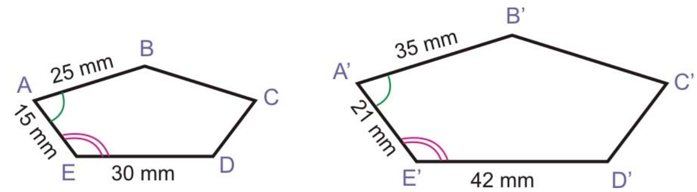
Similar Figures and Scale Ratio

With polygons, we can express the "stretching equally in all directions" in mathematical terms.
Two polygons are similar IF both of these are true:

- The corresponding angles in the two polygons are congruent (the same);
- The corresponding sides in the two polygons are in the same ratio.

This ratio is called the **scale ratio** or just **scale**.

Example. The pentagons ABCDE and A'B'C'D'E' are similar. Their angles are congruent. This means that $\angle A = \angle A'$, $\angle B = \angle B'$, and so on.
Also, their corresponding sides are in the same ratio.

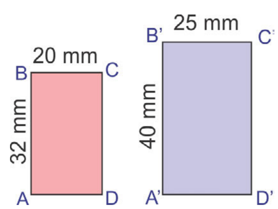


For example, the ratio $\overline{AB}$: $\overline{A'B'}$ is 25:35, which simplifies to 5:7. The ratio $\overline{AE}$: $\overline{A'E'}$ is 15:21, which simplifies to 5:7. And so on. Each pair of corresponding sides is in the ratio of 5:7.

So, the scale ratio between the smaller and the larger pentagon is 5:7. Note that we can also turn the ratio around, and say that the scale ratio between the *larger* and the *smaller* pentagon is 7:5.

1. Write the ratios between the corresponding sides of these similar figures. Simplify the ratios.

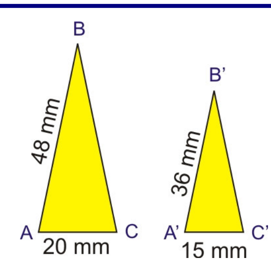
a.



$\overline{AB}$: $\overline{A'B'}$ =

$\overline{BC}$: $\overline{B'C'}$ =

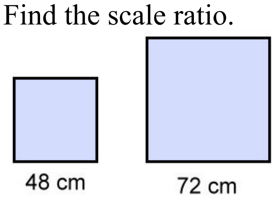
b.



$\overline{AB}$: $\overline{A'B'}$ =

$\overline{AC}$: $\overline{A'C'}$ =

2. Find the scale ratio.



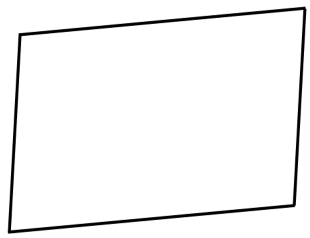
3. The sides of one triangle measure 5 ft., 3 ft., and 4 ft.
   The sides of another measure 10 ft, 5 ft, and 8 ft.
   Are the triangles similar?

Sometimes we write the scale ratio as a single decimal number.

Example. The scale between two similar figures is 7:5 = 7/5 = 1.4. Writing the scale as the single number 1.4 means that the larger figure is 1.4 times the size of the smaller figure. The number 1.4 is easy to use in calculations of side lengths.

Or, if we compare the smaller figure to the larger figure, the scale ratio is 5:7 = 5/7 ≈ 0.71.
This means the smaller figure is 5/7 or about 0.71 times the size of the larger figure.

4. Draw a smaller copy of this parallelogram. Let each side be exactly 0.7 as long as the sides in the original.

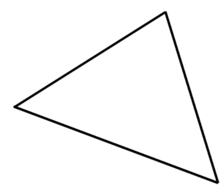


5. The scale ratio between two rectangles is 2:5.
The sides of the smaller one measure 6 cm and 8 cm.
How long are the sides of the larger one?

6. The scale ratio between two parallelograms is 2:3.
The sides of the larger one measure 36 mm and 48 mm.
How long are the sides of the smaller one?

7. The radius of one circle is 5.5 cm, and the radius of another is 10.4 cm. Are the circles similar?

8. When you follow these instructions, is the resulting triangle similar to the original, congruent, or neither?



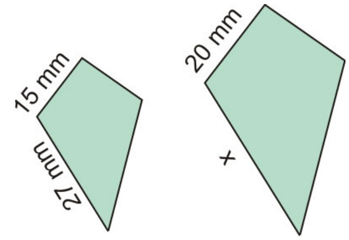
a. You draw a copy of this triangle, but while drawing, your paper is upside down.

b. You draw a new triangle using the same angle measurements as in this triangle but different side lengths.

c. You draw a new triangle using *one* angle measurement that is the same as in this triangle and two angle measurements that are different.

d. You draw a new triangle so that its sides are exactly 2.4 times as long as the sides in the original triangle.

Example. The two kites are similar. Find the side length marked with x.



Solution 1. The two ratios we get from the two pairs of corresponding sides are equal, so we can write a proportion:

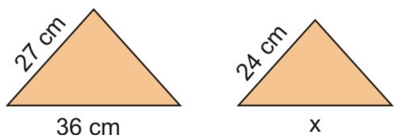
$$\begin{aligned}
\frac{20\text{ mm}}{15\text{ mm}} &= \frac{x}{27\text{ mm}} \\
15x &= 20 \times 27\text{ mm} \\
15x &= 540\text{ mm} \\
\frac{15x}{15} &= \frac{540\text{ mm}}{15} \\
x &= 36\text{ mm}
\end{aligned}$$

Solution 2.

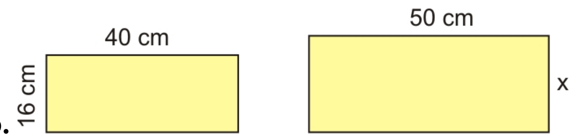
The scale ratio between the larger and smaller kite is 20:15 = 4:3. As a single number, this ratio is 4:3 = 4/3. Therefore, the unknown side length is 4/3 × 27 mm = 36 mm.

9. The figures are similar. Calculate the side length marked with x.

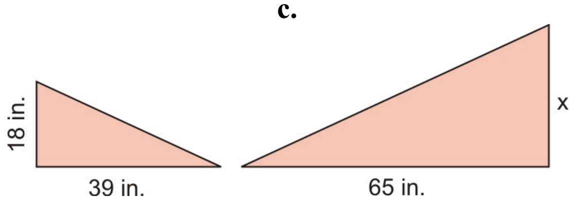
a.



b.



c.



d.