

Using Ratios to Convert Measuring Units

Consider the conversion factor **1 inch = 2.54 cm**. If we think of it as an equation and divide both sides by “1 inch,” then we will get 1 on the left side, and a RATIO on the right side:

$$1 \text{ inch} = 2.54 \text{ cm}$$

This is the conversion factor, but we will think of it as an equation now.

$$\frac{1 \text{ inch}}{1 \text{ inch}} = \frac{2.54 \text{ cm}}{1 \text{ inch}}$$

Divide both sides by “1 inch.” Yes, we do include the unit *inch* in this.

$$1 = \frac{2.54 \text{ cm}}{1 \text{ inch}}$$

We get a plain 1 on the left side (something divided by itself equals 1).

What we get on the right side is the ratio 2.54 cm per 1 inch (or 2.54 cm to 1 inch), and that ratio equals 1.

We can also do this the other way around:

$$1 \text{ inch} = 2.54 \text{ cm}$$

This is the conversion factor, but we will think of it as an equation now.

$$\frac{1 \text{ inch}}{2.54 \text{ cm}} = \frac{2.54 \text{ cm}}{2.54 \text{ cm}}$$

Divide both sides by “2.54 cm”. Yes, we do include the unit *cm* in this.

$$\frac{1 \text{ inch}}{2.54 \text{ cm}} = 1$$

We get a plain 1 on the right side (something divided by itself equals 1).

What we get on the left side is the ratio 1 inch per 2.54 cm (or 1 inch to 2.54 cm), and that ratio equals 1.

In fact, we can transform *any* conversion factor between measuring units into a ratio that is equal to 1.

$$1 \text{ qt} = 0.946 \text{ L}$$

$$\downarrow$$

$$\frac{1 \text{ qt}}{0.946 \text{ L}} = 1$$

$$0.946 \text{ L} = 1 \text{ qt}$$

$$\downarrow$$

$$\frac{0.946 \text{ L}}{1 \text{ qt}} = 1$$

$$1 \text{ mi} = 1.6093 \text{ km}$$

$$\downarrow$$

$$\frac{1 \text{ mi}}{1.6093 \text{ km}} = 1$$

$$1 \text{ lb} = 0.454 \text{ kg}$$

$$\downarrow$$

$$\frac{1 \text{ lb}}{0.454 \text{ kg}} = 1$$

1. Think of the conversion factors as equations, and transform each one into a new equation of the form “1 = a ratio” or “a ratio = 1.”

$$1 \text{ ft} = 0.3048 \text{ m}$$

↓

$$1 \text{ ounce} = 28.35 \text{ g}$$

↓

$$1 \text{ mi} = 1,760 \text{ yd}$$

↓

$$1 \text{ m} = 1.0936 \text{ yd}$$

↓

We can use these ratios that equal one in **converting measuring units**.

How does that happen? Study the following example carefully. Mathematically speaking, we multiply the quantity we want to convert by 1. Multiplying it by 1 does not change its value. Then, we replace that 1 with one of the ratios of measuring units that equal 1. Next, we cross out the measuring units that cancel out. Lastly, we multiply/divide the numbers involved.

$$56 \text{ cm} = 56 \text{ cm} \cdot 1 = 56 \text{ cm} \cdot \frac{1 \text{ in.}}{2.54 \text{ cm}} = \cancel{56 \text{ cm}} \cdot \frac{1 \text{ in.}}{\cancel{2.54 \text{ cm}}} = \frac{56 \cdot 1 \text{ in.}}{2.54} = 22.047 \text{ in.} \approx 22 \text{ in.}$$

Multiply the quantity by 1. Replace that 1 with a ratio. Cancel out the cm units. Calculate. Round.

Notice that we **keep the the units of measure** in the calculation! The “cm” units cancel out, and we end up with only the unit “in” (which is what we wanted: to convert the given quantity into *inches*).

Another example, of converting 8.9 quarts into liters:

$$8.9 \text{ qt} = 8.9 \text{ qt} \cdot 1 = 8.9 \text{ qt} \cdot \frac{0.946 \text{ L}}{1 \text{ qt}} = \cancel{8.9 \text{ qt}} \cdot \frac{0.946 \text{ L}}{\cancel{1 \text{ qt}}} = \frac{8.9 \cdot 0.946 \text{ L}}{1} = 8.4194 \text{ L} \approx 8.4 \text{ L.}$$

Multiply the quantity by 1. Replace that 1 with a ratio. Cancel out the qt units. Calculate. Round.

2. Use the given ratios to convert the measuring units.

a. Use $1 = \frac{2.54 \text{ cm}}{1 \text{ in.}}$ to convert 79 in. to centimeters.

79 in. =

b. Use $1 = \frac{1 \text{ mi}}{1.6093 \text{ km}}$ to convert 56 km to miles.

56 km =

c. Use $1 = \frac{1.6093 \text{ km}}{1 \text{ mi}}$ to convert 2.8 mi to kilometers.

2.8 mi =

d. Use $1 = \frac{0.946 \text{ L}}{1 \text{ qt}}$ to convert 4 qt to liters.

4 qt =

How do you know whether to use the ratio $\frac{1 \text{ in.}}{2.54 \text{ cm}}$ or the ratio $\frac{2.54 \text{ cm}}{1 \text{ in.}}$ when converting 7 inches into centimeters?

If the quantity you start with has inches, then you will need to cancel out the unit “inches” in the conversion. Therefore, choose the ratio that has inches in the denominator.

Here is an example of using the *wrong* ratio:

$$7 \text{ in.} = 7 \text{ in.} \cdot 1 = 7 \text{ in.} \cdot \frac{1 \text{ in.}}{2.54 \text{ cm}} = 7 \text{ in.} \cdot \frac{1 \text{ in.}}{2.54 \text{ cm}} = \frac{7 \text{ in.} \cdot 1 \text{ in.}}{2.54 \text{ cm}} = 2.7559 \text{ in.}^2 / \text{cm}$$

Replace 1
with a ratio.

Nothing cancels.

Calculate.

The answer is not reasonable. Since inches are the longer units, 7 inches should convert to a bigger number of cm. The units didn't work out, either.

Here are some conversion factors you will need in the following problems.

$1 \text{ inch} = 2.54 \text{ cm}$

$1 \text{ yard} = 0.9144 \text{ m}$

$1 \text{ quart} = 0.946 \text{ L}$

$1 \text{ lb} = 0.454 \text{ kg}$

$1 \text{ foot} = 0.3048 \text{ m}$

$1 \text{ mile} = 1.6093 \text{ km}$

$1 \text{ ounce} = 28.35 \text{ g}$

$1 \text{ kg} = 2.2 \text{ lb}$

3. Use ratios to convert the measuring units.

a. 89 cm into inches

b. 15 kg into pounds

c. 78 miles into km

d. 89 feet into meters

e. 365 g into ounces

Chaining (optional). We can use TWO (or more) ratios in the conversion, and “chain” them together.

Example. Convert 0.9 liters into liquid ounces.

We have TWO conversion factors: 1 quart = 0.946 L and 1 quart = 32 oz. From these, we can write *four* ratios: $\frac{1 \text{ qt}}{0.946 \text{ L}}$, $\frac{0.946 \text{ L}}{1 \text{ qt}}$, $\frac{32 \text{ oz}}{1 \text{ qt}}$, and $\frac{32 \text{ oz}}{1 \text{ qt}}$, all equaling 1. We can use TWO of those four, “chaining” them together, to go from 0.9 liters to however many ounces:

$$0.9 \text{ L} = 0.9 \text{ L} \cdot \frac{1 \text{ qt}}{0.946 \text{ L}} \cdot \frac{32 \text{ oz}}{1 \text{ qt}} = \cancel{0.9 \text{ L}} \cdot \frac{\cancel{1 \text{ qt}}}{0.946 \cancel{\text{ L}}} \cdot \frac{32 \text{ oz}}{1 \cancel{\text{ qt}}} = \frac{0.9 \cdot 32 \text{ oz}}{0.946} = \approx 30.4 \text{ oz.}$$

Write the two ratios that equal 1. Cancel out the liters and quarts. Calculate. Round.

How do you choose which two of the possible four ratios to use? Since you start out with LITERS, you want a ratio where LITERS are in the denominator. And since you want to end up with OUNCES, you want a ratio where OUNCES are NOT in the denominator. The quarts and liters cancel out in the process, leaving the ounces.

4. Convert the measuring units as indicated.

a. Use the ratios (2.54 cm / 1 in.) and (12 in. / 1 ft) to convert 5 ft into centimeters.

5 ft =

b. Use the ratios (1 qt / 32 oz) and (0.946 L / 1 qt) to convert 24 oz into liters.

c. Convert 700 yards into meters.

d. Convert 8 kg into ounces (weight).

e. Convert 371 ounces into grams.

f. Convert 15 pints into liters.