math MAMMOTH

Grade 5-B Worktext

Decimals, part 2

Fractions: add and subtract

Fractions: multiply and divide

Geometry

By Maria Miller

Sample worksheet from
https://www.mathmammoth.com

Contents

Foreword

Math Mammoth Grade 5-B Worktext covers the second half of fifth grade mathematics studies. In part 5-A, students have studied the four operations with whole numbers, large numbers, problem solving, decimal arithmetic, and statistical graphs. In this part, 5-B, we study more about decimals, a lot about fractions and fraction arithmetic, and geometry.

Chapter 6 continues our study of decimals. The focus is on multiplying decimals by decimals, dividing decimals by decimals, and conversions between measuring units.

Chapter 7 covers the addition and subtraction of fractions—another topic of focus for 5th grade, besides decimals. The most difficult topic of this chapter is adding and subtracting unlike fractions, which is done by first converting them to equivalent fractions with a common denominator.

In chapter 8, we study the multiplication and division of fractions from various angles.

Chapter 9 takes us to geometry, starting with a review of angles and polygons. From there, students will learn to draw circles, to classify triangles and quadrilaterals, and the concept of volume in the context of right rectangular prisms (boxes).

I wish you success with teaching math!

Maria Miller, the author

Chapter 6: Decimals, Part 2
Introduction

In this chapter, we focus on decimal multiplication and division, and conversions between measurement units.

We start out with the topic of multiplying decimals by decimals. This is typically a fairly easy topic, as long as students remember the rule concerning the decimal digits in the answer. This rule could be confused with the other rules of decimal arithmetic that we also study in this chapter. In 6th grade, I provide a proof for the rule using fraction multiplication. I didn't include it here, because the chapter already contains so many new topics for students that including the justification for the rule may just cause an overload, plus, students haven't studied fraction multiplication yet.

Then we learn about multiplication as *scaling*. We cannot view decimal multiplications, such as 0.4×1.2, as repeated addition. Instead, they are viewed as scaling — shrinking or enlarging — the number or quantity by a scaling factor. So, 0.4×1.2 is thought of as scaling 1.2 by 0.4, or as four-tenths *of* 1.2. You may recognize this as the same as 40% of 1.2.

Next, we learn about decimal divisions that can be done with mental math. Students divide decimals by whole numbers (such as $0.8 \div 4$ or $0.45 \div 4$) by relating them to equal sharing. They divide decimals by decimals in situations where the divisor goes evenly into the dividend, thus yielding a whole-number quotient (e.g. $0.9 \div 0.3$ or $0.072 \div 0.008$).

In the next lesson, *More Division with Decimals*, we simply review long division with decimals, when the divisor is a whole number.

The following topic is multiplying and dividing decimals by powers of ten. This is presented with the help of place value charts. The actual concept is that the number being multiplied or divided *moves* in the place value chart, as many places as there are zeros in the power of ten. As a shortcut, we can move the decimal point. However, the movement of the decimal point is an "illusion" — that is what seems to happen — but in reality, the number itself got bigger or smaller; thus, its digits actually changed positions in the place value system.

Next, we study the metric system and how to convert various metric units (within the metric system), such as converting kilograms to grams, or dekaliters to hectoliters. The first of the two lessons mainly deals with very commonly used metric units, and we use the meaning of the prefix to do the conversion. For example, centimeter is a hundredth part of a meter, since the prefix "centi" means 1/100. Knowing that gives us a means of converting between centimeters and meters.

The second lesson deals with more metric units, even those not commonly used, such as dekaliters and hectograms, and teaches a method for conversions using a chart. These two methods for converting measuring units within the metric system are sensible and intuitive, and help students not to rely on mechanical formulas.

Then we turn our attention to dividing decimals by decimals, which then completes our study of all decimal arithmetic. The principle here is fairly simple, but it is easy to forget (multiply both the dividend and the divisor by a power of ten, until you have a whole-number divisor).

After learning that, students practice measurement conversions within the customary system, rounding measurements, and some generic problem solving with decimals. Recall that not all students need all the exercises; use your judgment.

Problems accompanied by a small picture of a calculator are meant to be solved with the help of a calculator. Otherwise, a calculator should not be allowed.

The Lessons in Chapter 6

Helpful Resources on the Internet

Decimal Arithmetic - Videos by Maria
These are my videos where I explain all about decimal arithmetic: adding, subtracting, multiplying, dividing, comparing and rounding decimals, plus some problem solving. Suitable for grades 5-6.
https://www.mathmammoth.com/videos/grade_5/5th-grade-videos.php#decimals

MULTIPLICATION

Exploring Multiplication of Decimals
Enter two numbers with one decimal digit, and you will see the product as a rectangular area.
http://www.hbschool.com/activity/elab2004/gr6/1.html

Decimals Workshop
Practice adding, subtracting, multiplying, or dividing decimals with this customizable interactive exercise.
http://mrnussbaum.com/decimals-workshop/

Multiply Decimals by Whole Numbers
Polish your decimal multiplication skills with this interactive online exercise.
https://www.khanacademy.org/math/cc-fifth-grade-math/cc-5th-arith-operations/cc-5th-mult-decimals/e/multiplying-decimals-without-the-standard-algorithm-2

Decimal Times — Mental Multiplication
Practice mental and written methods for multiplying and dividing decimal numbers in this interactive online exercise.
http://www.transum.org/Maths/Activity/Decimals/

Multiply with Decimals: Simple Word Problems
Practice multiplying decimals by whole numbers with these interactive word problems.
https://www.studyladder.com/games/activity/multiply-decimals-by-whole-numbers-22247

DIVISION (AND MULTIPLICATION)

Divide Decimals by Whole Numbers In Your Head
Practice mental division of decimals by whole numbers in this interactive exercise.
https://www.khanacademy.org/math/cc-fifth-grade-math/cc-5th-arith-operations/cc-5th-dividing-decimals/e/dividing-decimals-without-the-standard-algorithm-3

Divide Decimals by Decimals in Your Head
Practice mental division of tenths by tenths in this online exercise.
https://www.khanacademy.org/math/cc-fifth-grade-math/cc-5th-arith-operations/cc-5th-dividing-decimals/e/dividing-decimals-without-the-standard-algorithm-6

Long Division to Decimal Places
This is a short tutorial about dividing whole numbers to decimal digits. At the bottom of the page you will find a short online quiz.
https://www.mathsisfun.com/long_division3.html

Dividing Decimals
This is a short tutorial about dividing decimals, whether by whole numbers or by decimals. At the bottom of the page you will find a short online quiz.
https://www.mathsisfun.com/dividing-decimals.html

Divide and Multiply Decimals Quiz
Practice multiplying and dividing decimals by powers of ten in this 10-question quiz.
https://www.thatquiz.org/tq-3/?-j12c-l5-p0

Multiply and Divide Decimals Quiz
Use this interactive exercise to polish your decimal multiplication and division skills.
https://campus.mangahigh.com/en/px/38/0/0

Long Division - choose "Decimal division"
Enter the numbers that you want to divide and click on the button "Do division" to see a step-by-step illustration of the solution.
http://www.mathsonline.org/pages/longdiv.html

Decimal Long Division Worksheets
Use these randomly generated worksheets for extra practice.
https://www.homeschoolmath.net/worksheets/decimal_division.php#long

Multiply and Divide Decimals by Powers of Ten
Practice multiplying and dividing by powers of ten with this multiple-choice quiz.
http://www.mathgames.com/skill/6.46-multiply-and-divide-decimals-by-powers-of-ten

UNITS OF MEASUREMENT

Conversion Quizzes - ThatQuiz.org
A customizable online quiz about conversions between measuring units. The options include both metric and customary systems and six different difficulty levels.
http://www.thatquiz.org/tq-n/science/metric-system/

Horrendous Soup Game
Make a recipe for the most disgusting soup you can imagine in this fun game that practices conversion between metric units of measurement.
http://mrnussbaum.com/soup

Metric System Conversions Quiz
Practice converting between different units of measurement in the metric system with this 10-question online quiz.
https://www.thatquiz.org/tq-n/?-j17v-l4-p0

Metric Millionaire
Practice the metric system with these interactive multiple-choice word problems.
https://www.quia.com/rr/30535.html?AP_rand=837779130

Word Problems Involving Measurement Conversions
Solve word problems that involve converting between metric measures of distance, volume, and mass, as well as measures of time.
https://www.khanacademy.org/math/on-sixth-grade-math/on-measurement/on-unit-conversion/e/converting-measurements-word-problems

Inches Measurement Worksheets
Measurement worksheets for identifying specific points on an imperial inch ruler including whole inch units and fractional inch units.
https://www.dadsworksheets.com/worksheets/inches-measurement-inches-on-ruler.html

Common Conversion Factors Practice
Practice memorizing the common conversion factors in this interactive exercise.
http://www.sporcle.com/games/12121/measurement-conversion

Convert Mixed Customary Units
Practice converting customary units of measurement in this interactive online exercise.
http://www.mathgames.com/skill/5.10-convert-mixed-customary-units

Convert Customary Units
Fill in the tables to convert between US customary measures of distance, volume, and mass.
https://www.khanacademy.org/math/cc-fifth-grade-math/cc-5th-measurement-topic/cc-5th-unit-conversion/e/converting-units--us-customary-

Customary Unit Conversion Printable Worksheets
Use these randomly generated worksheets for extra practice. Refresh the page (F5) to get a different worksheet.
https://www.homeschoolmath.net/worksheets/measuring-customary.php#grade5

Converting Units - Word Problems
Solve word problems that involve converting between US customary measures of distance, volume, and mass in this interactive exercise from Khan Academy.
https://www.khanacademy.org/math/cc-fifth-grade-math/cc-5th-measurement-topic/cc-5th-unit-word-problems/e/converting-units-word-problems--us-customary-

PROBLEM SOLVING

Burnside's Billions Game
Mr. Burnsides is leaving you his fortune, but under one condition... You need to buy up his 27 favorite world landmarks in the next 40 days! This game involves foreign currency, exchange rates, and large-scale calculations involving decimals.
https://www.mrnussbaum.com/billions/

**Sample worksheet from
https://www.mathmammoth.com**

FOR REVIEW

Decimals Challenge Board
Click on the the numbers on the "Challenge Board" to get questions and earn points by answering them correctly.
https://www.quia.com/cb/95593.html

Decimal and Whole Number Jeopardy
Review place value, comparing, and rounding decimals. Also, practice number patterns.
http://www.quia.com/cb/8142.html

Multiply Decimals by Decimals

Multiplying decimals is easy! You simply multiply as if there were no decimal points. Then place the decimal point in the answer following this rule:

The answer will have **as many decimal places/digits** as there are, IN TOTAL, in all of the factors.

Example 1. 0.05×0.7	**Example 2.** $12 \times 2 \times 0.3 \times 0.2$
Multiply in your head: $5 \times 7 = 35$. The factor 0.05 has **two** and 0.7 has **one** decimal digit. The answer has to have **three**, so the answer is 0.035.	Multiply mentally: $12 \times 2 \times 3 \times 2 = 144$. The factors have 0, 0, 1, and 1 decimal digits—a total of 2. The answer has to have 2 decimal digits/places, so the answer is 1.44.

1. Multiply first as if there were no decimal points. Then add the decimal point to the answer.

a. $0.5 \times 0.3 =$ _____	**c.** $0.4 \times 0.08 =$ _____	**e.** $8 \times 0.3 =$ _____
b. $0.9 \times 0.6 =$ _____	**d.** $0.7 \times 0.02 =$ _____	**f.** $0.1 \times 2.7 =$ _____
g. $0.2 \times 0.1 =$ _____	**i.** $0.9 \times 0.01 =$ _____	**k.** $0.7 \times 0.3 =$ _____
h. $0.8 \times 0.1 =$ _____	**j.** $9 \times 0.06 =$ _____	**l.** $7 \times 0.03 =$ _____

The answer to a decimal multiplication may end in one or more *decimal* zeros. That is no problem. You may **simplify the final answer** by dropping the ending decimal zeros.

Example 3. To solve 50×0.006, first multiply in your head $50 \times 6 = 300$. The factors (50 and 0.006) have 0 and 3 decimal places, so the answer will have **3**. Therefore, the answer is 0.300, but it *simplifies* to 0.3.	**Example 4.** To solve 400×0.05, we first multiply $400 \times 5 = 2000$. The factors have 0 and 2 decimal digits, so the answer will have **2**. The answer is 20.00. You can simplify that to 20.

2. Multiply. Simplify your final answer.

a. $0.4 \times 0.5 =$ _0.20_ = _0.2_	**e.** $40 \times 0.05 =$ _____ = _____
b. $20 \times 0.06 =$ _____ = _____	**f.** $0.6 \times 0.2 \times 0.5 =$ _____ = _____
c. $3 \times 0.2 \times 0.5 =$ _____ = _____	**g.** $600 \times 0.004 =$ _____ = _____
d. $300 \times 0.08 =$ _____ = _____	**h.** $0.4 \times 0.5 \times 60 =$ _____ = _____

The shortcut to decimal multiplication

1) Multiply as if there were no decimal points.

2) Place the decimal point in the answer. The **number of decimal digits** in the answer is the **SUM** of the number of decimal digits in the factors.

Example 5. To solve 0.81×2.5, multiply **as if it was 81 × 25**. In other words, ignore the decimal points. (Also, "081" is the same as 81 so we can ignore the zero, too.)

Since 0.81 has *two* decimal digits, and 2.5 has *one*, the answer will have *three*. The final answer is therefore 2.025.

```
        8 1
    ×   2 5
    ───────
      4 0 5
  + 1 6 2 0
  ─────────
    2 0 2 5
```

Does that make sense? We can estimate:

$0.81 \times 2.5 \approx 0.8 \times 3 = 2.4$
or $0.81 \times 2.5 \approx 0.8 \times 2.5 = 2$.

Yes, a final answer of 2.025 makes sense, since it is close to our estimates.

Example 6. This time, we include the decimal points when writing calculating 1.49×0.7, but even so, we multiply **as if it was 149 × 7**. Imagine the decimal points are not there! And there is NO need to align the decimal points like in addition and subtraction.

The final answer has *three* decimal places, since the factors have two and one, respectively.

```
      3 6
    1.4 9        ← two decimal places
  ×   0.7        ← one decimal place
  ───────
  1.0 4 3        ← three decimal places
```

Estimate: $1.49 \times 0.7 \approx 1.5 \times 0.7 = 1.05$.

If the estimate was *not* close to our final answer, there would probably be an error somewhere.

3. Solve with long multiplication. Also, estimate. Write the longer number on top.

a. 0.3×1.19

Estimate: _____

b. 0.9×51.7

Estimate: _____

c. 204.5×0.4

Estimate: _____

d. 2.2×0.72

Estimate: _____

e. 5.6×2.8

Estimate: _____

f. 3.34×4.2

Estimate: _____

Example 7. Strawberries cost $5.15/kg, and you buy 0.7 kg. What is the total cost?

If you were to buy 3 kilograms of strawberries, you would multiply 3 × $5.15 to find the total price. When you buy 0.7 kg, you do the SAME: **multiply the price by 0.7.** The total cost is **0.7 × $5.15 = 3.605** (see the multiplication on the right).

Since this is a money amount, it needs rounded to two decimals: $3.61.

Note that the answer also makes sense: 0.7 kg of strawberries should cost less than 1 kg of strawberries, which was $5.15.

```
    1 3
    5 1 5
  ×     7
  _____
  3 6 0 5
```

4. Find the total cost. Write a multiplication.

   a. Ribbon costs $1.10 per meter, and you buy 0.4 meters.

   Cost: _____ × _____ = _____

   b. Nuts cost $8 per pound. You buy 0.3 pounds.

   Cost: _____ × _____ = _____

   c. A phone call costs $7 per hour. You talk for 1.2 hours.

   Cost: _____ × _____ = _____

   d. Lace costs $2.20 per meter, and you buy 1.5 meters.

   Cost: _____ × _____ = _____

5. Lauren went to a farmer's market, and bought 0.4 kg of blueberries at $9.20/kg, 3.5 kg of potatoes at $1.20/kg, and 1.2 kg of carrots at 1.85/kg. Find the total cost.

6. You're trying to find out how much it costs to feed your pet parrot for a <u>year</u>.

   You have figured that your parrot eats about 90 grams, which is 0.09 kg, of parrot feed in a day. Also, parrot feed costs $7.08/kg. So, what is the cost of feeding your parrot for a year (not just a day)? Round your answer to the nearest dollar.
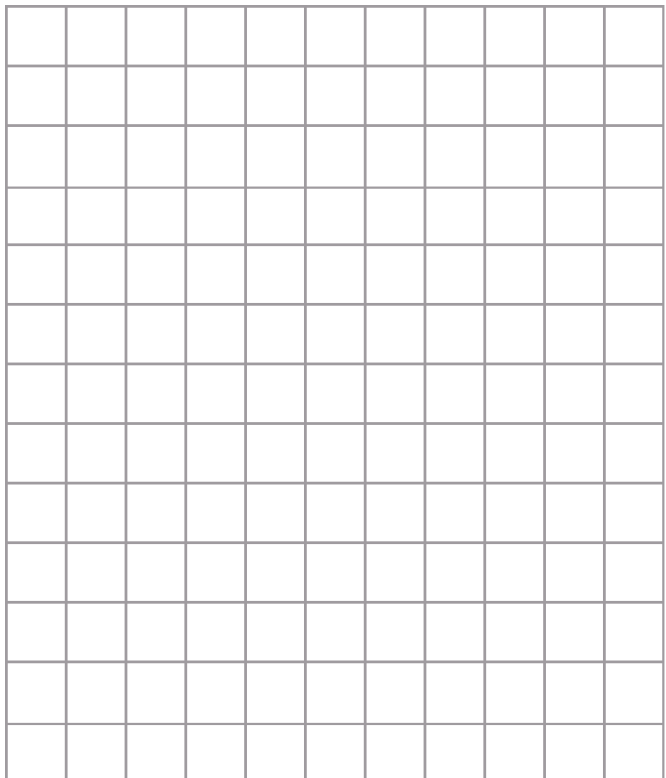
7. Multiply.

a. 0.7×1.1 = _____	**c.** 0.002×9 = _____	**e.** $0.4 \times 4 \times 0.2$ = _____
b. 0.02×0.5 = _____	**d.** 1.1×0.3 = _____	**f.** $6 \times 0.06 \times 0.2$ = _____

8. Find the missing factors.

a. $0.5 \times$ _____ = 0.35	**d.** $4 \times$ _____ = 2.4	**g.** $0.9 \times$ _____ = 7.2
b. $0.5 \times$ _____ = 0.035	**e.** $4 \times$ _____ = 0.024	**h.** $0.9 \times$ _____ = 0.081
c. $0.05 \times$ _____ = 0.035	**f.** $4 \times$ _____ = 0.24	**i.** $0.9 \times$ _____ = 0.54

9. Solve by long multiplication or in your head. Place the letter of each problem under the right answer, and solve the riddle.
   Hint: Sometimes you can figure out the correct answer by estimating.

0.8×2.7 U	10×0.042 S	2.09×1.7 R	200×0.008 O
1.6×2.05 D	1.5×1.5 E	70×0.05 S	$0.5 \times 0.4 \times 0.7$ Y
10×0.04 L	3.45×0.7 I	0.05×2.4 E	$0.01 \times 600 \times 0.2$ F
	1.06×0.4 E	15.2×0.18 B	

Riddle: *What did number 22 say to 21.21?* *"YOU'RE...*

2.736	0.424	0.42	2.415	3.28	2.25		0.14	1.6	2.16	3.553	3.5	0.12	0.4	1.2

Multiplication as Scaling

Scaling means expanding or shrinking something by some factor.
Multiplication can be thought of as scaling.

This red stick ⸺ is 40 pixels long.
Let's **scale** it to be four times as long:

⸺ → ⸺⸺⸺

We can write a multiplication "equation":

4 × ⸺ = ⸺⸺⸺

Using pixels, 4 × 40 px = 160 px.

Now let's scale the same red stick to be
0.4 (four-tenths) as long as it is at first:

⸺ → ⸺

Notice, it shrank! We write a multiplication:

0.4 × ⸺ = ⸺

In pixels, 0.4 × 40 px = 16 px.

The number we multiply by (4 and 0.4 above) is called the **scaling factor**.

- If the scaling factor is more than 1, such as 2.3, the resulting stick is *longer* than the original one.
- If the scaling factor is less than 1, such as 0.5 or 0.66, the resulting stick is *shorter*.

1. The stick is being *shrunk*. How long will it be in pixels? Compare the problems.

a. 0.1 × ⸺ = . 0.1 × 40 px = _____ px	**b.** 0.3 × ⸺ = ⸺ 0.3 × 40 px = _____ px	**c.** 0.6 × ⸺ = ⸺ 0.6 × 40 px = _____ px
d. 0.2 × ⸺ = ⸺ 0.2 × 40 px = _____ px	**e.** 0.5 × ⸺ = ⸺ 0.5 × 40 px = _____ px	**f.** 0.9 × ⸺ = ⸺ 0.9 × 40 px = _____ px

Let's **expand** this stick ⸺ (40 px) to be 1.2 times as long, like this:

1.2 × ⸺ = ⸺⸺

In pixels, it is now 1.2 × 40 = 48 pixels long.

2. The red stick ⸺ is 50 pixels long. It is being *expanded*. Fill in the blanks.

a. 1.5 × ⸺ = ⸺⸺ 1.5 × 50 px = _____ px	**b.** 1.3 × ⸺ = ⸺⸺ 1.3 × 50 px = _____ px
c. 2.2 × ⸺ = ⸺⸺ 2.2 × 50 px = _____ px	**d.** 3.3 × ⸺ = ⸺⸺⸺ 3.3 × 50 px = _____ px

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Chapter 7: Fractions: Add and Subtract
Introduction

In 5th grade, students study most aspects of fraction arithmetic: addition, subtraction, multiplication, and then in some special cases, division. Division of fractions is studied in more detail in 6th grade.

This chapter starts out with lessons on various ways to add and subtract mixed numbers. These are meant partially to review and partially to develop speed in fraction calculations. The lesson *Subtracting Mixed Numbers 2* presents an optional way to subtract, where we use a negative fraction. This is only meant for students who can easily grasp subtractions such as $(1/5) - (4/5) = -3/5$, and is not intended to become a "stumbling block." Simply skip the method if your student does not understand it easily.

Students have already added and subtracted *like* fractions in fourth grade. Now it is time to "tackle" the more complex situation of *unlike* fractions. To that end, we first review how to convert fractions into other equivalent fractions. These lessons use a visual model of splitting pie pieces further, and from that, we develop the common procedure for equivalent fractions.

This skill is used immediately in the next lessons about adding and subtracting unlike fractions. We begin this topic by using visual models, and then gradually advance toward the abstract. Several lessons are devoted to understanding and practicing the basic concept, and also to applying this new skill to mixed numbers.

The lesson *Comparing Fractions* reviews some mental math methods for comparing fractions. Students also learn a "brute force" method based on converting fractions to equivalent fractions. This chapter ends with a lesson on measuring in inches, using units as small as 1/16 of an inch.

The Lessons in Chapter 7

Helpful Resources on the Internet

MIXED NUMBERS

Clara Fraction Ice Cream Shop
Convert improper fractions to mixed numbers, while scooping various ice cream flavors onto the cone.
https://mrnussbaum.com/clarafraction/

Fraction Models
Explore improper fractions, mixed numbers, decimals, and percentages using several models: bar, area, pie, and set. Adjust numerators and denominators to see how they alter the models.
http://illuminations.nctm.org/Activity.aspx?id=3519

Fractions Workshop
Choose "Add mixed fractions with like denominators" and the number of problems you would like to do.
https://mrnussbaum.com/fractions-workshop-ipad.html

Subtracting Mixed Fractions Quiz (Like Denominators)
Drag and drop each answer to the corresponding subtraction problem.
http://www.fractions4kids.com/subtracting-mixed-fractions-quiz/

Subtracting Mixed Numbers with Borrowing
Learn how to borrow mixed fractions with this animation.
https://www.wisc-online.com/learn/formal-science/mathematics/abm701/subtracting-mixed-number-fractions-with-borro

EQUIVALENT FRACTIONS

Equivalent Fractions
You are given a fraction that is shown with a visual model and on a number line, and you need to construct two *other* fractions that are equivalent to the given fraction. Drag two sliders to choose the denominators for your fractions and then click pieces to color them.
http://illuminations.nctm.org/Activity.aspx?id=3510

Fresh Baked Fractions
Practice equivalent fractions by clicking on a fraction that is not equal to others.
http://www.funbrain.com/fract/

Triplets: Equivalent Fractions
Sort the space teams by equivalent fractions to make sure all the athletes get to the correct starting place before the games begin.
https://www.mathplayground.com/Triplets/index.html

Fishy Fractions
Feed Ulani the pelican by choosing the fish with the correct equivalent fraction.
http://streaming.discoveryeducation.com/braingames/iknowthat/Fractions/FractionGame.cfm?Topic=equivalentfractions

Fraction Worksheets: Equivalent Fractions with Visual Models
Create custom-made worksheets for equivalent fractions. Choose to include pie images or not.
https://www.homeschoolmath.net/worksheets/equivalent_fractions.php

Sample worksheet from
https://www.mathmammoth.com

Fraction Worksheets: Equivalent Fractions, Simplifying, Convert to Mixed Numbers
Create custom-made worksheets for these fraction operations.
https://www.homeschoolmath.net/worksheets/fraction-b.php

ADDITION AND SUBTRACTION

Fraction Videos 1: Addition and Subtraction
A set of videos by the author that cover topics in this chapter.
https://www.mathmammoth.com/videos/fractions_1.php

Adding Fractions with Uncommon Denominators Tool at Conceptua Fractions
A tool that links a visual model to the procedure of adding two unlike fractions.
https://www.conceptuamath.com/app/tool/adding-fractions-with-uncommon-denominators

Add Unlike Fractions with Number Line Models
Practice adding unlike fractions. Click "EXPLAIN" to see a visual illustration and the answer.
http://www.visualfractions.com/AddUnlike/

Drop Zone
Practice making a sum of one using fractions in this interactive online activity.
https://www.brainpop.com/games/dropzone/

Add Mixed Numbers with Unlike Denominators - Quiz
Use this simple online quiz for extra practice.
http://www.mathgames.com/skill/5.72-add-mixed-numbers-with-unlike-denominators

Fruit Shoot Fractions
This game practices addition of fractions. There are several different levels to choose from.
http://www.sheppardsoftware.com/mathgames/fractions/FruitShootFractionsAddition.htm

Fruit Splat
Practice finding the least common denominator. This game has three different levels to choose from.
http://www.sheppardsoftware.com/mathgames/fractions/LeastCommonDenomimator.htm

Fraction Word Problems
Practice adding and subtracting fractions with these interactive word problems.
https://mrnussbaum.com/grade5standards/568-2/

Math Balloons: Fractions
Answer whether the fraction additions are true or false in this timed activity.
http://www.mathnook.com/math/math-balloons-fractions.html

Fraction Bars Blackjack
The computer gives you two fraction cards. You have the option of getting more or "holding". The object is to get as close as possible to 2, without going over, by adding the fractions on your cards.
http://fractionbars.com/Fraction_Bars_Black_Jack/

Old Egyptian Fractions
Puzzles to solve: add fractions like a true Old Egyptian Math Cat!
http://www.mathcats.com/explore/oldegyptianfractions.html

Fraction Worksheets: Addition, Subtraction, Multiplication, and Division
Create custom-made worksheets for the four operations with fractions and mixed numbers.
https://www.homeschoolmath.net/worksheets/fraction.php

ORDERING AND COMPARING

Comparing Fractions Tool at Conceptua Fractions
An interactive tool where students place numbers, visual models, and decimals on a number line.
http://www.conceptuamath.com/app/tool/comparing-fractions

Comparison Shoot Out
Choose level 2 or 3 to compare fractions and shoot the soccer ball to the goal.
http://www.fuelthebrain.com/games/comparison-shootout/

Comparing Fractions—XP Math
Simple timed practice with comparing two fractions.
http://xpmath.com/forums/arcade.php?do=play&gameid=8

Visual Fractions Game
Find a fraction between two given fractions with the help of this visual tool.
https://www.mathplayground.com/visual_fractions.html

Fractional Hi Lo
The computer has selected a fraction. You make guesses and it tells if your guess was too high or low.
http://www.theproblemsite.com/games/hilo.asp

My Closest Neighbor
A neat card game where you need to make a fraction that is as close as possible to the given fraction.
https://denisegaskins.com/2014/08/06/fraction-game-my-closest-neighbor/

Comparing/Ordering Fractions Worksheets
Create customizable worksheets for comparing or ordering fractions. You can include pie images.
https://www.homeschoolmath.net/worksheets/comparing_fractions.php

MEASURING & GENERAL

Measure It!—Practice measuring lines in inches.
https://www.funbrain.com/measure/

Measuring—Practice measuring with a virtual ruler. Choose the category "Inches, Sixteenths".
http://www.abcya.com/measuring.htm

Sal's Sub Shop—Cut the subs to the given measurements.
https://mrnussbaum.com/sal/

Fraction Word Problems
Practice your fraction skills with this interactive jeopardy game.
https://jeopardylabs.com/play/fraction-word-problems

Who Wants Pizza?—A tutorial and interactive exercises about fraction addition and multiplication.
http://math.rice.edu/~lanius/fractions/

Fraction Lessons—Tutorials, examples, and videos explaining all the basic fraction topics.
http://www.mathexpression.com/learning-fractions.html

Online Fraction Calculator
https://www.homeschoolmath.net/worksheets/fraction_calculator.php

Fraction Terminology

As we study fraction operations, it is important that you understand the terms, or words, that we use. This page is for reference. You can post it on your wall or even make your own fraction poster based on it. Some of the terms below you already know; some we will study in this chapter.

$\dfrac{3}{11}$ The top number is the **numerator**. It *enumerates,* or numbers (counts), *how many* pieces there are.
The bottom number is the **denominator**. It *denominates,* or names, *what kind* of parts they are.

A mixed number has two parts: a whole-number part and a fractional part.

For example, $2\dfrac{3}{7}$ is a mixed number. Its whole-number part is 2, and its fractional part is $\dfrac{3}{7}$.

The mixed number $2\dfrac{3}{7}$ actually means $2 + \dfrac{3}{7}$.

Like fractions have the same denominator. They have the same kind of parts. It is easy to add and subtract like fractions, because all you have to do is look at *how many* of that kind of part there are.
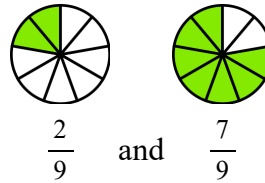
$\dfrac{2}{9}$ and $\dfrac{7}{9}$ are like fractions.

Unlike fractions have a different denominator. They have different kinds of parts. It is a little more complicated to add and subtract unlike fractions. You need to first change them into like fractions. Then you can add or subtract them.
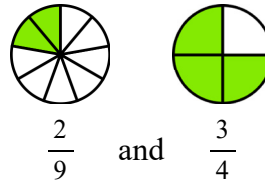
$\dfrac{2}{9}$ and $\dfrac{3}{4}$ are unlike fractions.

A proper fraction is a fraction that is less than 1 (less than a whole pie). 2/9 is a proper fraction.

$\dfrac{2}{9}$ is a proper fraction.

An improper fraction is more than 1 (more than a whole pie). Being a *fraction*, it is written as a fraction and *not* as a mixed number.
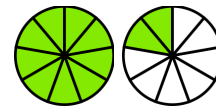
$\dfrac{11}{9}$ is an improper fraction.

Equivalent fractions are equal in value. If you think in terms of pies, they have the same amount of "pie to eat," but they are written using different denominators, or are "cut into different kinds of slices."
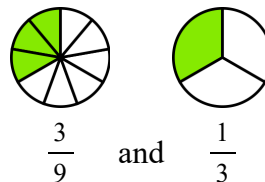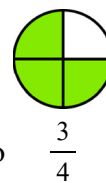
$\dfrac{3}{9}$ and $\dfrac{1}{3}$ are equivalent fractions.

Simplifying or reducing a fraction means that, for a given fraction, you find an equivalent fraction that has a "simpler," or smaller, numerator and denominator. (It has fewer but bigger slices.)
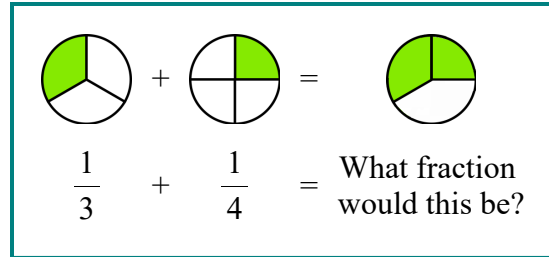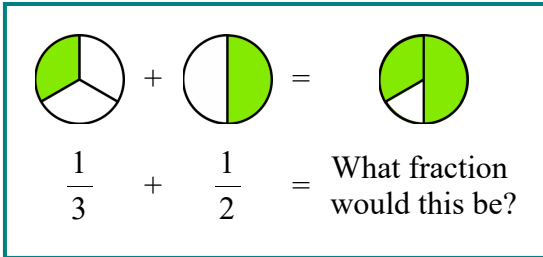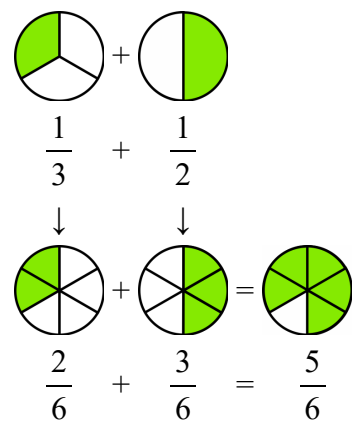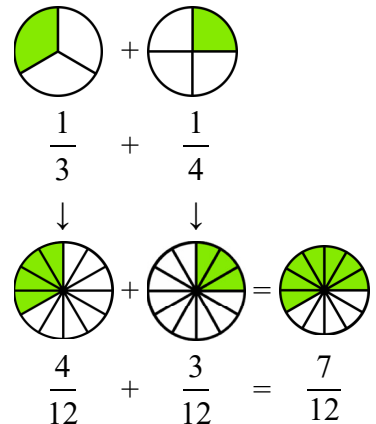
$\dfrac{9}{12}$ simplifies to $\dfrac{3}{4}$.

**Sample worksheet from
https://www.mathmammoth.com**

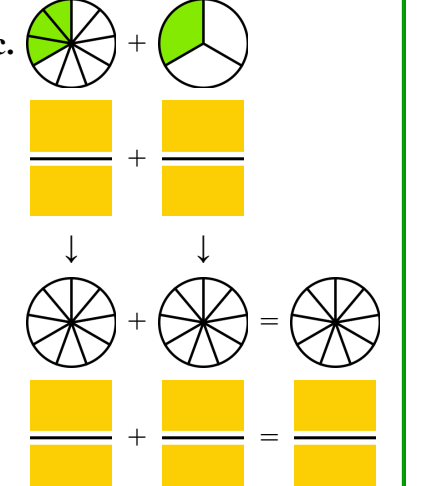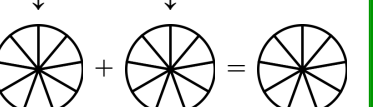(This page intentionally left blank.)
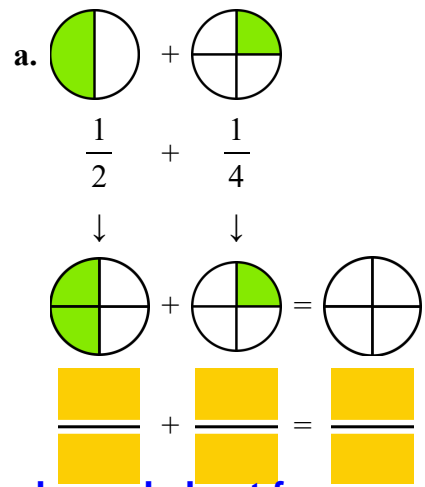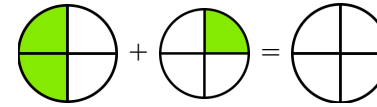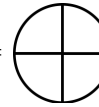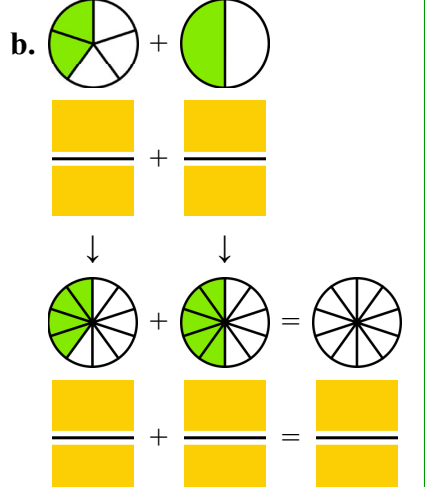
Adding and Subtracting Unlike Fractions

Cover the page below the black line. Then try to figure out the addition problems below.

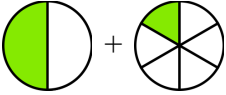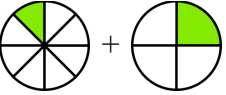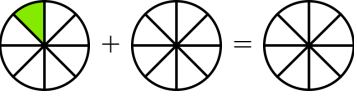$$\frac{1}{3} + \frac{1}{2} = \text{What fraction would this be?}$$

$$\frac{1}{3} + \frac{1}{4} = \text{What fraction would this be?}$$
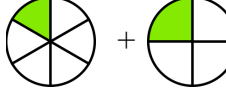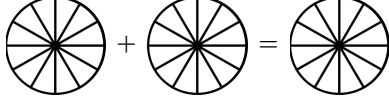
$$\frac{1}{3} + \frac{1}{2}$$
$$\downarrow \qquad \downarrow$$
$$\frac{2}{6} + \frac{3}{6} = \frac{5}{6}$$

$$\frac{1}{3} + \frac{1}{4}$$
$$\downarrow \qquad \downarrow$$
$$\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$$

Did you solve the problems above?

The solution is this:

We convert the fractions so that they become *like* fractions (with a same denominator), using equivalent fractions.

Then we can add or subtract.

1. Write the fractions shown by the pie images. Convert them into equivalent fractions with the same denominator (like fractions), and then add them. Color the missing parts.

a.
$$\frac{1}{2} + \frac{1}{4}$$
$$\downarrow \qquad \downarrow$$

b.

c.

2. Convert the fractions to like fractions first, then add or subtract. In the bottom problems (d-f), you need to figure out what kind of pieces to use, but the *top* problems (a-c) will help you do that!
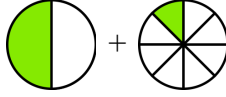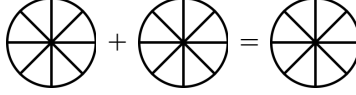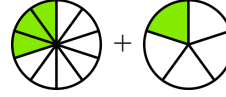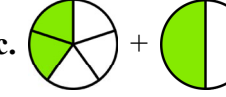
a. $\dfrac{1}{2} + \dfrac{1}{6}$

$$\rule{0.6cm}{0.4pt} + \dfrac{1}{6} = \rule{0.6cm}{0.4pt}$$

b. $\dfrac{1}{8} + \dfrac{1}{4}$

$$\dfrac{1}{8} + \rule{0.6cm}{0.4pt} = \rule{0.6cm}{0.4pt}$$

c. $\dfrac{1}{6} + \dfrac{1}{4}$

$$\rule{0.6cm}{0.4pt} + \rule{0.6cm}{0.4pt} = \rule{0.6cm}{0.4pt}$$

d. $\dfrac{5}{6} - \dfrac{1}{2}$

$$\dfrac{5}{6} - \rule{0.6cm}{0.4pt} = \rule{0.6cm}{0.4pt}$$

e. $\dfrac{5}{8} - \dfrac{1}{4}$

$$\rule{0.6cm}{0.4pt} - \rule{0.6cm}{0.4pt} = \rule{0.6cm}{0.4pt}$$

f. $\dfrac{5}{6} - \dfrac{1}{4}$

$$\rule{0.6cm}{0.4pt} - \rule{0.6cm}{0.4pt} = \rule{0.6cm}{0.4pt}$$

3. Convert the fractions to like fractions first, then add or subtract. In the bottom problems (d-f), you need to figure out what kind of pieces to use, but the *top* problems (a-c) will help you do that!

a. $\dfrac{1}{2} + \dfrac{1}{8}$

$$\rule{0.6cm}{0.4pt} + \rule{0.6cm}{0.4pt} = \rule{0.6cm}{0.4pt}$$

b. $\dfrac{3}{10} + \dfrac{1}{5}$

$$\rule{0.6cm}{0.4pt} + \rule{0.6cm}{0.4pt} = \rule{0.6cm}{0.4pt}$$

c. $\dfrac{2}{5} + \dfrac{1}{2}$

$$\rule{0.6cm}{0.4pt} + \rule{0.6cm}{0.4pt} = \rule{0.6cm}{0.4pt}$$

d. $\dfrac{1}{2} + \dfrac{3}{8}$

$$\rule{0.6cm}{0.4pt} + \rule{0.6cm}{0.4pt} = \rule{0.6cm}{0.4pt}$$

e. $\dfrac{9}{10} - \dfrac{2}{5}$

$$\rule{0.6cm}{0.4pt} - \rule{0.6cm}{0.4pt} = \rule{0.6cm}{0.4pt}$$

f. $\dfrac{4}{5} - \dfrac{1}{2}$

$$\rule{0.6cm}{0.4pt} - \rule{0.6cm}{0.4pt} = \rule{0.6cm}{0.4pt}$$

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Comparing Fractions

Sometimes it is easy to know which fraction is the greater of the two. Study the examples!

$$\frac{7}{9} > \frac{2}{9}$$
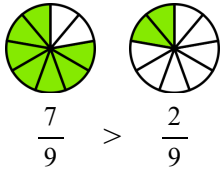
With **like fractions**, all you need to do is to check **which fraction has more "slices,"** and that fraction is greater.

$$\frac{3}{9} < \frac{3}{8}$$
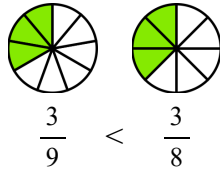
If both fractions have the **same number of pieces**, then the one with bigger pieces is greater.

$$\frac{4}{7} > \frac{5}{12}$$
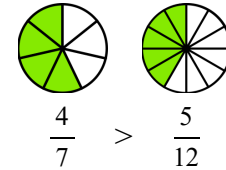
Sometimes you can **compare to 1/2**. Here, 4/7 is clearly more than 1/2, and 5/12 is clearly less than 1/2.

1. Find the fractions that are more than 1/2.

$$\frac{2}{5} \qquad \frac{5}{8} \qquad \frac{5}{6} \qquad \frac{5}{12} \qquad \frac{10}{21} \qquad \frac{8}{14} \qquad \frac{4}{10} \qquad \frac{28}{50}$$

2. Compare the fractions, and write > , < , or = .

a. $\frac{1}{8}$ $\frac{1}{10}$	**b.** $\frac{4}{9}$ $\frac{1}{2}$	**c.** $\frac{6}{10}$ $\frac{1}{2}$	**d.** $\frac{3}{9}$ $\frac{3}{7}$
e. $\frac{4}{7}$ $\frac{6}{13}$	**f.** $\frac{7}{4}$ $\frac{7}{6}$	**g.** $\frac{5}{14}$ $\frac{5}{9}$	**h.** $\frac{4}{20}$ $\frac{2}{20}$
i. $\frac{2}{11}$ $\frac{2}{5}$	**j.** $\frac{13}{27}$ $\frac{5}{8}$	**k.** $\frac{12}{24}$ $\frac{1}{2}$	**l.** $\frac{1}{20}$ $\frac{1}{8}$

A fraction that is more than one (like 6/5) must be bigger than a fraction that is less than one.

$$\frac{6}{5} > \frac{9}{10}$$

In some cases, you might be able to imagine pie pictures in your mind, and "see" which fraction is bigger.

$$\frac{2}{5} > \frac{1}{4}$$
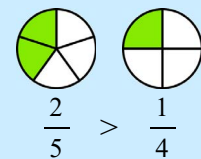
3. Compare the fractions, and write > , < , or = .

a. $\frac{3}{4}$ $\frac{8}{5}$	**b.** $\frac{8}{7}$ $\frac{3}{3}$	**c.** $\frac{49}{100}$ $\frac{61}{100}$	**d.** $\frac{7}{9}$ $\frac{8}{7}$
e. $\frac{9}{10}$ $\frac{3}{4}$	**f.** $\frac{6}{5}$ $\frac{9}{12}$	**g.** $\frac{4}{4}$ $\frac{9}{11}$	**h.** $\frac{1}{3}$ $\frac{3}{9}$
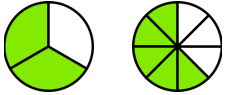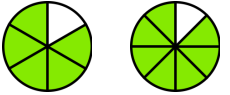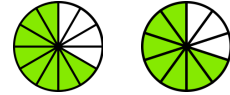
Sometimes none of the "tricks" explained in the previous page work, but we do have one more up our sleeve!

Convert both fractions into like fractions. Then compare.

In the picture on the right, it is hard to be sure if 3/5 is really more than 5/9. Convert both into 45th parts, and then it is easy to see that 27/45 is more than 25/45. Not by much, though!

$$\frac{3}{5} \qquad \frac{5}{9}$$

$$\downarrow \qquad \downarrow$$

$$\frac{27}{45} \quad > \quad \frac{25}{45}$$
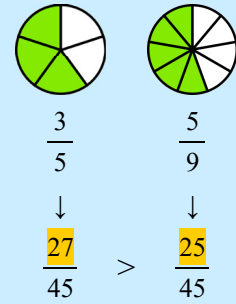
4. Convert the fractions into like fractions, and then compare them.

a.	$\dfrac{2}{3}$ $\downarrow$	$\dfrac{5}{8}$ $\downarrow$	**b.**	$\dfrac{5}{6}$ $\downarrow$	$\dfrac{7}{8}$ $\downarrow$	**c.**	$\dfrac{1}{3}$ $\downarrow$ $\qquad$ $\dfrac{3}{10}$ $\downarrow$

| **c.** | $\dfrac{1}{3}$ $\downarrow$ | $\dfrac{3}{10}$ $\downarrow$ | **d.** | $\dfrac{8}{12}$ $\downarrow$ | $\dfrac{7}{10}$ $\downarrow$ |

| **e.** | $\dfrac{5}{8}$ $\downarrow$ | $\dfrac{7}{12}$ $\downarrow$ | **f.** | $\dfrac{11}{8}$ $\downarrow$ | $\dfrac{14}{10}$ $\downarrow$ | **g.** | $\dfrac{6}{10}$ $\downarrow$ | $\dfrac{58}{100}$ $\downarrow$ | **h.** | $\dfrac{6}{5}$ $\downarrow$ | $\dfrac{11}{9}$ $\downarrow$ |

| **i.** | $\dfrac{7}{10}$ $\downarrow$ | $\dfrac{5}{7}$ $\downarrow$ | **j.** | $\dfrac{43}{100}$ $\downarrow$ | $\dfrac{3}{10}$ $\downarrow$ | **k.** | $\dfrac{9}{8}$ $\downarrow$ | $\dfrac{8}{7}$ $\downarrow$ | **l.** | $\dfrac{7}{10}$ $\downarrow$ | $\dfrac{2}{3}$ $\downarrow$ |

5. One cookie recipe calls for 1/2 cup of sugar. Another one calls for 2/3 cup of sugar. Which uses more sugar, a triple batch of the first recipe, or a double batch of the second?

How much more?

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Chapter 8: Fractions: Multiply and Divide
Introduction

This is another long chapter devoted solely to fractions. It rounds out our study of fraction arithmetic. (If you feel that your student(s) would benefit from taking a break from fractions, you can optionally have them study chapter 9 on geometry in between chapters 7 and 8.)

We start out by simplifying fractions. Since this process is the opposite of making equivalent fractions, studied in chapter 7, it should be relatively simple for students to understand. We also use the same visual model, just backwards: This time the pie pieces are joined together instead of split apart.

Next comes multiplying a fraction by a whole number. Since this can be solved by repeated addition, it is not a difficult concept at all.

Multiplying a fraction by a fraction is first explained as taking a certain part of a fraction, in order to teach the concept. After that, students are shown the usual shortcut for the multiplication of fractions.

Simplifying before multiplying is a process that is not absolutely necessary for fifth graders. I have included it here because it prepares students for the same process in future algebra studies and because it makes fraction multiplication easier. I have also tried to include explanations of *why* we are allowed to simplify before multiplying. These explanations are actually *proofs*. I feel it is a great advantage for students to get used to mathematical reasoning and proof methods well before they start high school geometry.

Then, we find the area of a rectangle with fractional side lengths, and show that the area is the same as it would be found by multiplying the side lengths. Students multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Students also multiply mixed numbers, and study how multiplication can be seen as resizing or scaling. This means, for example, that the multiplication $(2/3) \times 18$ km can be thought of as finding two-thirds of 18 km.

Next, we study division of fractions in special cases. The first one is seeing fractions *as* divisions; in other words recognizing that 5/3 is the same as $5 \div 3$. This of course gives us a means of dividing whole numbers and getting fractional answers (for example, $20 \div 6 = 3 \ 2/6$).

Then students encounter the shortcut or rule for fraction division: each division is actually changed into a *multiplication* by the reciprocal of the divisor. While this rule is presented here in 5th grade, students are not required to master fraction division in all cases (such as with mixed numbers). We focus on divisions where the divisor and dividend are whole numbers unit fractions. However, I have also included some problems with non-unit fractions.

After introducing the shortcut, students study two main contexts and real-life applications for fraction division: sharing divisions and so- called "measurement divisions", where we think how many times the divisor "fits into" the dividend.

The Lessons in Chapter 8

Helpful Resources on the Internet

Fraction Videos 2: Multiplication and Division
My own videos that cover multiplying and dividing fractions.
https://www.mathmammoth.com/videos/fractions_2.php

REDUCING/SIMPLIFYING FRACTIONS

Canceling Demonstration
Watch a movie that uses circles to demonstrate how to rename to lowest terms with canceling.
http://www.visualfractions.com/cancel/

Reduce Fractions Shoot
Reduce the fraction on the screen to the lowest terms by clicking the correct answer.
http://www.sheppardsoftware.com/mathgames/fractions/reduce_fractions_shoot.htm

Fraction Worksheets: Simplifying and Equivalent Fractions
Create custom-made worksheets for fraction simplification and equivalent fractions.
https://www.homeschoolmath.net/worksheets/fraction.php

Speedway Fractions
Add and subtract fractions, simplifying your answer. Power up your race car and win first place!
https://www.mathplayground.com/ASB_Speedway.html

Reducing Fractions to Lowest Terms
Simple worksheet exercise that you can use for extra practice.
http://www.mathgames.com/skill/3.46-reducing-fractions-to-lowest-terms

118

Fractions America
Turn the United States ablaze in color, then determine what fraction of the states are each color. Choose from 2, 3, 4, 5, 6, 7, or 8 fractions.
https://mrnussbaum.com/fraction-america/

Frosty Fractions
Add together the two fractions given. If the answer is available on the board, place a snowflake token over it. The winner is the first player to get a straight line of three snowflakes, either horizontally, vertically, or diagonally (this game is for two players).
http://www.counton.org/games/map-fractions/frosty/

FRACTION MULTIPLICATION

Multiply Fractions by Whole Numbers
Use this simple online exercise for additional practice as needed.
http://www.mathgames.com/skill/4.67-multiply-fractions-by-whole-numbers

Multiply Fractions and Whole Numbers
Practice multiplying a whole number times a fraction in this online exercise.
https://www.khanacademy.org/math/pre-algebra/fractions-pre-alg/multiplying-fractions-pre-alg/e/multiplying_fractions_by_integers

Interactive Model for the Multiplication of Fractions
In this interactive activity, you will see how to use area models to multiply fractions.
https://www.learner.org/courses/learningmath/number/session9/part_a/try.html

Fraction Multiplication TeacherTool
Students multiply two fractions together and use an area model to represent the product. Scroll down to "Fifth Grade Multiplication and Division" and click on "Fraction Multiplication 2".
http://www.dreambox.com/teachertools

Soccer Math - Multiplying Fractions
Answer the multiple-choice fraction multiplication problems and play soccer in between the questions.
http://www.math-play.com/soccer-math-multiplying-fractions-game/multiplying-fractions-game.html

Snow Sprint Fractions
Practice fraction multiplication while participating in a snowmobile race!
http://www.mathplayground.com/ASB_SnowSprint.html

Multiply Fractions with Models
Use this simple online exercise for additional practice as necessary.
http://www.mathgames.com/skill/5.109-multiply-fractions-with-models

Product Fractions Card Activity (p. 36 of the PDF)
In this activity, players work in pairs to multiply fractions. This is not a "game", as such, but rather an opportunity for students to work collaboratively and manipulate the problems.
http://www.pepnonprofit.org/uploads/2/7/7/2/2772238/acing_math.pdf

Multiply Mixed Numbers Quiz
Practice multiplying mixed numbers. Express the answers as mixed numbers and in lowest terms.
https://www.thatquiz.org/tq-3/?-j304-la-p0

**Sample worksheet from
https://www.mathmammoth.com**

Fraction Multiplication Quiz
Practice multiplying like and unlike fractions in this 10-question interactive quiz.
http://www.thegreatmartinicompany.com/Math-Quick-Quiz/fraction-multiply-quiz.html

Multiplying Fractions Word Problems
Solve and interpret fraction multiplication word problems in this interactive exercise from Khan Academy.
https://www.khanacademy.org/math/arithmetic/fractions/multiplying-fractions-word-probl/e/multiplying-fractions-by-fractions-word-problems

Fraction Multiplication as Scaling
Interpret how multiplying by a fraction greater or less than 1 affects the product in this interactive online exercise.
https://www.khanacademy.org/math/pre-algebra/fractions-pre-alg/multiplying-fractions-pre-alg/e/fraction-multiplication-as-scaling

Top Transfer
Begin at the "Start" with 100 on your calculator. What is the largest and smallest total you can end up with at the "Finish"?
https://www.transum.org/Software/SW/Starter_of_the_day/starter_December7.ASP

Who Wants Pizza?
A tutorial that explains fraction multiplication using a pizza, followed by some interactive exercises.
http://math.rice.edu/~lanius/fractions/

Multiply Fractions Jeopardy
Jeopardy-style game. Choose a question by clicking on the tile that shows the points you will win.
http://www.quia.com/cb/95583.html

Multiplying Fractions
Multiply the fractions shown and reduce to the answer to the lowest terms.
http://www.mathplayground.com/fractions_mult.html

Fraction of a Number
Practice finding fractional parts of various numbers in this interactive online exercise.
https://www.mathplayground.com/fractions_fractionof.html

FRACTION DIVISION

Divide Fractions by Whole Numbers - Word Problems
These are simple word problems which can be used to reinforce the topic of division with fractions.
http://www.mathgames.com/skill/5.94-divide-fractions-by-whole-numbers

Dividing Unit Fractions by Whole Numbers
Use this simple interactive exercise to reinforce fraction division skills.
https://www.khanacademy.org/math/arithmetic/fractions/dividing-fractions-tutorial/e/dividing_fractions_0.5

Dividing Whole Numbers by Unit Fractions
Practice dividing a whole number by a unit fraction in this interactive exercise.
https://www.khanacademy.org/math/arithmetic/fractions/dividing-fractions-tutorial/e/dividing_fractions

Fractions as Divisions
In this educational video, Sal shows how a/b and a÷b are equivalent. That is, the fraction bar and the division symbol mean the same thing.
https://www.khanacademy.org/math/cc-fifth-grade-math/cc-5th-fractions-topic/tcc-5th-fractions-as-division/v/fractions-as-division

Partitive Division of Fractions Tool
This tool provides you with a connection between a story problem (context), paraphrase, model, and procedure where a fractional value is shared equally. Students complete phrases like: 5 4/5 shared equally among 12. They use a double number line or an area model to visualize the answer, and they connect this understanding to the numeric procedure.
https://www.conceptuamath.com/app/tool/divide-partitive

Fractions as Divisions Word Problems
Practice word problems that involve using the fraction bar as division in this interactive exercise.
https://www.khanacademy.org/math/cc-fifth-grade-math/cc-5th-fractions-topic/tcc-5th-fractions-as-division/e/understanding-fractions-as-division--word-problems

Dividing Fractions: Word Problems
Solve word problems by dividing fractions by fractions in this interactive exercise.
https://www.khanacademy.org/math/arithmetic/fractions/div-fractions-fractions/e/dividing-fractions-by-fractions-word-problems

Seven Cookies for Grampy
Make seven whole cookies for Grampy by rearranging the fractional parts of other whole cookies.
http://www.visualfractions.com/sevencookies/

Fraction Worksheets: Addition, Subtraction, Multiplication, and Division
Create custom-made worksheets for fraction addition, subtraction, multiplication, and division.
http://www.homeschoolmath.net/worksheets/fraction.php

Thinking Blocks: Ratio Word Problems
Model and solve word problems with ratios using this interactive bar model tool.
http://www.mathplayground.com/tb_ratios/index.html

GENERAL

Visual Fractions
A great site for studying all aspects of fractions, including: identifying, renaming, comparing, addition, subtraction, multiplication, division.
http://www.visualfractions.com/

Visual Fraction Calculator
This fraction calculator can perform addition, subtraction, multiplication, or division of two fractions. The values for the calculation can be simple or mixed fractions, or consist of only wholes. Input of improper fractions is also allowed.
http://www.dadsworksheets.com/fraction-calculator.html

Conceptua Math Fractions Tools
Free and interactive fraction tools. Each activity uses several models, such as circles, horizontal and vertical bars, number lines, etc. that allow students to develop a conceptual understanding of fractions.
http://www.conceptuamath.com/app/tool-library

Fraction Lessons at MathExpression.com
Tutorials, examples, and videos explaining all the basic fraction topics.
http://www.mathexpression.com/learning-fractions.html

Online Fraction Calculator
Add, subtract, multiply, or divide fractions and mixed numbers.
https://www.homeschoolmath.net/worksheets/fraction_calculator.php

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Multiply Fractions by Fractions 1

We have studied how to find a fractional part of a whole number using multiplication.

For example, $\dfrac{3}{5}$ of 80 is written as the multiplication $\dfrac{3}{5} \times 80 = \dfrac{240}{5} = 48$.

REMEMBER: The word "of" translates here into **multiplication**.

We can use the same idea to find a fractional part of a fraction.

Example 1. One-half of ⬤ is ⬤ .

As a multiplication, $\dfrac{1}{2} \times \dfrac{1}{3} = \dfrac{1}{6}$.

Example 2. One-fourth of ⬤ is ⬤ .

As a multiplication, $\dfrac{1}{4} \times \dfrac{1}{3} = \dfrac{1}{12}$.

1. Find a fractional part of the given fraction. You can think of a leftover pizza piece, which you must share equally with one, two, or three other people. Write a multiplication sentence.

a. Find $\dfrac{1}{2}$ of

$$\dfrac{1}{2} \times \dfrac{1}{4} =$$

b. Find $\dfrac{1}{2}$ of

$$\underline{} \times \underline{} = \underline{}$$

c. Find $\dfrac{1}{2}$ of

$$\underline{} \times \underline{} = \underline{}$$

d. Find $\dfrac{1}{3}$ of

e. Find $\dfrac{1}{3}$ of

f. Find $\dfrac{1}{3}$ of

g. Find $\dfrac{1}{4}$ of

h. Find $\dfrac{1}{4}$ of

i. Find $\dfrac{1}{4}$ of

Did you notice a shortcut?

Shortcut: multiplying fractions of the type 1/n

To multiply fractions of the form 1/n where n is a whole number, simply multiply the denominators to get the new denominator. $\rightarrow \quad \dfrac{1}{4} \times \dfrac{1}{5} = \dfrac{1}{20}$ or $\dfrac{1}{2} \times \dfrac{1}{6} = \dfrac{1}{12}$

2. Multiply.

a. $\dfrac{1}{9} \times \dfrac{1}{2}$

b. $\dfrac{1}{13} \times \dfrac{1}{3}$

c. $\dfrac{1}{5} \times \dfrac{1}{20}$

We have now studied how to find 1/2 or 1/3 or 1/5 of some fractions. What about finding some other kind of fractional part? Let's again compare this to finding fractional parts of whole numbers.
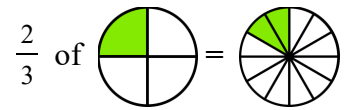
Review: To find $\dfrac{3}{4}$ of 16, or in other words $\dfrac{3}{4} \times 16$, you can first find $\dfrac{1}{4}$ of 16, which is 4.

Then just take that three times, which is 12. In other words, $\dfrac{3}{4} \times 16 = 12$.

We can use the same idea when finding a fractional part of another fraction.

Example 3. Find $\dfrac{2}{3}$ of $\dfrac{1}{4}$. First, we find $\dfrac{1}{3}$ of $\dfrac{1}{4}$, which is $\dfrac{1}{12}$.

Then, $\dfrac{2}{3}$ of $\dfrac{1}{4}$ is double that much, or $\dfrac{2}{12}$.

$\dfrac{2}{3}$ of =

Example 4. Find $\dfrac{4}{5}$ of $\dfrac{1}{7}$.

First, we find $\dfrac{1}{5}$ of $\dfrac{1}{7}$, which is $\dfrac{1}{35}$. Then, $\dfrac{4}{5}$ of $\dfrac{1}{7}$ is four times that much, or $\dfrac{4}{35}$.

Multiplying a fraction by a fraction means taking that fractional part *of* the fraction.
It is just like taking a certain part of the leftovers, when what is left over is a fraction.

3. The pictures show how much pizza is left, and you get a certain part of the leftovers.
How much will you get? Color in a picture to show the answer.

a. $\dfrac{3}{4} \times$ =

b. $\dfrac{2}{3} \times$ =

c. $\dfrac{3}{4} \times$ =

d. $\dfrac{2}{3} \times$ =

e. $\dfrac{2}{5} \times$ =

f. $\dfrac{4}{5} \times$ =

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Chapter 9: Geometry
Introduction

This chapter includes many problems that involve drawing geometric figures, because drawing is an excellent help towards achieving a conceptual understanding of geometry. Most of those are marked with "📓", meaning the exercise is to be done in a notebook or on blank paper.

The chapter starts out with lessons that review topics from previous grades, such as measuring angles, the vocabulary of basic shapes, how to draw a perpendicular line through a given point on a line, and how to draw a triangle with given angle measurements. Some fun is included, too, with star polygons.

In the lesson about circles, we learn the terms circle, radius, and diameter. Students are introduced to a compass, and they draw circles and circle designs using a compass.

Then we go on to classify quadrilaterals and learn the seven different terms used for them. The focus is on understanding the classification, and understanding that attributes defining a certain quadrilateral also belong to all the "children" (subcategories) of that type of quadrilateral. For example, squares are also rhombi, because they have four congruent sides (the defining attribute of a rhombus).

Next, we study and classify different triangles. Students are now able to classify triangles both in terms of their sides and also in terms of their angles. The lesson also includes several drawing problems where students draw triangles that satisfy the given information.

The last focus of this chapter is volume. Students learn that a cube with the side length of 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. They find the volume of right rectangular prisms by "packing" them with unit cubes and by using formulas. They recognize volume as additive and solve both geometric and real-word problems involving volume of right rectangular prisms.

The Lessons in Chapter 9

Helpful Resources on the Internet

FOR REVIEW OF ANGLES AND POLYGONS

Measuring Angles
Rotate the protractor into position and give your measurement to the nearest whole number.
https://www.mathplayground.com/measuringangles.html

Turtle Pond
Guide a turtle to a pond using commands that include turning him through certain angles and moving him specific distances.
http://illuminations.nctm.org/Activity.aspx?id=3534

Interactive Polygon Crossword Puzzle
Use the clues to help you guess the words that go in the puzzle, and fill it in.
http://www.mathgoodies.com/puzzles/crosswords/ipolygon3.html

Types of Polygons Vocabulary Quiz
In this interactive quiz you have to quickly name different types of polygons based on given clues. For each question you will have only 30 seconds to write your answer!
http://www.math-play.com/types-of-poligons.html

Polygon Matching Game
Many of the polygons included are quadrilaterals.
https://www.mathplayground.com/matching_shapes.html

Free Worksheets for Area and Perimeter
Create worksheets for the area and the perimeter of rectangles/squares with images, word problems, or problems where the student writes an expression for the area using the distributive property. Options also include area and perimeter problems for irregular rectangular areas, and more.
https://www.homeschoolmath.net/worksheets/area_perimeter_rectangles.php

Areas of Compound Shapes
Practice finding the perimeter and area of compound shapes.
https://www.cimt.org.uk/projects/mepres/book7/bk7i9/bk7_9i4.htm

Circle
This page includes a detailed lesson about circles, as well as interactive exercises to practice the topic.
http://www.mathgoodies.com/lessons/vol2/geometry.html

How to use a Compass to Draw a Circle
Simple instructions on how to use a compass.
https://www.youtube.com/watch?v=02XRad7s1Io

QUADRILATERALS

Interactive Quadrilaterals
See all the different kinds of quadrilateral "in action." You can drag the corners, see how the angles change, and observe what properties do not change.
https://www.mathsisfun.com/geometry/quadrilaterals-interactive.html

Properties of Quadrilaterals
Investigate the properties of a kite, a rhombus, a rectangle, a square, a trapezoid, and a parallelogram in this dynamic, online activity.

180

Complete the Quadrilateral
This is a hands-on activity (printable worksheets) where students join the dots to complete quadrilaterals, which helps students learn about the different types of quadrilaterals.
http://fawnnguyen.com/don-stewards-complete-quadrilateral/

Types of Quadrilaterals Quiz
Identify the quadrilaterals that are shown in the pictures in this interactive multiple-choice quiz.
http://www.softschools.com/math/geometry/quadrilaterals/types_of_quadrilaterals/

Quadrilateral Types Practice at Khan Academy
Identify quadrilaterals based on pictures or attributes in this interactive quiz.
https://www.khanacademy.org/math/basic-geo/basic-geo-shapes/basic-geo-classifying-shapes/e/quadrilateral_types

Classify Quadrilaterals Worksheets
Make free printable worksheets for classifying (identifying, naming) quadrilaterals.
https://www.homeschoolmath.net/worksheets/classify_quadrilaterals.php

TRIANGLES

Triangle Shoot
Practice classifying triangles by their angles or by their sides, or identifying types of angles, with this "math splat" game.
http://www.sheppardsoftware.com/mathgames/geometry/shapeshoot/triangles_shoot.htm

Rags to Riches: Classify Triangles by Sides and Angles
Answer multiple-choice questions about classifying triangles by their angles and sides and about angle measures of a triangle in a quest for fame and fortune.
http://www.quia.com/rr/457498.html

Identify Triangles Quiz
A simple multiple-choice quiz about identifying (classifying) triangles either by their sides or angles. You can modify some of the quiz parameters, such as the number of problems in it.
http://www.thatquiz.org/tq-A/?-j1-l34-p0

Interactive Triangles Activity
Play with different kinds of triangles (scalene, isosceles, equilateral, right, acute, obtuse). Drag the vertices and see how the triangle's angles and sides change.
https://www.mathsisfun.com/geometry/triangles-interactive.html

Classify Triangles Worksheets
Make free printable worksheets for classifying triangles by their sides, angles, or both.
https://www.homeschoolmath.net/worksheets/classify_triangles.php

Angles and Their Measures Matching Game
Learn to better estimate angles by matching angles to their angle measures.
https://www.mathmammoth.com/practice/angles-matching

VOLUME

Geometric Solids
Rotate various geometric solids by dragging with the mouse. Count the number of faces, edges, and vertices.
https://illuminations.nctm.org/Activity.aspx?id=3521

Cuboid Exploder and Isometric Shape Exploder
These interactive demonstrations let you see either various cuboids (a.k.a. boxes or rectangular prisms) or various shapes made of unit cubes, and then "explode" them to the unit cubes, illustrating volume.
http://www.teacherled.com/resources/cuboidexplode/cuboidexplodeload.html and
http://www.teacherled.com/resources/isoexplode/isoexplodeload.html

3-D Boxes Activity
Identify how many cubes are in the 3-D shapes in this interactive activity.
http://www.interactivestuff.org/sums4fun/3dboxes.swf

Rectangular Prisms Interactive Activity
Fill a box with cubes, rows of cubes, or layers of cubes. Can you determine a rule for finding the volume of a box if you know its width, depth, and height?
http://illuminations.nctm.org/Activity.aspx?id=4095

Interactivate: Surface Area and Volume
Explore or calculate the surface area and volume of rectangular prisms and triangular prisms. You can change the base, height, and depth interactively.
http://www.shodor.org/interactivate/activities/SurfaceAreaAndVolume/

Decompose Figures To Find Volume - Practice at Khan Academy
Find the volume of irregular 3-D figures by dividing the figures into rectangular prisms and finding the volume of each part.
https://www.khanacademy.org/math/cc-fifth-grade-math/cc-5th-measurement-topic/cc-5th-volume/e/decompose-figures-to-find-volume

Volume Word Problems
Practice solving word problems that involve volume of rectangular prisms.
https://www.khanacademy.org/math/pre-algebra/measurement/volume-introduction-rectangular/e/volume_2

Worksheets for the Volume and Surface Area of Rectangular Prisms
Customizable worksheets for volume or surface area of cubes and rectangular prisms. Includes the option of using fractional edge lengths.
https://www.homeschoolmath.net/worksheets/volume_surface_area.php

FOR FUN

Patch Tool
An online activity where the student designs a pattern using geometric shapes.
http://illuminations.nctm.org/Activity.aspx?id=3577

Shape Guess - Elimination Game
Have fun with shapes while playing this interactive online guessing game!
http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.SHAP&ID2=AB.MATH.JR.SHAP.SHAP&lesson=html/object_interactives/shape_classification/use_it.html

Interactivate! Tessellate
An online, interactive tool for creating your own tessellations. Choose a shape, then edit its corners or edges. The program automatically changes the shape so that it will tessellate (tile) the plane. Then push the tessellate button to see your creation! Requires Java.
http://www.shodor.org/interactivate/activities/Tessellate

Sample worksheet from
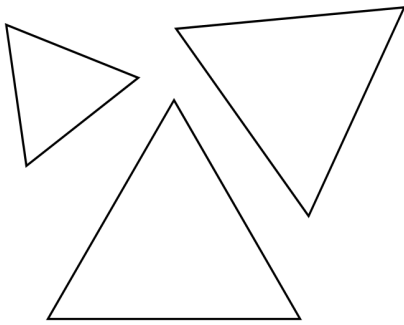**https://www.mathmammoth.com**

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Equilateral, Isosceles, and Scalene Triangles

Classification according to sides

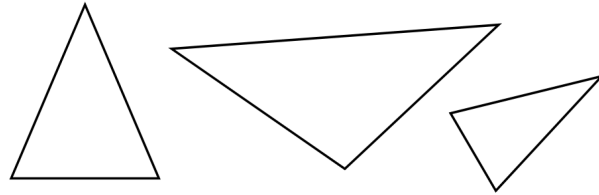If all three sides of a triangle are congruent (the same length), it is called an **equilateral triangle**.

"*Equi-*" refers to things that are the same or equal, and "*lateral*" refers to sides. Think of it as a "same-sided" triangle.

If only *two* of a triangle's sides are congruent, then it is called an **isosceles triangle**.

Think of it as a "same-legged" triangle, the "legs" being the two sides that are the same length.

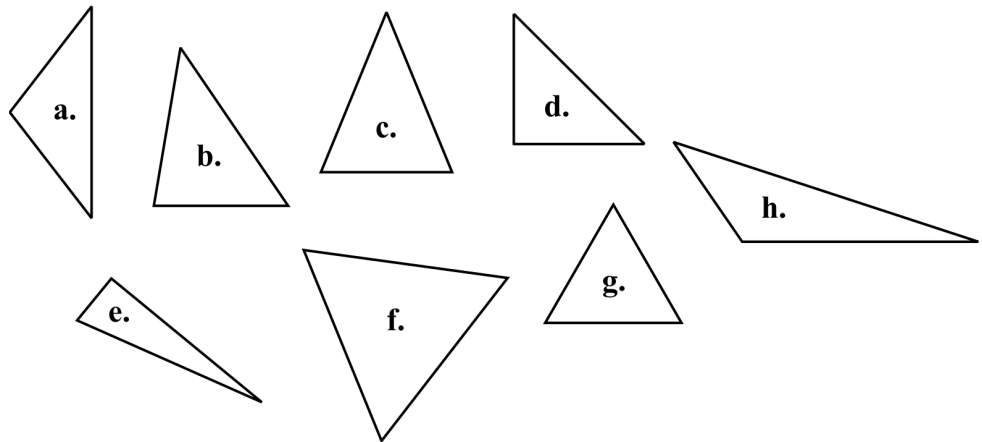MARK the two congruent sides of each isosceles triangle:

Lastly, if none of the sides of a triangle are congruent (all are different lengths), it is a **scalene triangle**.
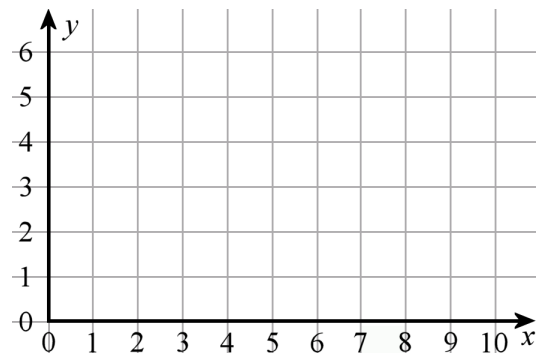
1. Classify the triangles by the lengths of their sides as either equilateral, isosceles, or scalene.

    You can mark each triangle with an "*e*," "*i*," or "*s*" correspondingly.

   a.
    b.
    c.
    d.
    e.
    f.
    g.
    h.

2. Plot the points (0, 0), (3, 5), (0, 5) , and connect them with line segments to form a triangle.

    Classify your triangle by its sides.
    Is it equilateral, isosceles, or scalene?

    _____

> **Classification according to angles**
>
> Remember, we can also classify a triangle according to its <u>angles</u>.
>
> - A **right triangle** has one right angle.
> - An **obtuse triangle** has one obtuse angle.
> - An **acute triangle** has three acute angles.

3. Classify the triangles as "acute," "right," or "obtuse" (by their angles), and also as "equilateral," "isosceles," or "scalene" (by their sides).
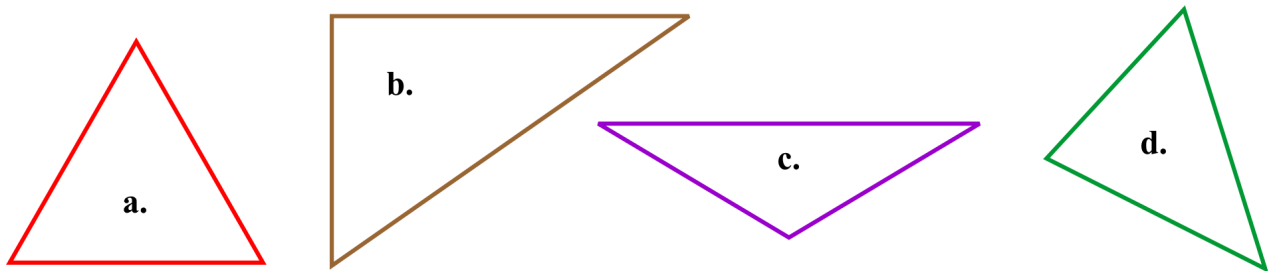
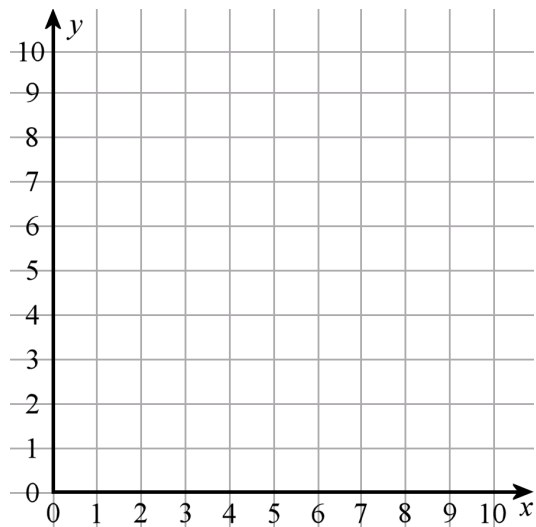| Triangle	Classification by the angles	Classification by the sides
a.		
b.		
c.		
d.		

4. Plot the points, and connect them with line segments to form two triangles. Classify the triangles by their angles <u>and</u> sides.

Triangle 1: (0, 0), (4, 0), (0, 4)

_____ and

_____

Triangle 2: (5, 5), (1, 8), (9, 4)

_____ and

_____

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Area and Perimeter Problems

Example 1. Find the area of the shaded figure.

The easiest way to do this is:
(1) Find the area of the larger outer rectangle.
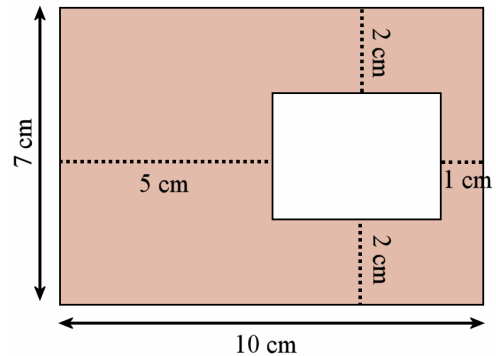(2) Find the area of the white inner rectangle.
(3) Subtract the two.

1. The area of the large rectangle is

   $7 \text{ cm} \times 10 \text{ cm} = 70 \text{ cm}^2$.

2. First we find the *sides* of the white rectangle by subtracting. The longer side of the white rectangle is $10 \text{ cm} - 5 \text{ cm} - 1 \text{ cm} = 4 \text{ cm}$.
   The shorter side is $7 \text{ cm} - 2 \text{ cm} - 2 \text{ cm} = 3 \text{ cm}$.

   So, the area of the white rectangle is $4 \text{ cm} \times 3 \text{ cm} = 12 \text{ cm}^2$.

3. Lastly we subtract the two areas to find the shaded area: $70 \text{ cm}^2 - 12 \text{ cm}^2 = 58 \text{ cm}^2$.

Note: In the following problems, record your work—all the calculations—carefully. That way you are much less likely to make mistakes! Use a notebook, if necessary, for additional space.

1. **a.** Find the area of the *white* rectangle.
      All lines meet at right angles.
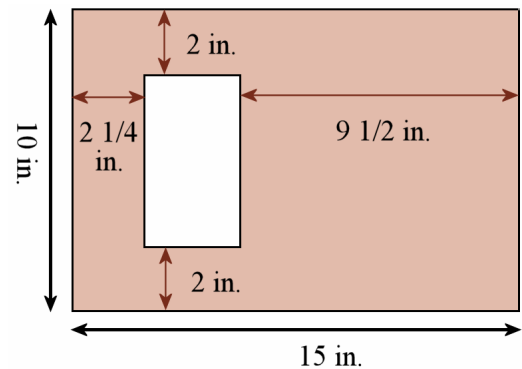
   b. Find the area of the shaded figure.

2. The image on the right shows a picture frame.
   Find the area of the actual frame (that is, of the shaded part).
   Give your answer to the nearest whole square centimeter.
   (All lines meet at right angles.)
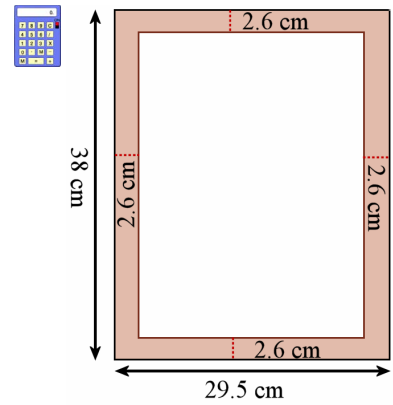
3. The perimeter of a rectangle is 42 cm.
   If the long side of the rectangle is 11 cm,
   how long is the shorter side?

4. The perimeter of a square is ½ mile.

   a. How long is one side of the square, in miles?
   Draw a sketch to help you.

   b. How long is one side of the square, in *feet*?
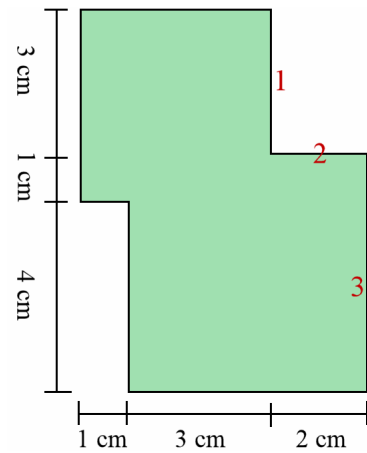   (1 mile = 5,280 ft)

Example 2. Find the perimeter of the figure.

We need to find the length of *each* side and then add the lengths. Start, for example, at the side marked with 1, then go to the side marked with 2, then to side 3, and so on, until you have "traveled" all the way around the figure.

Side 1 is 3 cm. Side 2 is 2 cm. Side 3 is 5 cm.
The total perimeter is:

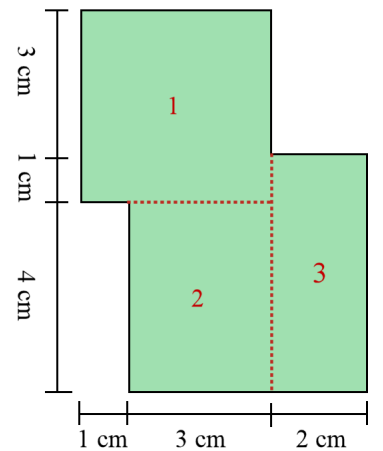3 cm + 2 cm + 5 cm + 5 cm + 4 cm + 1 cm + 4 cm + 4 cm = 28 cm.

Example 3. Find the area of the figure.

Divide the figure into rectangles by drawing in it some additional lines.

Rectangle 1 has an area of 4 cm × 4 cm = 16 cm^2.

Rectangle 2 has an area of 3 cm × 4 cm = 12 cm^2.
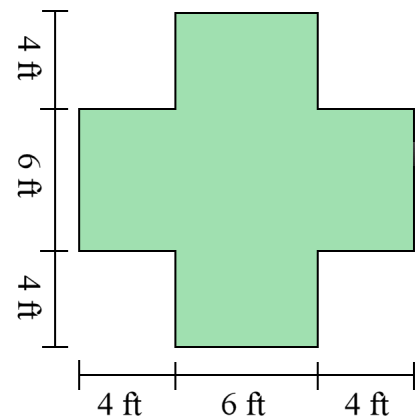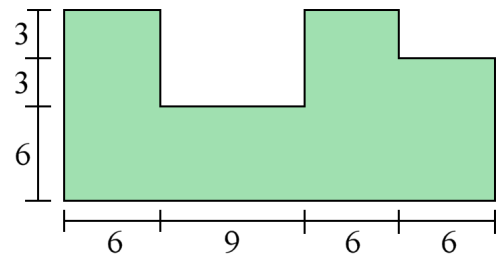
Rectangle 3 has an area of 2 cm × 5 cm = 10 cm^2.

The total area is: 16 cm^2 + 12 cm^2 + 10 cm^2 = 38 cm^2.

5. Find the area and the perimeter of this figure.
   All lines meet at right angles.

6. Find the area and the perimeter of this figure.
   All lines meet at right angles.
   The dimensions are given in centimeters.

7. A farmer fenced a rectangular field with 910 ft of fencing.
   One side of that field measures 330 ft.
   How long is the other side of the field?

8. Find the area and the perimeter of this figure.
   All lines meet at right angles.
   The dimensions are given in inches.