ interquartile range ________

c. Let’s say the data sets in (1a) and (1b) are the quiz scores of two groups (A and B) of students.

Which group did better in general? _______ In which group did the quiz scores vary more? _______

Make bar graphs for the quiz scores of the two groups. Note how the graphs, too, show the answers to the above questions.





2. Find the asked statistical measures of the data sets.

a. The height of some children in centimetres:

136 138 139 139 140 140 140 140 140 141 141 141 142 144 144 145 147

1st quartile _________ median _________ 3rd quartile _________

interquartile range _________ range _________

b. The number of paid vacation days in a year of the employees in a small firm:

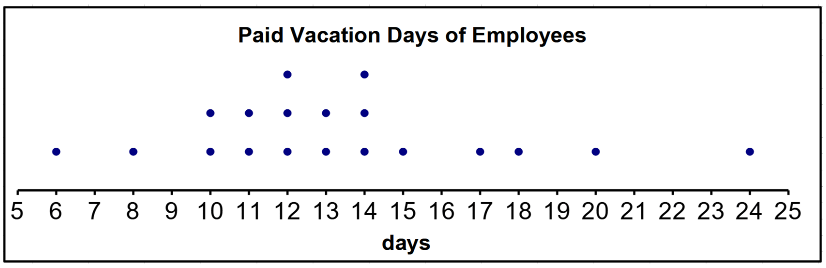
6 8 10 10 11 11 12 12 12 13 13 14 14 14 15 17 18 20 24

1st quartile _________ median _________ 3rd quartile _________

interquartile range _________ range _________

Use the measures you just found, and fill in:

Half of the employees have from _____ to _____ vacation days in a year.



Boxplots

Boxplots or **box-and-whisker plots** are simple graphs on a number line that use a box with "whiskers" to visually show the quartiles of the data. Boxplots show us a **five-number summary** of the data: the minimum, the 1st quartile, the median, the 3rd quartile and the maximum.

Example 1. These are prices of a hair brush in three stores (in rand).

140 150 190 200 200 200 210 240 250 340 340 350 350 370 420 450 550

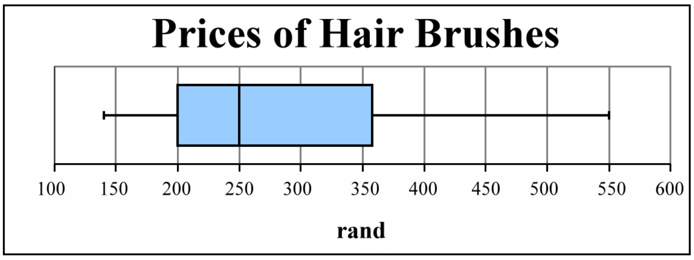
Five-number summary:

Minimum: R140

First quartile: R200

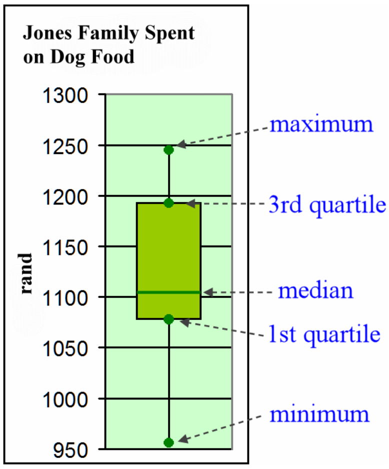
Median: R250

Third quartile: R360
Maximum: R550



The box itself starts at the 1st quartile and ends at the 3rd quartile. Therefore, its width is the interquartile range. We draw a line in the box marking the median (R250).

The boxplot also has two "whiskers". The first whisker starts at the minimum (R140) and ends at the first quartile. The other whisker is drawn from the third quartile to the maximum (R550).



Example 2. This boxplot shows the Jones family's monthly expenditures over a 12-month period for dog food. This time the boxplot is drawn vertically.

The box extends over half the data. This means that half the time, they spend from about R1080 to R1190 monthly. But sometimes they spend only about R950, and sometimes up to R1245 in a month.

<u>Five-number summary:</u>

Minimum: R956

First quartile: R1078

Median: R1105

Third quartile: R1193

Maximum: R1245

1. **a.** Read the five-number summary from the boxplot.

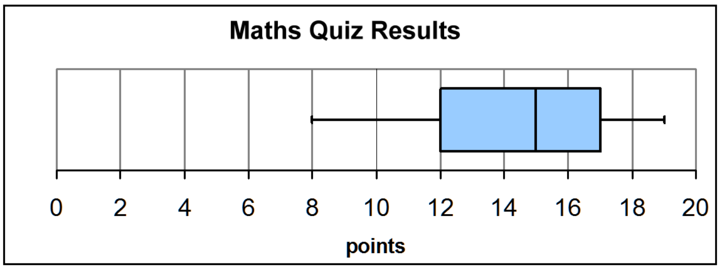
Minimum:

First quartile:

Median:

Third quartile:

Maximum:



b. Look at the box and fill in: Half the students got between _____ and _____ points in the quiz.

c. Do you think the quiz went well?