The Sieve of Eratosthenes and Prime Factorization

To find all the prime numbers less than 100 we can use the *sieve of Eratosthenes.*

1. Cross out 1, as it is not considered a prime.
2. Cross out all the even numbers except 2.
3. Cross out all the multiples of 3 except 3.
4. You do not have to check multiples of 4. Why?
5. Cross out all the multiples of 5 except 5.
6. You do not have to check multiples of 6. Why?
7. Cross out all the multiples of 7 except 7.
8. You do not have to check multiples of 8 or 9 or 10.
9. The numbers left are primes.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

List the primes between 0 and 100 below:

2, 3, 5, 7, ____

Why do you not have to check numbers that are bigger than 10? Let's think about multiples of 11. The following multiples of 11 have already been crossed out: 2 × 11, 3 × 11, 4 × 11, 5 × 11, 6 × 11, 7 × 11, 8 × 11, and 9 × 11. The multiples of 11 that have not been crossed out are 10 × 11 and onward... but they are not on our chart! Similarly, the multiples of 13 that are less than 100 are 2 × 13, 3 × 13, ..., 7 × 13, and all of those have already been crossed out when you crossed out multiples of 2, 3, 5, and 7.

Use the divisibility tests for 2, 3, 4, 5, 6, 9, and 10 when building a factor tree.

```
135
/ \
5 × ?
```

```
     2 7
5 ) 1 3 5
  - 1 0
      3 5
    - 3 5
        0
```

```
135
/ \
5 × 27
    / \
    3 × 9
        / \
        3 × 3
```

We start out by noticing that 135 is **divisible by 5.** From long division, we know that 135 = 5 × 27. The final factorization is 135 = 3 × 3 × 3 × 5 or $3^3 \times 5$.

```
441
/ \
9 × ?
```

```
     4 9
9 ) 4 4 1
  - 3 6
      8 1
    - 8 1
        0
```

```
    441
    / \
  9  ×  49
 / \   / \
3 × 3 × 7 × 7
```

Adding the digits of 441, we get 9, so it is **divisible by 9.** We divide to get 441 = 9 × 49. The end result is 441 = 3 × 3 × 7 × 7 or $3^2 \times 7^2$.

```
912
/ \
4 × ?
```

```
     2 2 8
4 ) 9 1 2
   -8
    1 1
   - 8
      3 2
    - 3 2
        0
```

```
      912
      / \
   4  ×  228
  / \    / \
2 × 2 × 4 × 57
```

```
        912
        / \
     4  ×  228
    / \    /  \
2 × 2  ×  4  ×  57
          / \   / \
          2 × 2 × 3 × 19
```

The last two digits of 912 are "12" so it is **divisible by 4**.

228, too, is **divisible by 4** (its last digits are "28").

Lastly, 57 is 3 × 19. The prime factorization of 912 is $2^4 \times 3 \times 19$.

Sample worksheet from
www.mathmammoth.com

1. Find the prime factorization of these numbers. Use a notebook for long divisions.

a. 124 / \\ 2 × ___ / \\	**b.** 260 / \\ 10 × ___ / \\ / \\	**c.** 96 / \\ 3 × ___ / \\
d. 90	**e.** 165	**f.** 95
g. 80	**h.** 240	**i.** 272
j. 76	**k.** 126	**l.** 104

Sample worksheet from
www.mathmammoth.com

2. Find the prime factorization of the following numbers.

a. 196	**b.** 380	**c.** 336
d. 306	**e.** 116	**f.** 720
g. 675	**h.** 990	**i.** 945

Puzzle Corner Find all the primes between 0 and 200. Use the sieve of Eratosthenes again (you need to make a grid in your notebook).

This time, you need to cross out 1, and then every even number except 2, every multiple of 3 except 3, every multiple of 5 except 5, every multiple of 7 except 7, every multiple of 11 except 11, and every multiple of 13 except 13.

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