Contents

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

Chapter 3: Decimals

Chapter 4: Ratios

Chapter 5: Percent

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

Powers and Exponents

Exponents are a “shorthand” for writing repeated multiplications by the same number.

For example, $2 \times 2 \times 2 \times 2 \times 2$ is written 2^5.
$5 \times 5 \times 5 \times 5 \times 5 \times 5$ is written 5^6.

The tiny raised number is called the **exponent**. It tells us how many times the *base* number is multiplied by itself.

exponent

$$12^4 = 12 \times 12 \times 12 \times 12$$
$$= 20\,736$$

base

The expression 2^5 is read as “two to the fifth power,” “two to the fifth,” or “two raised to the fifth power.”

Similarly, 7^9 is read as “seven to the ninth power,” “seven to the ninth,” or “seven raised to the ninth power.”

The “powers of 6” are simply expressions where 6 is raised to some power: For example, 6^3, 6^4, 6^{45} and 6^{99} are powers of 6. What would powers of 10 be?

Expressions with the exponent 2 are usually read as something “**squared.**” For example, 11^2 is read as “**eleven squared.**” That is because it gives us *the area of a square* with the side length of 11 units.

Similarly, if the exponent is 3, the expression is usually read using the word “**cubed.**” For example, 31^3 is read as “**thirty-one cubed**” because it gives the *volume of a cube* with the edge length of 31 units.

1. Write the expressions as multiplications, and then solve them in your head.

a. $3^2 = \underline{\ 3 \times 3 = 9\ }$ **b.** 1^6

c. 4^3 **d.** 10^4

e. 5^3 **f.** 10^2

g. 2^3 **h.** 8^2

i. 0^5 **j.** 10^5

k. 50^2 **l.** 100^3

2. Rewrite the expressions using an exponent, then solve them. You may use a calculator.

a. $2 \times 2 \times 2 \times 2 \times 2 \times 2$ **b.** $8 \times 8 \times 8 \times 8 \times 8$

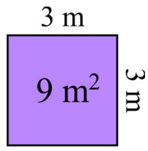
c. 40 squared **d.** $10 \times 10 \times 10 \times 10$

e. nine to the eighth power **f.** eleven cubed

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

You just learned that the expression 7^2 is read "seven *squared*" because it tells us the area of a *square* with a side length of 7 units. Let's compare that to square metres and other units of area.



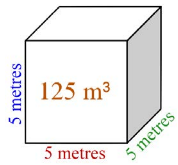
If the sides of a square are 3 m long, then its area is 3 m × 3 m = 9 m^2 or nine square metres.

Notice that the symbol for square metres is $\mathbf{m^2}$. This means "**metre × metre**." We are, in effect, squaring the unit *metre* (multiplying the unit of length *metre* by itself)!

The expression $(9\text{ cm})^2$ means 9 cm × 9 cm. We multiply 9 by itself, but we also multiply the unit *cm* by itself. That is why the result is **81 $\mathbf{cm^2}$**. The square centimetre (cm^2) comes from multiplying "**centimetre × centimetre**."

We do the same thing with any other unit of length to form the corresponding unit for area, such as square kilometres or square millimetres.

In a similar way, to calculate the volume of this cube, we multiply 5 m × 5 m × 5 m = 125 m^3. We not only multiply 5 by itself three times, but also multiply the unit *metre* by itself three times (metre × metre × metre) to get the unit of volume "cubic metre" or m^3.



3. Express the area (A) as a multiplication, and solve.

a. A square with a side of 12 kilometres: A = 12 km × 12 km = ____________	**b.** A square with sides 6 m long: A = ______________________________
c. A square with a side length of 6 centimetres: A = ______________________________	**d.** A square with a side with a length of 12 cm: A = ______________________________

4. Express the volume (V) as a multiplication, and solve.

a. A cube with an edge of 2 cm: V = 2 cm × 2 cm × 2 cm = ____________	**b.** A cube with edges 10 cm long each: V = ______________________________
c. A cube with edges 1 m in length: V = ______________________________	**d.** A cube with edges that are all 5 m long: V = ______________________________

5. **a.** The perimeter of a square is 40 centimetres. What is its area?

b. The volume of a cube is 64 cubic centimetres. How long is its edge?

c. The area of a square is 121 m^2. What is its perimeter?

d. The volume of a cube is 27 cm^3. What is the length of one edge?

Sample worksheet from https://www.mathmammoth.com

©2022 Taina Miller

The powers of 10 are very special —and very easy!

Notice that the exponent tells us *how many zeros* there are in the answer.

$10^1 = 10$

$10^2 = 10 \times 10 = 100$

$10^3 = 10 \times 10 \times 10 = 1\,000$

$10^4 = 10\,000$

$10^5 = 100\,000$

$10^6 = 1\,000\,000$

6. Fill in the patterns. In part (d), choose your own number to be the base.
Use a calculator in parts (c) and (d).

a.	**b.**	**c.**	**d.**
$2^1 =$	$3^1 =$	$5^1 =$	
$2^2 =$	$3^2 =$	$5^2 =$	
$2^3 =$	$3^3 =$	$5^3 =$	
$2^4 =$	$3^4 =$	$5^4 =$	
$2^5 =$	$3^5 =$	$5^5 =$	
$2^6 =$	$3^6 =$	$5^6 =$	

7. Look at the patterns above. Think carefully how each step comes from the previous one. Then answer.

a. If $3^7 = 2\,187$, how can you use that result to find 3^8?

b. Now find 3^8 without a calculator.

c. If $2^{45} = 35\,184\,372\,088\,832$, use that to find 2^{46} without a calculator.

8. Fill in.

a. 17^2 gives us the _____________ of a _______________ with sides ______ units long.

b. 101^3 gives us the _____________ of a _______________ with edges ______ units long.

c. 2×6^2 gives us the ____________ of two _______________ with sides ______ units long.

d. 4×10^3 gives us the ____________ of ____ _______________ with edges ______ units long.

Puzzle Corner

Make a pattern, called a **sequence**, with the powers of 2, starting with 2^6 and going *backwards* to 2^0. At each step, *divide* by 2. What is the logical (though surprising) value for 2^0 from this method?

Make another, similar, sequence for the powers of 10. Start with 10^6 and divide by 10 until you reach 10^0. What value do you calculate for 10^0?

Try this same pattern for at least one other base number, n. What value do you calculate for n^0? Do you think it will come out this way for every base number?

Why or why not?

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

The Distributive Property

The **distributive property** states that $a(b + c) = ab + ac$

It may look like a meaningless or difficult equation to you now, but don't worry, it will become clearer!

The equation $\boldsymbol{a(b + c) = ab + ac}$ means that you can *distribute* the multiplication (by a) over the sum $b + c$ so that you multiply the numbers b and c separately by a, and add last.

You have already used the distributive property! When you separated $3 \cdot 84$ into $3 \cdot (80 + 4)$, you then multiplied 80 and 4 *separately* by 3, and added last: $3 \cdot 80 + 3 \cdot 4 = 240 + 12 = 252$.
We called this using "partial products" or "multiplying in parts."

Example 1. Using the distributive property, we can write the product $2(x + 1)$ as $2x + 2 \cdot 1$, which simplifies to $2x + 2$.

Notice what happens: Each term in the sum $(x + 1)$ gets multiplied by the factor 2! Graphically:

$$2(x + 1) = \underline{2x} + \underline{2 \cdot 1}$$

Example 2. To multiply $s \cdot (3 + t)$ using the distributive property, we need to multiply *both* 3 and t by s:

$s \cdot (3 + t) = s \cdot 3 + s \cdot t$, which simplifies to $3s + st$.

1. Multiply using the distributive property.

a. $3(90 + 5) = 3 \cdot$ ___ $+ 3 \cdot$ ___ $=$	**b.** $7(50 + 6) = 7 \cdot$ ___ $+ 7 \cdot$ ___ $=$
c. $4(a + b) = 4 \cdot$ ___ $+ 4 \cdot$ ___ $=$	**d.** $2(x + 6) = 2 \cdot$ ___ $+ 2 \cdot$ ___ $=$
e. $7(y + 3) =$	**f.** $10(s + 4) =$
g. $s(6 + x) =$	**h.** $x(y + 3) =$
i. $8(5 + b) =$	**j.** $9(5 + c) =$

Example 3. We can use the distributive property also when the sum has three or more terms. Simply multiply *each term* in the sum by the factor in front of the brackets:

$5(x + y + 6) = 5 \cdot x + 5 \cdot y + 5 \cdot 6$, which simplifies to $5x + 5y + 30$

2. Multiply using the distributive property.

a. $3(a + b + 5) =$	**b.** $8(5 + y + r) =$
c. $4(s + 5 + 8) =$	**d.** $3(10 + c + d + 2) =$

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

Example 4. Now one of the terms in the sum has a coefficient (the 2 in $2x$):

$$6(2x + 3) = 6 \cdot 2x + 6 \cdot 3 = 12x + 18$$

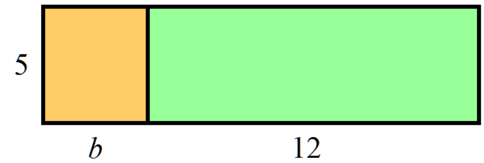
3. Multiply using the distributive property.

a. $2(3x + 5) =$	**b.** $7(7a + 6) =$
c. $5(4a + 8b) =$	**d.** $2(4x + 3y) =$
e. $3(9 + 10z) =$	**f.** $6(3x + 4 + 2y) =$
g. $11(2c + 7a) =$	**h.** $8(5 + 2a + 3b) =$

To understand even better why the the distributive property works, let's look at an area model (this, too, you have seen before!).

The area of the whole rectangle is 5 times $(b + 12)$.

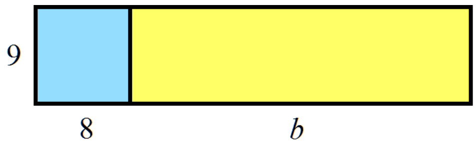
But if we think of it as *two* rectangles, the area of the first rectangle is $5b$, and of the second, $5 \cdot 12$.



Of course, these two expressions have to be equal:

$$5 \cdot (b + 12) = 5b + 5 \cdot 12 = 5b + 60$$

4. Write an expression for the area in two ways, thinking of one rectangle or two.

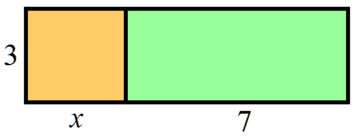


a. 9(_____ + _____) and

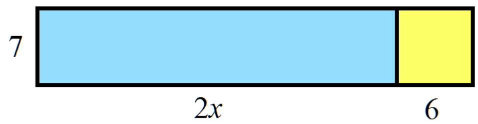
9 · _____ + 9 · _____ =

b. s(_____ + _____) and

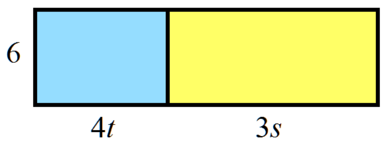
s · _____ + s · _____ =



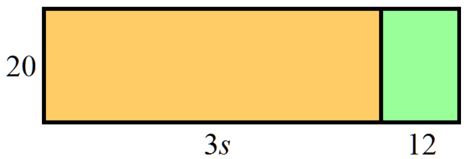
c. ____ (_____ + _____) and



d.



e.



f.

Sample worksheet from
https://www.mathmammoth.com
©2022 Taina Miller

[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

Using Two Variables

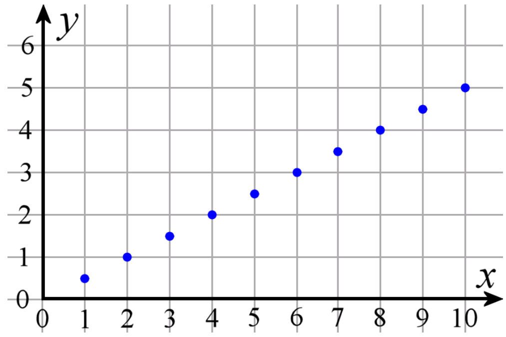
Often in mathematics—and in real life—we study the relationship between two variables.

Example 1. The equation $y = \frac{1}{2}x$ has two variables, *y* and *x*.

There are many values of *x* and *y* that make that equation true. For example, when *x* is 4, then *y* is (1/2) · 4 = 2. Some of the values of *x* and *y* are listed below.

x	1	2	3	4	5
y	½	1	1 ½	2	2 ½

x	6	7	8	9	10
y	3	3 ½	4	4 ½	5



We can plot or graph these (*x*, *y*) pairs as points in the coordinate grid.
These ordered pairs actually are a **function**. We will not study the exact definition of a function here, but you can think of a function as a relationship between two variables.

In this lesson, you will study only **linear functions**. The word "linear" comes from the fact that the graphs of those functions look like a *line*. There exist many other, different kinds of functions as well.

Example 2. One towel costs \$4. If you buy 17 towels, the cost is 17 · \$4 = \$68.
In this situation, we are interested in <u>two</u> <u>variables</u> whose values can change:

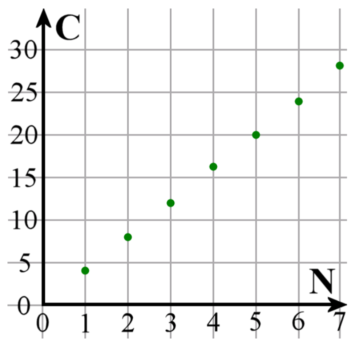
1. **The number of towels** a person buys is a variable. (It can vary!) Let's denote the number of towels by N.
2. **The total** cost varies according to how many towels are bought. Let C be the cost.

There is a very simple relationship between N and C: **C = N · \$4**
(This means the total cost *is* the number of towels times \$4.)

This is normally written as **C = 4N** because in algebra we write the number in front of the variable (not vice versa), and we omit the multiplication sign between a number and a variable.

The table below shows some *possible* values of C and N.

(x)	N	1	2	3	4	5	6	7	10	15	20
(y)	C	4	8	12	16	20	24	28	40	60	80



From this table, we get lots of number pairs. Some of them are plotted on the coordinate grid you see on the right.

You may have seen coordinate grids that have *x* and *y*-axis. This time we will label our axes N and C, according to the names of the variables. If this seems confusing, think of the variable N as the "*x*", and the variable C as the "*y*".

In this situation, we think of the variable N as the *independent variable*, and the variable C as the *dependent variable*, because its value *depends* on the value of N <u>according to the given equation</u> (C = 4N). In other words, we let the value of N vary (sort of independently), and the values of C are what we calculate or "observe," noticing how they depend on the value of N.

The independent variable is *always* plotted on the <u>horizontal axis</u>.

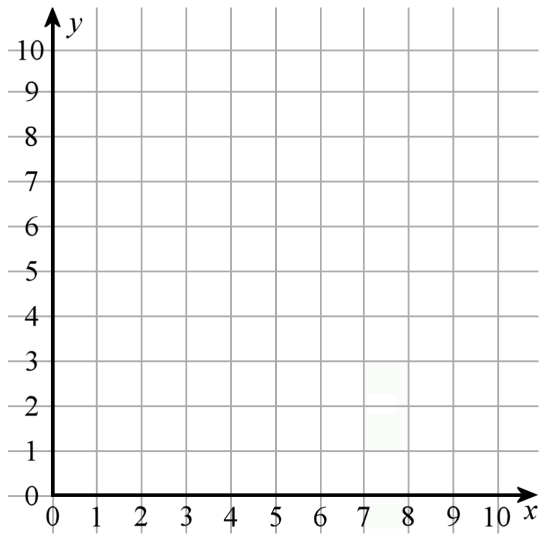
We *could* look at this situation just the opposite way also: let the cost be the independent variable, and study how the number of towels depends on that. Then, we would plot C on the horizontal axis, and calculate N using an equation that depends on C (it would be N = C/4).

Sample worksheet from
https://www.mathmammoth.com
©2022 Taina Miller

1. Calculate the values of y according to the equation $y = x + 2$.

x	0	1	2	3	4	5	6	7	8
y	2	3	4						

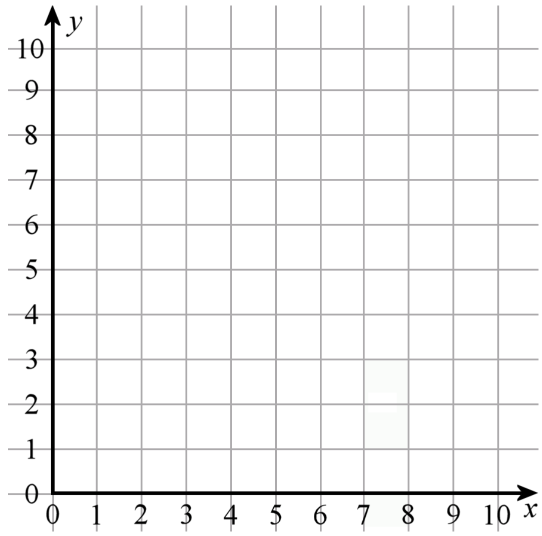
Now, plot the points.



2. Calculate the values of y according to the equation $y = 8 - x$.

x	0	1	2	3	4	5	6	7	8
y	8								

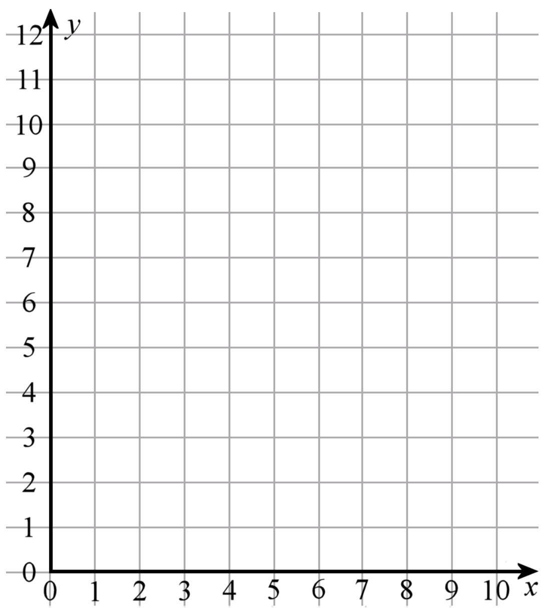
Now, plot the points.



3. Calculate the values of y according to the equation $y = 2x - 1$.

x	1	2	3	4	5	6
y						

Now, plot the points.



Sample worksheet from https://www.mathmammoth.com

©2022 Taina Miller

[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

Problem Solving with Decimals

Example 1. Martha jogs 0.8 kilometre every day. How many days will it take for her to jog a distance of 20 kilometres?

We could divide 20 km by 0.8 km to find out how many times 0.8 "fits" into 20. However, there is also another way: we can solve it with *mental maths*.

Notice, 0.8 goes evenly into 4, and 4 goes evenly into 20.

0.8 fits five times into 4 (because $5 \cdot 0.8 = 4$). And, 4 fits five times into 20.

So, 0.8 fits into 20 exactly $5 \cdot 5 = 25$ times.

Martha will have jogged 20 kilometres in 25 days.

Example 2. If you divide a paper that is 21.25 cm wide into three equally-wide columns. How wide are the columns?

We divide 21.25 cm by three. Since the width of 21.25 cm is given to *two* decimal places, it is reasonable to also give the answer to *two* decimal places, so divide until there are *three* decimals in the quotient, and then round to the nearest hundredth.

$$\begin{array}{rrrrrrr}
 & 0 & 7 & . & 0 & 8 & 3 \\ \hline
3\,) & 2 & 1 & . & 2 & 5 & 0 \\
 & & & & -2 & 4 & \\ \cline{5-6}
 & & & & & 1 & 0 \\
 & & & & & - & 9 \\ \cline{6-7}
 & & & & & & 1
\end{array}$$

$21.25 \div 3 \approx 7.08$, so the columns are about 7.08 cm wide.

If you have to actually measure these columns using a standard ruler, then it would be reasonable to give the answer as 7.1 cm.

In all of the problems, give your answer to a meaningful accuracy, especially when the division is not even.

1. Jack, John and Jerry shared a prize of $200 equally.
   How much did each one get?

2. A student textbook weighs 0.4 kg. How many of those can you pack
   into a suitcase so that the total weight is 18 kg?

Sample worksheet from
https://www.mathmammoth.com

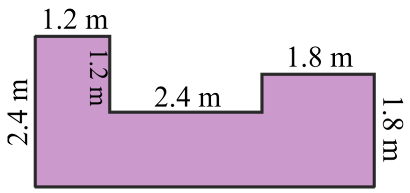
©2022 Taina Miller

3. These are the quiz results of the Spanish class:
21 15 18 29 19 34 39 21 11 8 15 28 15 11 12.
Find the average.

To calculate the average of a set of numbers:

1. Add all of the numbers.
2. Divide the sum by the number of the data entries.

4. Find the area and perimeter of this shape.



5. Kitchen Delight makes blenders. Each blender weighs 1.2 kg. The shipping company allows no more than 40 kg of weight in each shipping crate. How many blenders can be packed into each shipping crate?

6. Find the unit prices for the following items. Round to the nearest cent. Use the space below for calculations.

Item and price	Unit price	What would this cost…?	
5 L of orange juice for $15.65		2.3 L of orange juice	
3 kg of chicken for $13.25		5.7 kg of chicken	
4 kg of bananas for $7.90		2.5 kg of bananas	

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

Convert Metric Measuring Units

The metric system has one basic unit for each thing we might measure: For length, the unit is the **metre**. For weight, it is the **gram**. And for volume, it is the **litre**.

All of the other units for measuring length, weight, or volume are *derived* from the basic units using *prefixes*. The prefixes tell us what multiple of the basic unit the *derived unit* is.

For example, centilitre is 1/100 part of a litre (*centi* means 1/100).

Prefix	Abbreviated	Meaning
kilo-	k	1 000
hecto-	h	100
deka-	da	10
-	-	(the basic unit)
deci-	d	1/10
centi-	c	1/100
milli-	m	1/1000

Unit	Abbr	Meaning
kilometre	km	1 000 metres
hectometre	hm	100 metres
decametre	dam	10 metres
metre	m	(the basic unit)
decimetre	dm	1/10 metre
centimetre	cm	1/100 metre
millimetre	mm	1/1000 metre

Unit	Abbr	Meaning
kilogram	kg	1 000 grams
hectogram	hg	100 grams
dekagram	dag	10 grams
gram	g	(the basic unit)
decigram	dg	1/10 gram
centigram	cg	1/100 gram
milligram	mg	1/1000 gram

Unit	Abbr	Meaning
kilolitre	kl	1 000 litres
hectolitre	hl	100 litres
dekalitre	dal	10 litres
litre	L	(the basic unit)
decilitre	dl	1/10 litre
centilitre	cl	1/100 litre
millilitre	ml	1/1000 litre

1. Write these amounts using the basic units (metres, grams, or litres) by "translating" the prefixes. Use both fractions and decimals, like this: 3 cm = 3/100 m = 0.03 m (since "centi" means "hundredth part").

a. 3 cm = *3/100 m* = *0.03 m*	**b.** 2 cg = __________ g = __________ g
5 mm = __________ m = __________ m	6 ml = __________ L = __________ L
7 dl = __________ L = __________ L	1 dg = __________ g = __________ g

2. Write the amounts in basic units (metres, grams, or litres) by "translating" the prefixes.

a. 3 kl = __________ L	**b.** 2 dam = __________ m	**c.** 70 km = __________ m
8 dag = __________ g	9 hl = __________ L	5 hg = __________ g
6 hm = __________ m	7 kg = __________ g	8 dal = __________ L

3. Write the amounts with derived units (units with prefixes) and a single-digit number.

a. 3 000 g = <u>3</u> <u>kg</u>	**b.** 0.01 m = __________ ______	**c.** 0.04 L = __________ ______
800 L = <u>8</u> ______	0.2 L = __________ ______	0.8 m = __________ ______
60 m = <u>6</u> ______	0.005 g = __________ ______	0.007 L = __________ ______

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

[This page is intentionally left blank.]

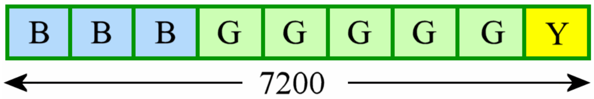
Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

Ratio Problems and Bar Models 1

Often, ratio problems become easy by drawing a **bar model**.

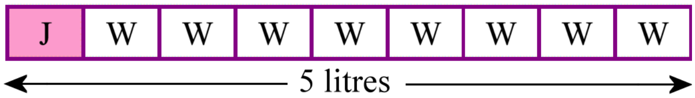
The ratio of blue shirts to green shirts to yellow shirts is 3 to 5 to 1. If there are 7 200 shirts in all, how many of them are of each colour?



Look at the bar model. There are a total of 7 200 shirts.
We draw 3 “blocks” for the blue shirts, 5 “blocks” for the green shirts and 1 “block” for the yellow shirts to show the ratio of **3 : 5 : 1**. It is obvious one “block” means 7 200 ÷ 9 = 800 shirts. So there are a total of 2 400 blue shirts, 4 000 green shirts and 800 yellow shirts.

Juice concentrate is mixed with water in a ratio of 1:8. If you want to make 5 litres of juice, how much concentrate and how much water do you need?

Let’s draw a bar model. (In reality, of course, the juice and water mix, but for the purpose of calculating, this model is helpful.)



There are a total of 9 equal parts, so we simply divide 5 litres by 9. First, change 5 litres to 5 000 millilitres, and then divide: 5 000 ml ÷ 9 ≈ 555.56 ml.

However, that is way too accurate. Measuring cups do not normally let us measure to the nearest millilitre, and not even to the nearest 10 millilitres, so let’s round this to the nearest 50 ml to get 550 ml.

So we need 550 ml of juice concentrate and 5 000 ml − 550 ml = 4 450 ml of water.

1. A factory makes shirts in a ratio of 1:3:3:1 for the sizes S, M, L and XL, respectively.

   a. Draw a bar model. What is the ratio of small (S) shirts to the total number of shirts?

   b. In a batch of 1 000 shirts, how many of them are of each size?

2. The instructions on a box of juice concentrate say to mix 2 parts of concentrate to 5 parts of water.

   a. If you want to make 3 litres of juice, how much concentrate and how much water do you need?

   b. Let’s say that you have 1/2 litre of concentrate left. According to the instructions, how much water would you need to add to that?

   How much diluted juice does this make?

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

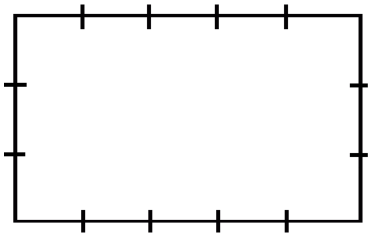
©2022 Taina Miller

Aspect Ratio

You might have heard about the aspect ratio of the screens of televisions, computer monitors and other monitors. The aspect ratio is simply **the ratio** of a **rectangle's width to its height or length.**

If the rectangle is "standing up," it is often easier to think and talk about width and height. If it is laid on the ground, then we usually talk about its width and length.

Example. A rectangle's width and height are in a ratio of 5:3. This means the aspect ratio is 5:3. If the rectangle's perimeter is 64 cm, what are its width and its height?



Let's draw the rectangle. Working from the 5:3 aspect ratio, let's divide the sides into "parts," or the same-sized segments, 5 for the width, and 3 for the height. We can see in the picture that perimeter is made up of 16 of these "parts." Since 64 ÷ 16 = 4, each part is 4 cm long.

Therefore, the rectangle's width is 5 · 4 cm = 20 cm, and its height is 3 · 4 cm = 12 cm.

1. The width and height of a rectangle are in a ratio of 9:2.

   a. Draw the rectangle, and divide its width and length into parts according to its aspect ratio.

   b. If the rectangle's perimeter is 220 cm, find its width and its height.

2. A rectangle's width is three times its length, and its perimeter is 120 mm. Find the rectangle's width and its length.

3. Find the aspect ratio of each rectangle:

   a. a rectangle whose height is 2/5 of its width

   b. a rectangle whose height is five times its width

   c. a square

4. The door of a refrigerator is 4/9 as wide as it is tall.

   a. What is the ratio of the door's width to its height?

   b. If the door is 54 cm wide, how tall is it?

Sample worksheet from https://www.mathmammoth.com

©2022 Taina Miller

[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

Percentage of a Number (Mental Maths)

100% of something means *all* of it. 1% of something means 1/100 of it.
Since one percent means "a hundredth part," calculating a percentage of a quantity is the same thing as finding a fractional part of it. So **percentages are really fractions**!
How much is 1% of 200 kg? This means how much is 1/100 of 200 kg? It is simply 2 kg. **To find 1% of something (1/100 of something), divide by 100.** Do you remember how to divide by 100 in your head? Just move the decimal point two places to the left. For example, 1% of 540 is 5.4, and 1% of 8.30 is 0.083.
To find 2% of some quantity, first find 1% of it, and double that. For example, let's find 2% of \$6. Since 1% of \$6 is \$0.06, then 2% of \$6 is \$0.12.
To find 10% of some quantity, divide by 10. Why does that work? It is because 10% is 10/100, which equals 1/10. So 10% is 1/10 of the quantity! For example, 10% of \$780 is \$78. And 10% of \$6.50 is \$0.65. (To divide by 10 in your head, just move the decimal point one place to the left.)
Can you think of a way to find 20% of a number? ____

1. Find 10% of these numbers.

a. 700 ________ **b.** 321 ________ **c.** 60 ________ **d.** 7 ________

2. Find 1% of these numbers.

a. 700 ________ **b.** 321 ________ **c.** 60 ________ **d.** 7 ________

3. One percent of Mother's pay cheque is \$22. How much is her total pay cheque?

4. Fill in the table. Use mental maths.

percentage ↓ number →	**1 200**	**80**	**29**	**9**	**5.7**
1% of the number					
2% of the number					
10% of the number					
20% of the number					

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

5. Fill in this guide for using mental maths with percentages:

Mental Maths and Percentage of a Number	
50% is $\frac{1}{2}$. To find 50% of a number, divide by _______.	50% of 244 is _________.
10% is $\frac{1}{\square}$. To find 10% of a number, divide by ________.	10% of 47 is __________.
1% is $\frac{1}{\square}$. To find 1% of a number, divide by ________.	1% of 530 is __________ .
To find 20%, 30%, 40%, 60%, 70%, 80%, or 90% of a number, • **First find __________% of the number, and** • **then multiply by 2, 3, 4, 6, 7, 8, or 9.**	10% of 120 is __________. 30% of 120 is __________. 60% of 120 is __________.

6. Find the percentages. Use mental maths.

a. 10% of 60 kg __________ 20% of 60 kg __________	**b.** 10% of $14 __________ 30% of $14 __________	**c.** 10% of 5 m __________ 40% of 5 m __________
d. 1% of $60 __________ 4% of $60 __________	**e.** 10% of 110 cm __________ 70% of 110 cm __________	**f.** 1% of $1 330 __________ 3% of $1 330 __________

7. David pays a 20% income tax on his $2 100 salary.

a. How many dollars is the tax?

b. How much money does he have left after paying the tax?

c. What percentage of his salary does he have left?

8. Nancy pays 30% of her $3 100 salary in taxes. How much money does she have left after paying the tax?

9. Identify the errors that these children made. Then find the correct answers.

a. Find 90% of $55. Peter's solution: 10% of $55 is $5.50 So, I subtract 100% − $5.50 = $94.50	**b.** Find 6% of $1 400. Patricia's solution: 1% of $1 400 is $1.40. So, 6% is six times that, or $8.40.

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

Finding the Total When the Percentage Is Known

Use a bar model to find the unknown total when you know the percentage and the quantity.

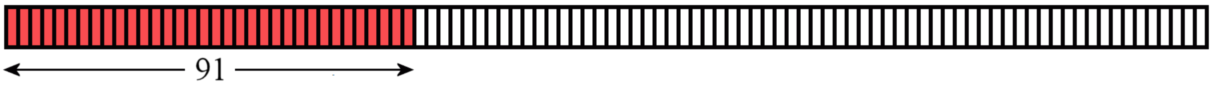
Example 1. If 32 red marbles make up 4/5 of the total number of marbles, how many marbles are there in all?

Look at the bar model. We have drawn the marbles as divided into 5 equal “blocks.” Four of those five blocks make up a total of 32 marbles. So, one block, or 1/5 of the marbles, is <u>8 marbles</u>. From that it is easy to calculate the total: $5 \cdot 8 = 40$ marbles.

The same reasoning works if the part of the marbles is given as a *percentage* instead of as a fraction:

Example 2. If 91 red marbles is 35% of the total number of marbles, how many marbles are there in all?

In the model, we need 100 little “blocks” with 35 of them coloured (since 35/100 of the marbles are red.)



The calculation is done the same way: If 35 “blocks” or 35% make up 91 marbles, then one “block”, or one percent, is $91 \div 35 = 2.6$. Then, to find the total, simply multiply that number by 100: $2.6 \cdot 100 = 260$.

1. Margie gave away 40 marbles, which was 20% of the marbles that she had.
   How many marbles did Margie have at first?
   Hint: Instead of 100 blocks, you can use 5 blocks, each representing 20% or 1/5.

2. Emma cut down the amount of sugar in a recipe by 75%.
   Now, she uses only 1/2 cup of sugar.
   How much sugar did the recipe call for originally?
   Hint: Instead of 100 blocks, you can use 4 blocks, each representing 25%.

3. When Eric bought a guitar for $90, he used up 12% of the money he had.
   How much money did he have at first?

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

Example 3. A phone was discounted by 40% and now costs $72. What was the price before the discount?

The cost now, $72, represents **60%** of the original total—not 40%.

We can find 10% of the original price by dividing $72 ÷ 6 = $12. And from that, 100% of the price is 10 times that, or $120. If this confuses you, draw a bar model with 10 parts, each representing 10% of the original price.

4. A dress was discounted by 20%.
   The discounted price is $24.
   What was the price before the discount?

5. A concert ticket was discounted by 60%.
   The discounted price is $21.60.
   What was the original price?

6. Joe spent 72% of his money, and now he has $56 left.
   How much did Joe have to begin with?

7. Crystal spent 52% of her money and now she has $120 left.
   How much did she spend?

8. Uncle Jack raises two different breeds of cows on his farm. Of his cows, 28% are Black Angus and the rest are Hereford. If he has 420 Black Angus cows, how many Herefords does he have?

9. A survey found out that 16% of the people who had bought a certain brand of coffee grinder were unhappy with it. If there were 126 people who *were* happy with it, then how many people in total had bought that brand?

Puzzle Corner

One calculator is discounted by 30% and now costs $42.
Another is discounted by 25% and now it also costs $42.
Which calculator had the cheaper original price? How much cheaper?

Sample worksheet from https://www.mathmammoth.com

©2022 Taina Miller

[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

Contents

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

Chapter 9: Geometry

Chapter 10: Statistics

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

The Sieve of Eratosthenes and Prime Factorisation

Remember? A number is a **prime** if it has no other factors besides 1 and itself.

For example, 13 is a prime, since the only way to write it as a multiplication is $1 \cdot 13$. In other words, 1 and 13 are its only factors.

And, 15 is not a prime, since we can write it as $3 \cdot 5$. In other words, 15 has other factors besides 1 and 15, namely 3 and 5.

To find all the prime numbers less than 100 we can use the *sieve of Eratosthenes.*

Here is an online interactive version: https://www.mathmammoth.com/practice/sieve-of-eratosthenes

1. Cross out 1, as it is not considered a prime.
2. Cross out all the even numbers except 2.
3. Cross out all the multiples of 3 except 3.
4. You do not have to check multiples of 4. Why?
5. Cross out all the multiples of 5 except 5.
6. You do not have to check multiples of 6. Why?
7. Cross out all the multiples of 7 except 7.
8. You do not have to check multiples of 8 or 9 or 10.
9. The numbers left are primes.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

List the **primes between 0 and 100** below:

2, 3, 5, 7, ____

Why do you not have to check numbers that are bigger than 10? Let's think about multiples of 11. The following multiples of 11 have already been crossed out: $2 \cdot 11$, $3 \cdot 11$, $4 \cdot 11$, $5 \cdot 11$, $6 \cdot 11$, $7 \cdot 11$, $8 \cdot 11$ and $9 \cdot 11$. The multiples of 11 that have not been crossed out are $10 \cdot 11$ and onward... but they are not on our chart! Similarly, the multiples of 13 that are less than 100 are $2 \cdot 13$, $3 \cdot 13$, ..., $7 \cdot 13$, and all of those have already been crossed out when you crossed out multiples of 2, 3, 5 and 7.

1. You learned this in 4th and 5th grades... find all the factors of the given numbers. Use the checklist to help you keep track of which factors you have tested.

a. 54

Check 1 2 3 4 5 6 7 8 9 10

factors: ________________________________

b. 60

Check 1 2 3 4 5 6 7 8 9 10

factors: ________________________________

c. 84

Check 1 2 3 4 5 6 7 8 9 10

factors: ________________________________

d. 97

Check 1 2 3 4 5 6 7 8 9 10

factors: ________________________________

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

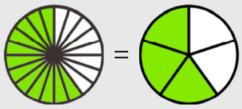
©2022 Taina Miller

Using Factoring When Simplifying Fractions

You have seen the process of **simplifying fractions** before.

In simplifying fractions, we divide both the numerator and the denominator by the same number. The fraction becomes *simpler*, which means that the numerator and the denominator are now *smaller* numbers than they were before.

However, this does NOT change the actual value of the fraction.
It is the "same amount of pie" as it was before. It is just cut differently.



Every four slices have been joined together.

$$\underset{\div 4}{\overset{\div 4}{\frac{12}{20} = \frac{3}{5}}}$$

Why does this work?

It is based on finding common factors and on how fraction multiplication works. In our example above, the fraction $\frac{12}{20}$ can be written as $\frac{4 \cdot 3}{4 \cdot 5}$. Then we can **cancel out** those fours: $\frac{\cancel{4} \cdot 3}{\cancel{4} \cdot 5} = \frac{3}{5}$.

The reason this works is because $\frac{4 \cdot 3}{4 \cdot 5}$ is equal to the fraction multiplication $\frac{4}{4} \cdot \frac{3}{5}$. And in that, 4/4 is equal to 1, which means we are only left with 3/5.

Example 1. Often, the simplification is simply written or indicated this way →
Notice that here, the 4's that were cancelled out do ***not*** get indicated in any way!
You only think it: "I divide 12 by 4, and get 3. I divide 20 by 4, and get 5."

$$\frac{\overset{3}{\cancel{12}}}{\underset{5}{\cancel{20}}} = \frac{3}{5}$$

Example 2. Here, 35 and 55 are both divisible by 5. This means we can cancel out those 5's, but notice this is not shown in any way. We simply cross out 35 and 55, think of dividing them by 5, and write the division result above and below.

$$\frac{\overset{7}{\cancel{35}}}{\underset{11}{\cancel{55}}} = \frac{7}{11}$$

1. Simplify the fractions, if possible.

a. $\frac{12}{36}$	**b.** $\frac{45}{55}$	**c.** $\frac{15}{23}$	**d.** $\frac{13}{6}$
e. $\frac{15}{21}$	**f.** $\frac{19}{15}$	**g.** $\frac{17}{24}$	**h.** $\frac{24}{30}$

2. Leah simplified various fractions like you see below. She did not get them right though.
Explain to her what she is doing wrong.

$\frac{24}{84} = \frac{20}{80} = \frac{1}{4}$ $\qquad$ $\frac{27}{60} = \frac{7}{40}$ $\qquad$ $\frac{14}{16} = \frac{10}{12} = \frac{6}{8} = \frac{3}{4}$

**Sample worksheet from
https://www.mathmammoth.com**

©2022 Taina Miller

[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

The Greatest Common Factor (GCF)

Let's take two whole numbers. We can then list all the factors of each number, and then find the factors that are common in both lists. Lastly, we can choose the greatest or largest among those "common factors." That is the **greatest common factor** of the two numbers. The term itself really tells you what it means!

Example 1. Find the greatest common factor of 18 and 30.

The factors of 18: 1, 2, 3, 6, 9 and 18.
The factors of 30: 1, 2, 3, 5, 6, 10, 15 and 30.

Their common factors are 1, 2, 3 and 6. The greatest common factor is 6.

Here is a **method to find all the factors of a given number**.

Example 2. Find the factors (divisors) of 36.

We check if 36 is divisible by 1, 2, 3, 4 and so on. Each time we find a divisor, we write down *two* factors.

- 36 is divisible by 1. We write $36 = 1 \cdot 36$, and that equation gives us two factors of 36: both the smallest (**1**) and the largest (**36**).
- 36 is also divisible by 2. We write $36 = 2 \cdot 18$, and that equation gives us two more factors of 36: the second smallest (**2**) and the second largest (**18**).
- Next, 36 is divisible by 3. We write $36 = 3 \cdot 12$, and now we have found the third smallest factor (**3**) and the third largest factor (**12**).
- Next, 36 is divisible by 4. We write $36 = 4 \cdot 9$, and we have found the fourth smallest factor (**4**) and the fourth largest factor (**9**).
- Finally, 36 is divisible by 6. We write $36 = 6 \cdot 6$, and we have found the fifth smallest factor (**6**) which is also the fifth largest factor.

We know that we are done because the list of factors from the "small" end (1, 2, 3, 4, 6) has met the list of factors from the "large" end (36, 18, 12, 9, 6).

Therefore, all of the factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18 and 36.

1. List all of the factors of the given numbers.

a. 48	**b.** 60
c. 42	**d.** 99

2. Find the greatest common factor of the given numbers. Your work above will help!

a. 48 and 60	**b.** 42 and 48	**c.** 42 and 60	**d.** 99 and 60

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

Revision: Multiplying Fractions 1

The **shortcut for multiplying fractions** is:

- Multiply the numerators.
- Multiply the denominators.

$$\frac{6}{7} \cdot \frac{5}{2} \cdot \frac{1}{3} = \frac{6 \cdot 5 \cdot 1}{7 \cdot 2 \cdot 3} = \frac{30}{42} = \frac{5}{7}$$

To **multiply mixed numbers**, *first* write them as fractions, then multiply.

$$2\,\frac{1}{3} \cdot 1\,\frac{1}{10} = \frac{7}{3} \cdot \frac{11}{10} = \frac{7 \cdot 11}{3 \cdot 10} = \frac{77}{30} = 2\,\frac{17}{30}$$

If one of the factors is a whole number, write it as a fraction with a denominator of 1.

$$6 \cdot \frac{11}{12} = \frac{6}{1} \cdot \frac{11}{12} = \frac{66}{12} = \frac{11}{2} = 5\,\frac{1}{2}$$

1. Multiply. Give your answer in lowest terms, and as a mixed number, if applicable.

a. $5 \cdot \frac{7}{8}$	**b.** $\frac{2}{7} \cdot \frac{5}{6}$
c. $\frac{9}{10} \cdot \frac{6}{7} \cdot \frac{1}{2}$	**d.** $1\,\frac{1}{3} \cdot 2\,\frac{2}{3}$
e. $\frac{1}{10} \cdot 3\,\frac{1}{5}$	**f.** $2\,\frac{5}{6} \cdot 10 \cdot \frac{1}{2}$

2. Find the area of a square with sides 1 ¼ units. Use fractions.

3. A biscuit recipe calls for 1½ cups of buttermilk, and Mary plans to make the recipe one and a half times, three times a week, in order to sell biscuits. How much buttermilk does she need in a week?

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

[This page is intentionally left blank.]

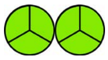
Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

Dividing Fractions: Reciprocal Numbers

One interpretation of division is **measurement division**, where we think: *How many times does one number go into another?* For example, to solve how many times 11 fits into 189, we divide 187 ÷ 11 = 17.

(The other interpretation is equal sharing; we will come to that later.)

Let's apply that to fractions. How many times does  go into  ?

We can solve this just by looking at the pictures: three times. We can write the division: $2 \div \frac{2}{3} = 3$.

To check the division, we multiply: $3 \cdot \frac{2}{3} = \frac{6}{3} = 2$. Since we got the original dividend, it checks.

We can use measurement division to check whether an answer to a division is reasonable.

For example, if I told you that $7 \div 1\frac{2}{3}$ equals $14\frac{1}{3}$, you can immediately see it doesn't make sense:

1 2/3 surely does not fit into 7 that many times. Maybe three to four times, but not 14!

You could also multiply to see that: 14-*and-something* times 1-*and-something* is way more than 14, and closer to 28 than to 14, instead of 7.

1. Find the answers that are unreasonable without actually dividing.

a. $\frac{4}{5} \div 6 = \frac{2}{15}$

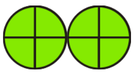
b. $2\frac{3}{4} \div \frac{1}{4} = \frac{7}{12}$

c. $\frac{7}{9} \div 2 = \frac{7}{18}$

d. $8 \div 2\frac{1}{3} = 18\frac{1}{3}$

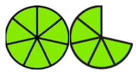
e. $5\frac{1}{4} \div 6\frac{1}{2} = 3\frac{1}{8}$

2. Solve with the help of the visual model, checking how many times the given fraction fits into the other number. Then write a division. Lastly, write a multiplication that checks your division.

a. How many times does  go into  ?

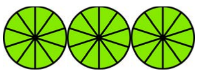
$2 \div \frac{3}{4} =$

Check: _____ $\cdot \frac{3}{4} =$

b. How many times does  go into  ?

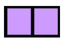
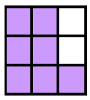
÷ =

Check:

c. How many times does  go into  ?

3 ÷ =

Check:

d. How many times does  go into  ?

÷ =

Check:

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

3. Solve. Think how many times the fraction goes into the whole number. Can you find a *pattern* or a *shortcut*?

a. $3 \div \frac{1}{6} =$	**b.** $4 \div \frac{1}{5} =$	**c.** $3 \div \frac{1}{10} =$	**d.** $5 \div \frac{1}{10} =$
e. $7 \div \frac{1}{4} =$	**f.** $4 \div \frac{1}{8} =$	**g.** $4 \div \frac{1}{10} =$	**h.** $9 \div \frac{1}{8} =$

The shortcut is this:

$$\begin{array}{ccc} 5 & \div & \frac{1}{4} \\ \downarrow & & \downarrow \\ 5 & \cdot & 4 \end{array} = 20 \qquad \begin{array}{ccc} 3 & \div & \frac{1}{8} \\ \downarrow & & \downarrow \\ 3 & \cdot & 8 \end{array} = 24 \qquad \begin{array}{ccc} 9 & \div & \frac{1}{7} \\ \downarrow & & \downarrow \\ 9 & \cdot & 7 \end{array} = 63$$

Notice that 1/4 inverted (upside down) is 4/1 or simply 4. We call 1/4 and 4 reciprocal numbers, or just reciprocals. So the shortcut is: multiply by the reciprocal of the divisor.

Does the shortcut make sense to you? For example, consider the problem 5 ÷ (1/4). Since 1/4 goes into 1 exactly four times, it must go into 5 exactly 5 · 4 = 20 times.

Two numbers are reciprocal numbers (or reciprocals) of each other if, when multiplied, they make 1.

$\frac{3}{4}$ is a reciprocal of $\frac{4}{3}$, because $\frac{3}{4} \cdot \frac{4}{3} = \frac{12}{12} = 1$.

$\frac{1}{7}$ is a reciprocal of 7, because $\frac{1}{7} \cdot 7 = \frac{7}{7} = 1$.

You can find the reciprocal of a fraction $\frac{m}{n}$ by flipping the numerator and denominator: $\frac{n}{m}$.

This works, because $\frac{m}{n} \cdot \frac{n}{m} = \frac{n \cdot m}{m \cdot n} = \frac{m \cdot n}{m \cdot n} = 1$.

To find the reciprocal of a mixed number or a whole number, first write it as a fraction, then "flip" it.

Since $2\frac{3}{4} = \frac{11}{4}$, its reciprocal number is $\frac{4}{11}$. And since $28 = \frac{28}{1}$, its reciprocal number is $\frac{1}{28}$.

4. Find the reciprocal numbers. Then write a multiplication with the given number and its reciprocal.

a. $\frac{5}{8}$	**b.** $\frac{1}{9}$	**c.** $1\frac{7}{8}$	**d.** 32	**e.** $2\frac{1}{8}$
$\frac{5}{8} \cdot \frac{\quad}{\quad} = 1$	$\frac{\quad}{\quad} \cdot \frac{\quad}{\quad} = 1$	$\frac{\quad}{\quad} \cdot \frac{\quad}{\quad} = 1$	$32 \cdot \frac{\quad}{\quad} = 1$	$\frac{\quad}{\quad} \cdot \frac{\quad}{\quad} = 1$

5. Write a division sentence to match each multiplication above.

a. $1 \div \frac{\quad}{\quad} = \frac{\quad}{\quad}$	**b.** $1 \div \frac{\quad}{\quad} = \frac{\quad}{\quad}$	**c.** $1 \div \frac{\quad}{\quad} = \frac{\quad}{\quad}$	**d.** ___ $\div \frac{\quad}{\quad} = \frac{\quad}{\quad}$	**e.** ___ $\div \frac{\quad}{\quad} = \frac{\quad}{\quad}$

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

Coordinate Grid

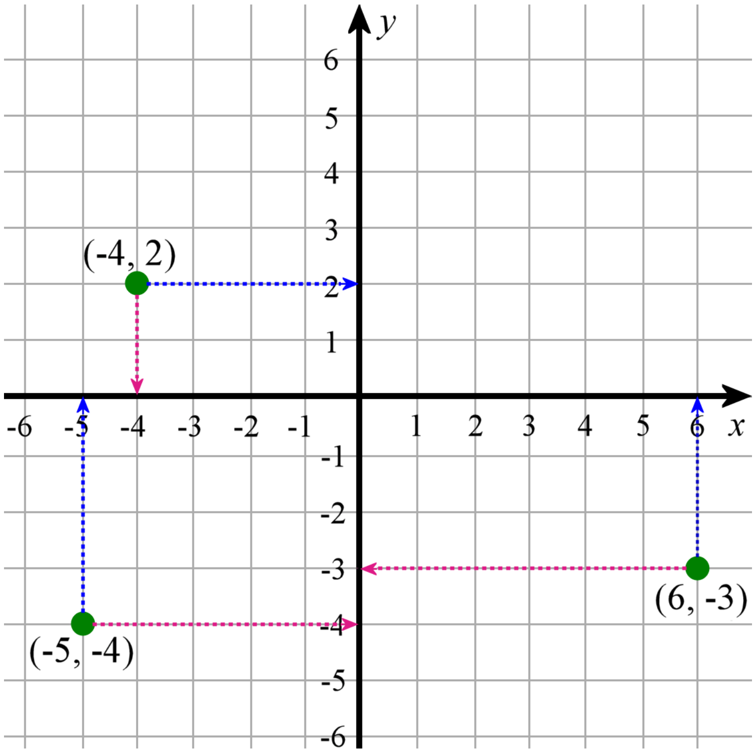
This is the *coordinate grid* or *coordinate plane*. We have extended the *x*-axis and the *y*-axis to include negative numbers now. The axes cross each other at the *origin*, or the point (0, 0).

The axes divide the coordinate plane into four parts, called *quadrants*. Previously, you have worked in only the so-called first quadrant, but now we will use all four quadrants.

The coordinates of a point are found in the same manner as before. Draw a vertical line (either up or down) from the point towards the *x*-axis. Where this line crosses the *x*-axis tells you the point's *x*-coordinate.

Similarly, draw a horizontal line (either right or left) from the point towards the *y*-axis. Where this line crosses the y-axis tells you the point's *y*-coordinate.

We list first the point's *x*-coordinate and then the *y*-coordinate. Look at the examples in the picture.



1. Write the *x*- and *y*-coordinates of the points.

A (____ , ____)

B (____ , ____)

C (____ , ____)

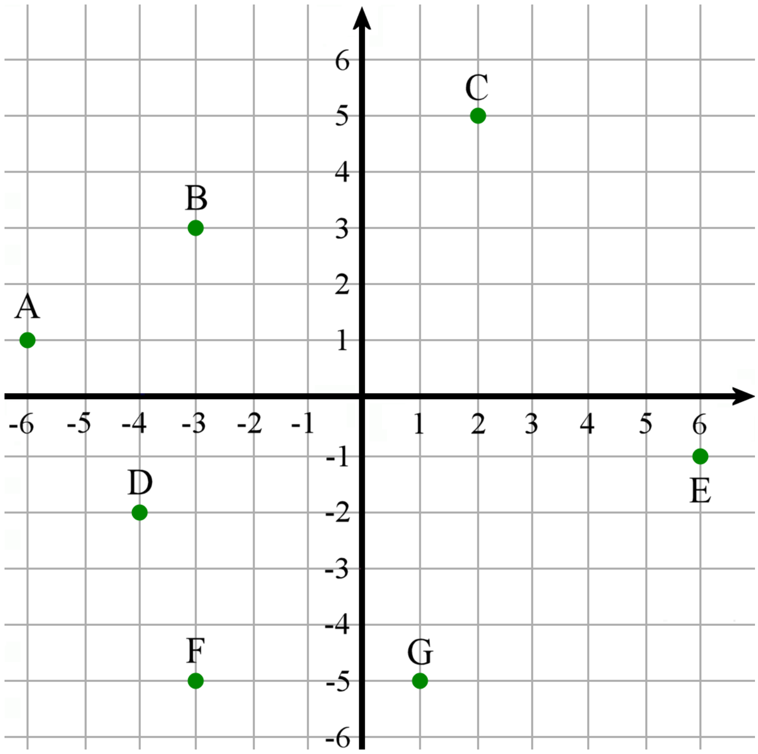
D (____ , ____)

E (____ , ____)

F (____ , ____)

G (____ , ____)

Self-check: Add the *x*-coordinates of all points. You should get −7.



Sample worksheet from https://www.mathmammoth.com

©2022 Taina Miller

[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

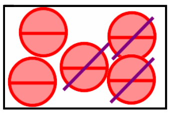
Subtracting a Negative Integer

We have already looked at such subtractions as 3 − 5 or −2 − 8, which you can think of as number line jumps. But what about **subtracting a negative integer?** What is 5 − (−4)? Or (−5) − (−3)?

Let's look at this kind of expression with a "double negative" in several different ways.

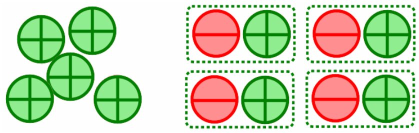
1. Subtraction as "taking away":

We can model subtracting a negative number using counters. (−5) − (−3) means we start with 5 negative counters, and then we *take away* 3 negative counters. That leaves 2 negatives, or −2.



5 − (−4) cannot easily be modelled that way, because it is hard to take away 4 negative counters when we do not have any negative counters to start with. But you *could* do it this way:

Start out with 5 positives. Then *add* four positive-negative pairs, which is just adding zero! Now you can take away four negatives. You are left with nine positives.



Start out with 5.

Add four positive-negative pairs, which amount to zero.

Lastly, cross out four negatives. You are left with nine positives.

2. Subtracting a negative number as a number line jump:

5 − (−4) is like standing at 5 on the number line, and getting ready to subtract, or go to the left. But, since there is a minus sign in front of the 4, it "turns you around" to face the positive direction (to the right), and you take 4 steps to the right instead. So, 5 − (−4) = 5 + 4 = 9.

(−5) − (−3) is like standing at −5, ready to go to the left, but the minus sign in front of 3 turns you "about face," and you take 3 steps to the right instead. You end up at −2.

3. Subtraction as a difference/distance:

To find the difference or distance between 76 and 329, subtract 329 − 76 = 253 (the smaller-valued number from the bigger-valued one). If you subtract the numbers the other way, 76 − 329, the answer is −253.

By the same analogy, we can think of 5 − (−4) as meaning the difference or distance between 5 and −4. From the number line we can see the distance is **9**.

(−5) − (−3) *could* be the distance between −5 and −3, except it has the larger number, −3, subtracted from the smaller number, −5.

If we turn them around, (−3) − (−5) would give us the distance between those two numbers, which is 2. Then, (−5) − (−3) would be the opposite of that, or −2.

Two negatives make a positive!

You have probably already noticed that, any way you look at it, we can, in effect, replace those two minuses in the middle with a + sign.
In other words, 5 − (−4) has the same answer as 5 + 4.
And (−5) − (−3) has the same answer as −5 + 3.
It may look a bit strange, but it works out really well.

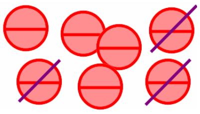
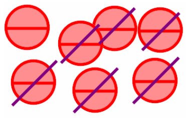
5 − (−4)
5 + 4 = 9

(−5) − (−3)
(−5) + 3 = −2

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

1. Write a subtraction sentence to match the pictures.

a. 	**b.** 

2. Write an addition or subtraction sentence to match the number line movements.

a. You are at −2. You jump 6 steps to the left.

b. You are at −2. You get ready to jump 6 steps to the left, but turn around at the last minute and jump 6 steps to the right instead.

3. Find the distance between the two numbers. Then, write a matching subtraction sentence. To get a positive distance, remember to *subtract the <u>smaller</u> number from the <u>bigger</u> number*.

a. The distance between 3 and −7 is _______. Subtraction: _______ − _______ = _______	**b.** The distance between −3 and −9 is _______. Subtraction: _______ − _______ = _______
c. The distance between −2 and 10 is _______. Subtraction: _______ − _______ = _______	**d.** The distance between −11 and −20 is _______. Subtraction: _______ − _______ = _______

4. Solve. Remember the shortcut: you can change each double minus "− −" into a plus sign.

a. −8 − (−4) =	**b.** −1 − (−5) =	**c.** 12 − (−15) =
8 − (−4) =	1 − (−5) =	−12 + 15 =
−8 + (−4) =	−1 − 5 =	−12 − 15 =
8 + (−4) =	1 − 5 =	12 + (−15) =

5. Connect with a line the expressions that are equal (have the same value).

a.		**b.**	
10 − (−3)	10 − 3	−9 + 2	−9 + (−2)
10 + (−3)	10 + 3	−9 − 2	−9 − (−2)

6. Write an integer addition or subtraction to describe the situations.

a. A roller coaster begins at 27 m above ground level. Then it descends 32 metres.

b. Matt has $25. He wants to buy a bicycle from his friend that costs $40. How much will he owe his friend?

Solve −1 + (−2) − (−3) − 4.

Puzzle Corner

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

Graphing

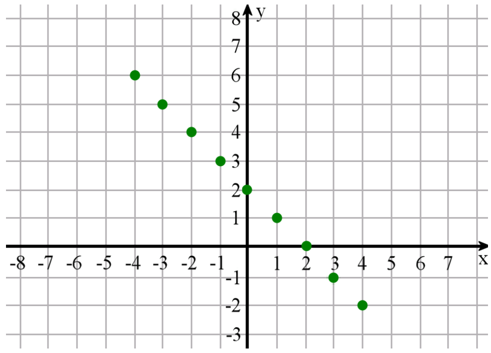
Remember? When an equation has two variables, there are many values of x and y that make that equation true.

Example. Note the equation $y = 2 - x$. If $x = 0$, then we can <u>calculate</u> the value of y using the equation: $y = 2 - 0 = 2$.

So, when $x = 0$ and $y = 2$, that equation is true. We can plot the number pair (0, 2) on the coordinate grid.

Some of the other (x, y) values that make the equation true are listed below, and they are plotted on the right.

x	−4	−3	−2	−1	0	1	2	3	4
y	6	5	4	3	2	1	0	−1	−2

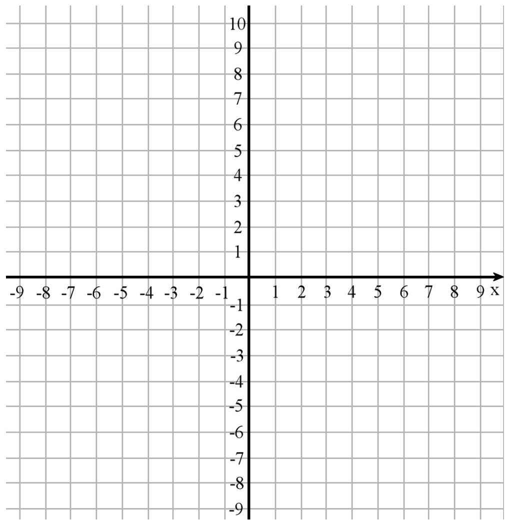


1. Plot the points from the equations. Graph both (b) and (c) in the same grid.

a. $y = x + 4$

x	−9	−8	−7	−6	−5	−4	−3	−2
y								

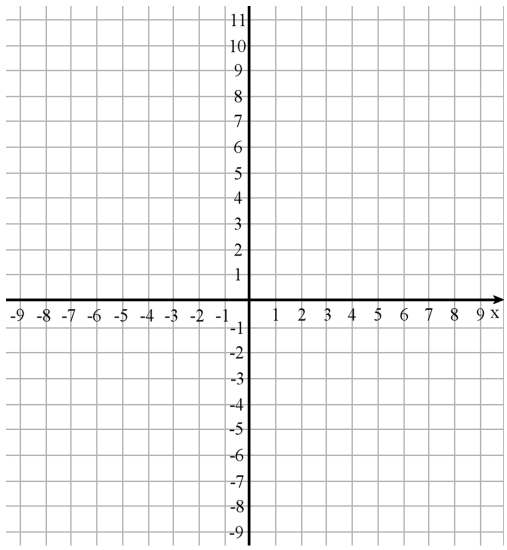
x	−1	0	1	2	3	4	5	6
y								



b. $y = 6 - x$

x	−3	−2	−1	0	1	2	3
y							

x	4	5	6	7	8	9
y						



c. $y = x - 2$

x	−5	−4	−3	−2	−1	0	1	2
y								

x	3	4	5	6	7	8	9
y							

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

Polygons in the Coordinate Grid

Here is a neat way to **find the area of any polygon whose vertices are points in a grid.**

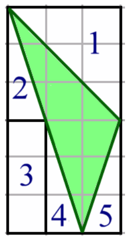
(1) Draw a rectangle around the polygon.

(2) Divide the area between the polygon and the rectangle into triangles and rectangles.

(3) Calculate those areas.

(4) Subtract the calculated areas from the total area of the large rectangle to find the area of the polygon.

Example. To find the area of the coloured triangle, we draw a rectangle around it that is 3 units by 6 units. Then we find the areas marked with 1, 2, 3, 4, and 5:

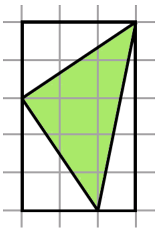


1: a triangle; $3 \cdot 3 \div 2 = 4.5$ square units
2: a triangle; $1 \cdot 3 \div 2 = 1.5$ square units
3: a rectangle; $1 \cdot 3 = 3$ square units
4: a triangle; $1 \cdot 3 \div 2 = 1.5$ square units
5: a triangle; $1 \cdot 3 \div 2 = 1.5$ square units
The total for the shapes 1, 2, 3, 4, and 5 is 12 square units.

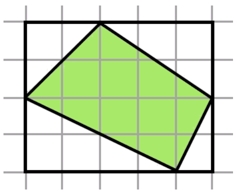
Therefore, the area of the coloured triangle is 18 square units − 12 square units = 6 square units.

1. Find the areas of the shaded figures.

a.

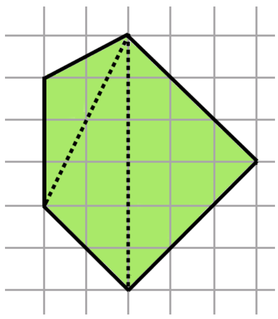


b.



2. This figure is called a _______________________.

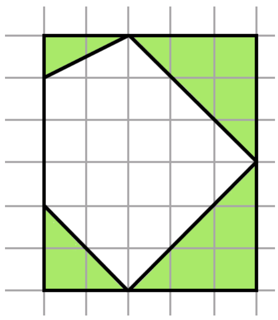
   Calculate its area using the three triangles.
   For each triangle, use the *vertical* side as the base.



3. Let's use another way of calculating the area of the same figure.

   1. Calculate the area of the rectangle that encloses the figure.
   2. Calculate the areas of the four shaded triangles.
   3. Subtract.

   Verify that you get the same result as in exercise #1.



Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

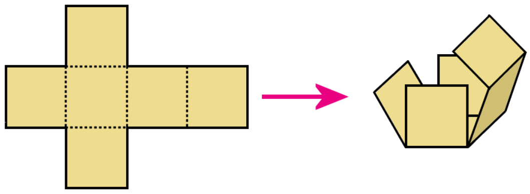
[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

Nets and Surface Area 1

This picture shows a flat figure, called a **net**, that can be folded up to form a solid, in this case a cube.

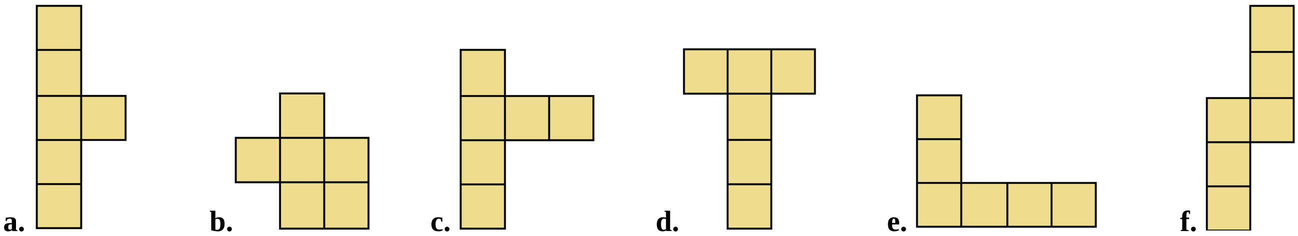


Each face of a cube is a square. If we find the total area of its faces, we will have found the **surface area** of the cube.

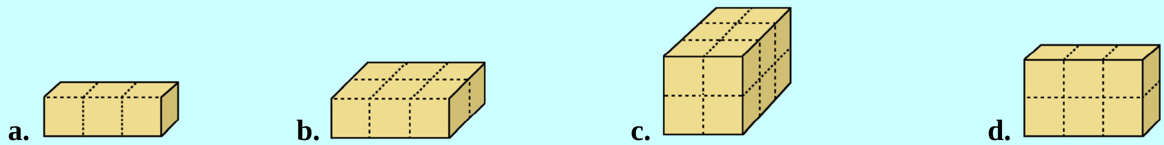
Let's say that each edge of this cube measures 2 cm. Then one face would have an area of 2 cm · 2 cm = 4 cm^2, and the total surface area of the six faces of the cube would be 6 · 4 cm^2 = 24 cm^2.

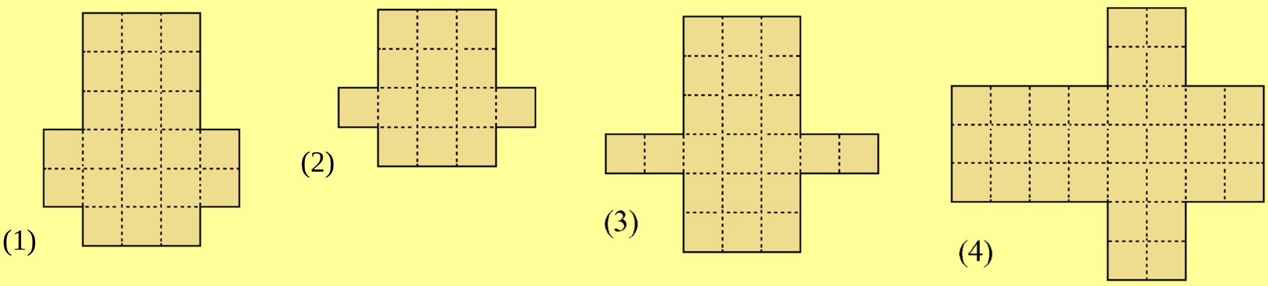
What is its volume? Remember, **volume** has to do with how much space a figure takes up, and not with "flat" area. Volume is measured in *cubic* units, whereas area is measured in *square* units. The volume of this cube is 2 cm · 2 cm · 2 cm = (2 cm)^3 = 8 cm^3.

1. Which of these patterns are nets of a cube? In other words, which ones can be folded into a cube? You can copy the patterns on paper, cut them out and fold them.



2. Match each rectangular prism (a), (b), (c) and (d) with the correct net (1), (2), (3) and (4). Again, if you would like, you can copy the nets onto paper, cut them out, and fold them.





3. Find the surface area (A) and volume (V) of each rectangular prism in problem #2 if the edges of the little cubes are 1 cm long.

a. A = ________ cm^2 **b.** A =________ cm^2 **c.** A =________ cm^2 **d.** A =________ cm^2

V = ________ cm^3 V = ________ cm^3 V = ________ cm^3 V = ________ cm^3

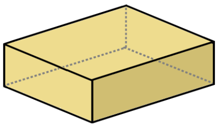
Sample worksheet from https://www.mathmammoth.com

©2022 Taina Miller

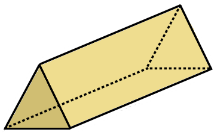
A **prism** has two identical polygons as its top and bottom faces. These polygons are called the *bases* of the prism. The bases are connected with faces that are parallelograms (and often rectangles).

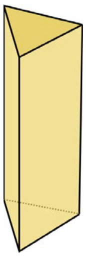
Prisms are named after the polygon used as the bases.

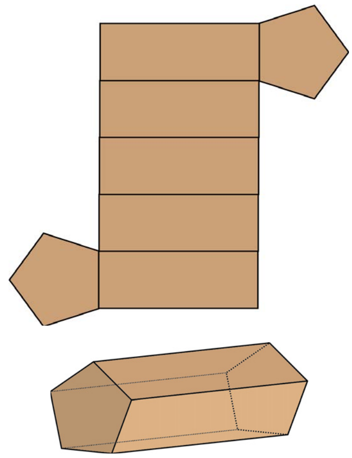
A rectangular prism.
The bases are rectangles.



Two **triangular prisms**.
One is lying down, where the base is facing you.
The other is "standing up".



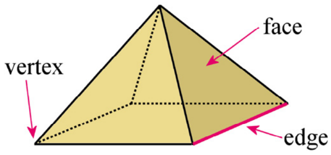


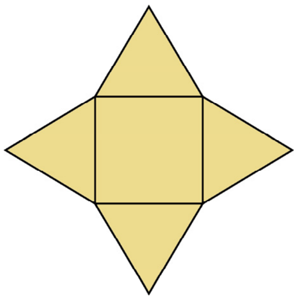


A pentagonal prism and its net. The bases are pentagons. Again, the base is not "on the bottom" but facing you.

A **pyramid** has a polygon as its bottom face (the base), and triangles as other faces.

Pyramids are named after the polygon at the base: a triangular pyramid, square pyramid, rectangular pyramid, pentagonal pyramid, and so on.



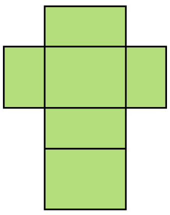


A **square pyramid** and its net.

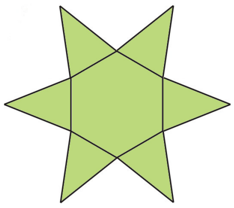
See interactive solids and their nets at the link below:
https://www.mathsisfun.com/geometry/polyhedron-models.html

4. Name the solid that can be built from each net.

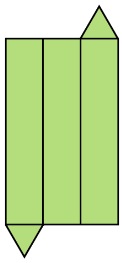
a.



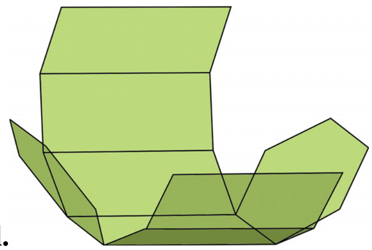
b.



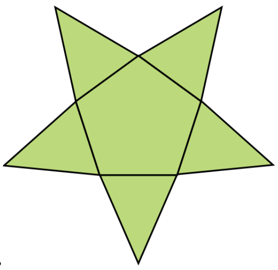
c.



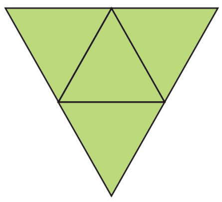
d.



e.



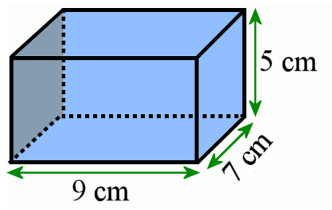
f.



Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

5. Which expression, (1), (2), or (3), can be used to calculate the surface area of this prism correctly? (You do *not* have to actually calculate the surface area.)

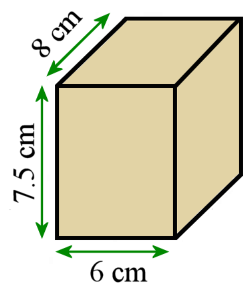


1. $2 \cdot 35\ \text{cm}^2 + 2 \cdot 63\ \text{cm}^2 + 2 \cdot 45\ \text{cm}^2$

2. $5\ \text{cm} \cdot 9\ \text{cm} \cdot 7\ \text{cm}$

3. $5\ \text{cm} \cdot 7\ \text{cm} + 9\ \text{cm} \cdot 7\ \text{cm} + 9\ \text{cm} \cdot 5\ \text{cm}$

6. Ryan organised the calculation of the surface area of this prism into three parts. Write down the intermediate calculations, and solve. This way, your teacher (or others) can follow your work. Remember also to include the units (cm or cm^2)!



Top and bottom:

Back and front:

The two sides:

Total:

7. The surface area of a cube is 96 square centimetres.

a. What is the area of one face of the cube?

b. How long is each edge of the cube?

c. Find the volume of the cube.

Puzzle Corner

Consider the rectangular prisms in problem #2. If the edges of the little cubes were double as long, how would that affect the surface area? Volume?

You can use the table below to investigate the situation.

Prism a.	**Prism b.**	**Prism c.**	**Prism d.**
A = ________ cm^2	A = ________ cm^2	A = ________ cm^2	A = ________ cm^2
V = ________ cm^3	V = ________ cm^3	V = ________ cm^3	V = ________ cm^3

Sample worksheet from
https://www.mathmammoth.com
©2022 Taina Miller

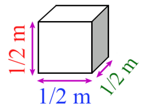
[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

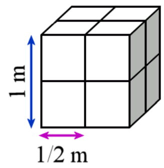
©2022 Taina Miller

Volume of a Rectangular Prism with Sides of Fractional Length

Example 1. Let's imagine that the edges of this little cube each measure ½ m.



If we stack eight of them so that we get a bigger cube… we get this:



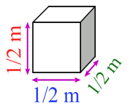
The bigger cube has <u>1 m edges, so its volume is 1 cubic metre</u>.

If eight identical little cubes make up this bigger cube, and its volume is 1 cubic metre, then <u>the volume of *one* little cube is 1/8 cubic metre</u>.

Notice: this is the same result that we get if we multiply the height, width and depth of the little cube:

$$\frac{1}{2}\text{ m} \cdot \frac{1}{2}\text{ m} \cdot \frac{1}{2}\text{ m} = \frac{1}{8}\text{ m}^3$$

1. The edges of each little cube measure ½ m. What is the total volume, in cubic metres, of these figures?



a.

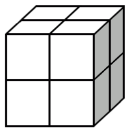
width = 1/2 m

height = ________ m

depth = ________ m

1 little cube,
1/8 m^3

V = __________ m^3



b.

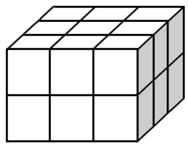
width = ________ m

height = ________ m

depth = ________ m

8 little cubes,
each 1/8 m^3

V = __________ m^3



c.

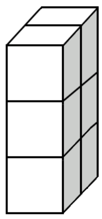
width = ________ m

height = ________ m

depth = ________ m

________ little cubes,
each 1/8 m^3

V = __________ m^3



d.

width = ________ m

height = ________ m

depth = ________ m

________ little cubes,
each 1/8 m^3

V = __________ m^3

2. Write a multiplication (width · depth · height) to calculate the volume of the figures (c) and (d) above, and verify that you get the same result as above.

a. V = ________ m · ________ m · ________ m

=

b. V = ________ m · ________ m · ________ m

=

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

Understanding Distributions

A **statistical question** is a question where we expect a range of *variability* in the answers to the question.

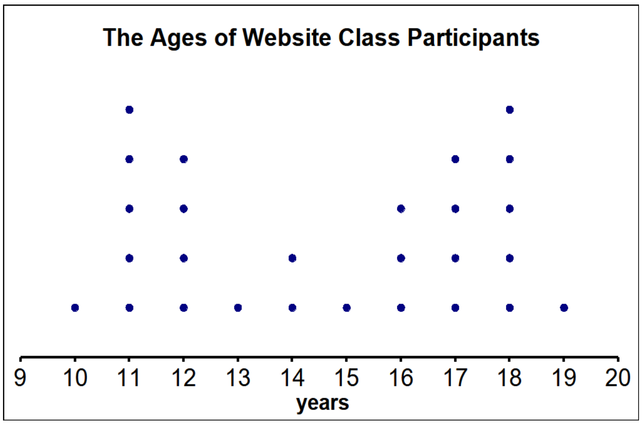
For example, “How old am I?” is *not* a statistical question (there is only one answer), but “How old are the students in my school?” *is* a statistical question because we expect the students’ ages not to be all the same.

“How much does this TV cost?” is *not* a statistical question because we expect there to be just one answer.

“How much does this TV cost in various stores around town?” *is* a statistical question, because we expect a number of different answers: the prices in different stores will vary.

To answer a statistical question we collect a set of **data** (many answers). The data can be displayed in some kind of a graph, such as a bar graph, a histogram, or a dot plot.

This is a **dot plot** showing the ages of the participants in a website-building class. Each dot in the plot signifies one observation. For example, we can see there was one 13-year old and two 14-year olds in the class.



The dot plot shows us the **distribution** of the data: it shows how many times (the frequency) each particular value (age in this case) occurs in the data.

This distribution is actually **bimodal,** or “double-peaked”. This means it has two “centres”: one around 11-12 years, and another around 17-18 years.

1. Are these statistical questions? If not, change the question so that it becomes a statistical question.

a. What colour are my teacher’s eyes?

b. How much money do the students in this university spend for lunch?

c. How much money do working adults in Romania earn?

d. How many children in the United States use a cell phone regularly?

e. What is the minimum wage in Ohio?

f. How many sunny days were there in August, 2020, in London?

g. How many pets does my friend have?

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

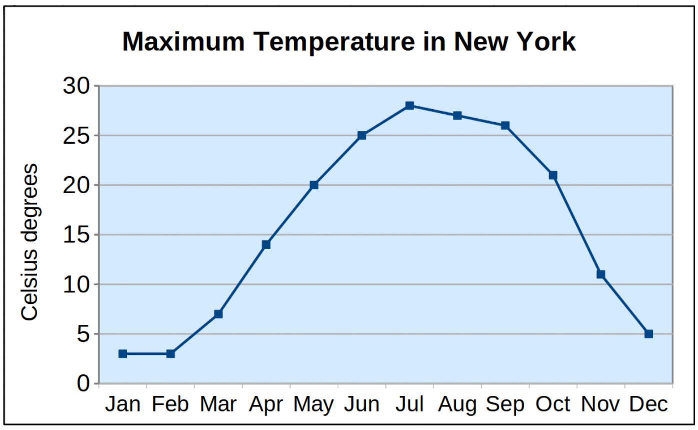
2. Is this graph based on a statistical question?

   Why or why not?

3. The line graph shows the maximum temperatures in New York for each month of a certain year.

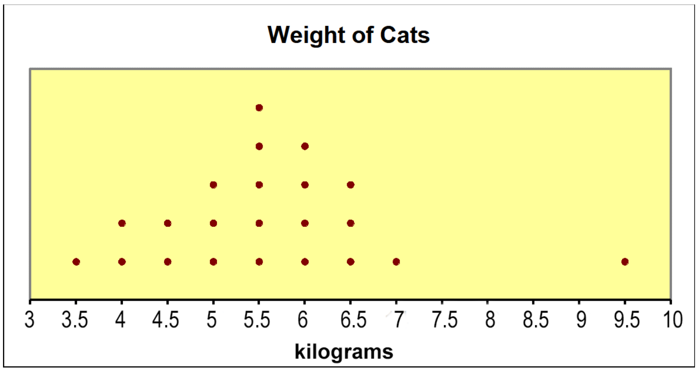
   Is this graph based on a statistical question?

   Why or why not?



4. The title of this dot plot is not the best. But, could the plot be based on a statistical question?

   If yes, give it a better, more specific, title. Imagine what situation and what question might have produced the data.



5. Change each question from a non-statistical question to a statistical question, and vice versa.

   a. What shampoo do you use?

   b. How cold was it yesterday where you live?

   c. How old are people in Germany when they marry (the first time)?

   d. How long does it take for our company’s packages to reach the customers?

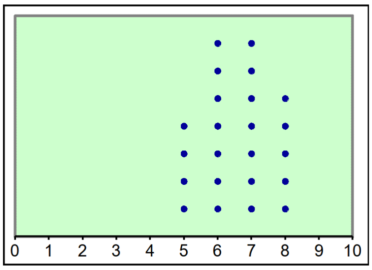
Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

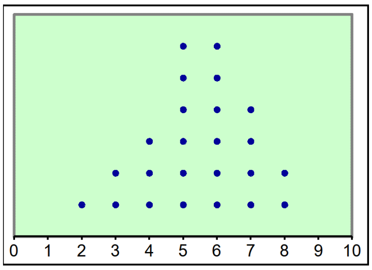
We are often interested in the **centre**, **spread** and **overall shape** of the distribution. Those three things can summarise for us what is important about the distribution.

The **centre** of a distribution has to do with where its peak is. We can use mean, median and mode to characterise the central tendency of a distribution. We will study those in detail in the next lesson.

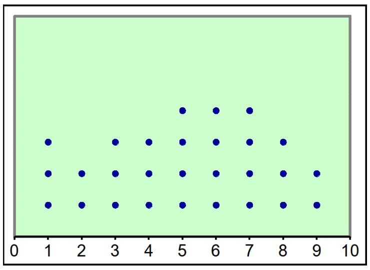
These three dot plots show how the **spread** of a distribution can vary. This means how the data items themselves are spread—whether they are "spread" all over, or tightly concentrated near some value, or somewhat concentrated around some value. We will study more about spread in another lesson.



little spread

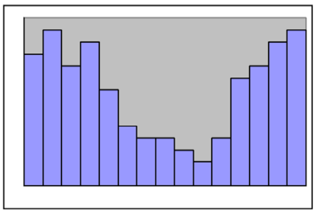


medium spread

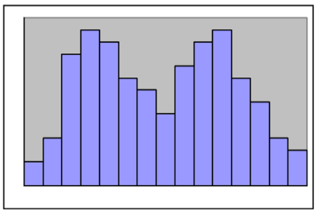


large spread

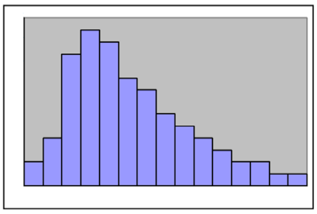
The distribution can have many varying overall **shapes.** For example:



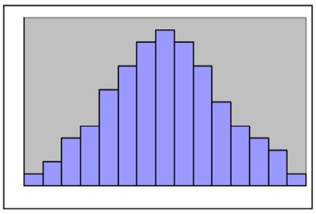
U-shaped



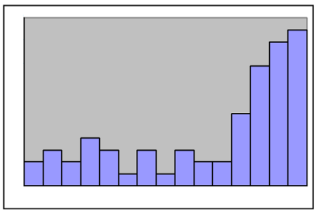
double-peaked (bimodal)



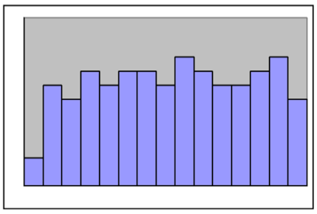
asymmetrical, right-tailed
(a.k.a. right-skewed)



bell-shaped



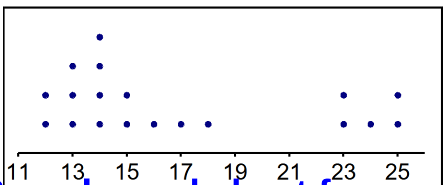
J-shaped
(can also be mirrored where most of the values are at the left)



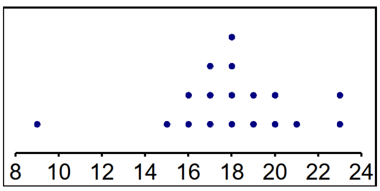
rectangular

In addition to its overall shape, a distribution may have a gap, an outlier, or a cluster:

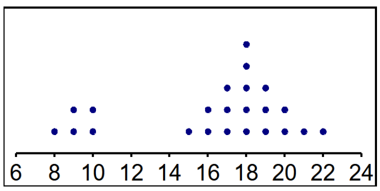
This distribution has a **gap** from 19 to 22:



In this distribution, 9 is an **outlier —** a data item whose value is considerably less or more than all the others.



This distribution has a bell shape overall (with a peak at 18), but also a **cluster** or a smaller peak at 8-10.



Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

Boxplots

Boxplots or **box-and-whisker plots** are simple graphs on a number line that use a box with “whiskers” to visually show the quartiles of the data. Boxplots show us a **five-number summary** of the data: the minimum, the 1st quartile, the median, the 3rd quartile and the maximum.

Example 1. These are prices of hair dryers in three stores (in dollars).

14 15 19 20 20 20 21 24 25 34 34 35 35 37 42 45 55

Five-number summary:

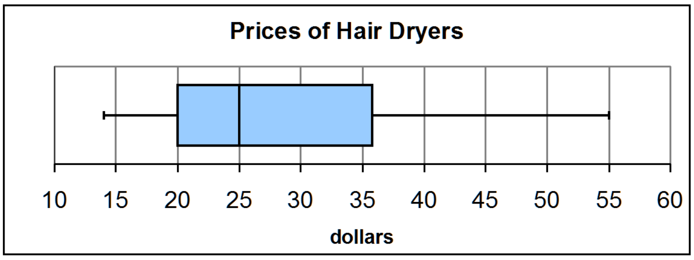
Minimum: $14

First quartile: $20

Median: $25

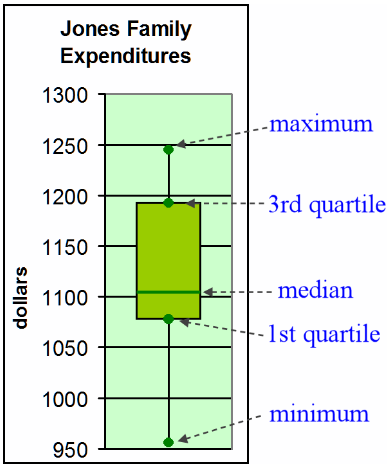
Third quartile: $36

Maximum: $55



The box itself starts at the 1st quartile and ends at the 3rd quartile. Therefore, its width is the interquartile range. We draw a line in the box marking the median ($25).

The boxplot also has two “whiskers”. The first whisker starts at the minimum ($14) and ends at the first quartile. The other whisker is drawn from the third quartile to the maximum ($55).



Example 2. This boxplot shows the Jones family’s monthly expenditures over a 12-month period. This time the boxplot is drawn vertically.

The box extends over half the data. This means that half the time, they spend from about $1 080 to $1 190 monthly. But sometimes they spend only about $950, and sometimes up to $1 245 in a month.

Five-number summary:

Minimum: $956

First quartile: $1 078

Median: $1 105

Third quartile: $1 193

Maximum: $1 245

1. **a.** Read the five-number summary from the boxplot.

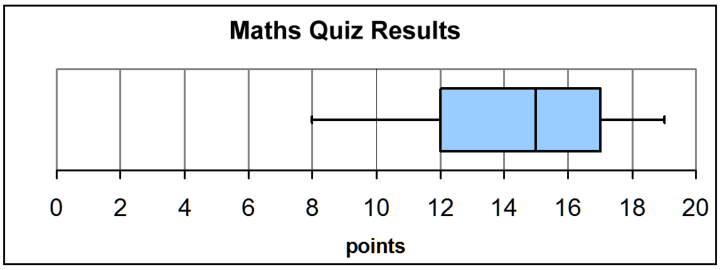
Minimum:

First quartile:

Median:

Third quartile:

Maximum:



b. Look at the box and fill in: Half the students got between _____ and _____ points in the quiz.

c. Do you think the quiz went well?

Sample worksheet from https://www.mathmammoth.com

©2022 Taina Miller

[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

Making Histograms

Histograms are like bar graphs, but the bars are drawn so they touch each other. Histograms are used only with numerical data.

Example. These are the prices of hair dryers in three stores again (in dollars). Make a histogram.

14 15 19 20 20 20 21 24 25 34 34 35 35 37 42 45 55

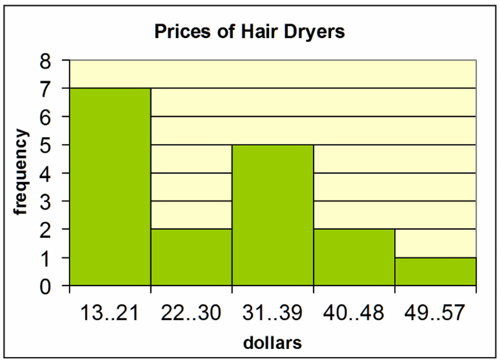
We need to decide how many bins to make and how "wide" they are. For that, we first calculate the **range**, or the difference between the greatest and smallest data item. It is 55 − 14 = 41. Then we divide the range into equal parts (bins) to get the *approximate* bin width.

If we make five bins, we get 41 ÷ 5 = 8.2 for the bin width. The bins would be 8.2 units apart. However, in this case it is nice to have the bins go by whole numbers, so we round 8.2 up to 9 and use 9 for the bin width.

The important part is that *each data item needs to be in one of the bins*. You may have to try out slightly different bin widths and starting points to see how it works. This time, starting the first bin at 13 makes the last bin to end at 57, which works, because the data will "fit" into the bins. (Starting at 14 would work, too.)

The **frequency** describes *how many data items fall into that bin*. Lastly, all we need is to draw the histogram, remembering that the bars touch each other.

Price ($)	Frequency
13..21	7
22..30	2
31..39	5
40..48	2
49..57	1



This is a **double-peaked** distribution and **skewed to the right** (the direction of skewness is where the "long tail" of the distribution is; in this case to the right). Since it definitely is *not* bell-shaped, the mean is *not* a good measure of centre. Therefore, the *median* is the better choice for measure of centre.

Consequently, we need to use *interquartile range*, and not mean absolute deviation, as a measure of variation.

The median is underlined below:

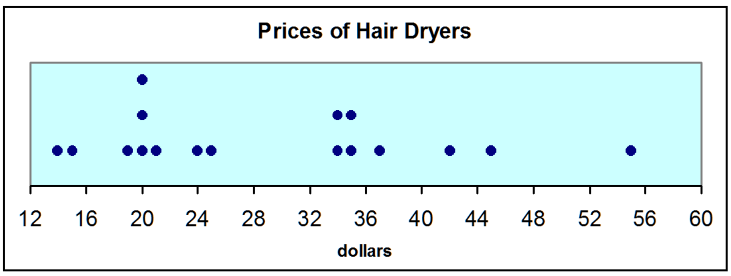
14 15 19 20 20 20 21 24 **<u>25</u>** 34 34 35 35 37 42 45 55

The 1st quartile is $20 and the 3rd quartile is $36 (verify those). So the interquartile range is $16.

This means that half of the data is found between 20 and 36 dollars. This price range is quite large for devices with a median price of only $25. A large range compared to the median describes data that is widely scattered. (We can also see that from the dot plot.)

Since the median is $25, which is nearer the low end of the interval from $20 to $36, the prices are somewhat more concentrated in the lower end of that interval.

For a comparison, look at the dot plot as well. It has a similar shape to the histogram.



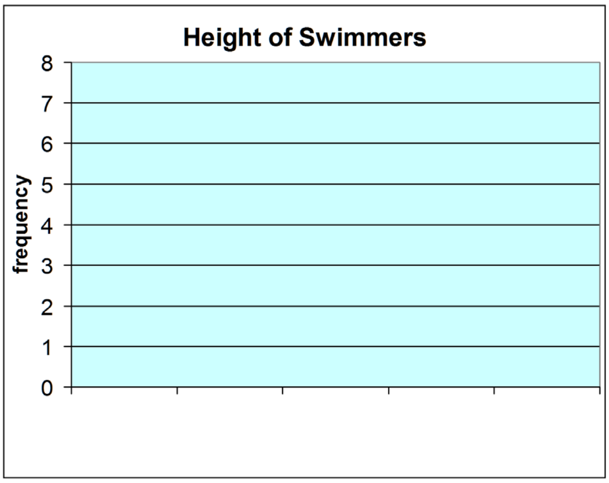
Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

1. This data lists the heights of 24 swimmers in centimetres. Make a histogram with <u>five</u> bins.

155 155 156 157 158 159 159 160 162 162 163 163
164 165 166 167 167 168 168 170 172 174 175 177

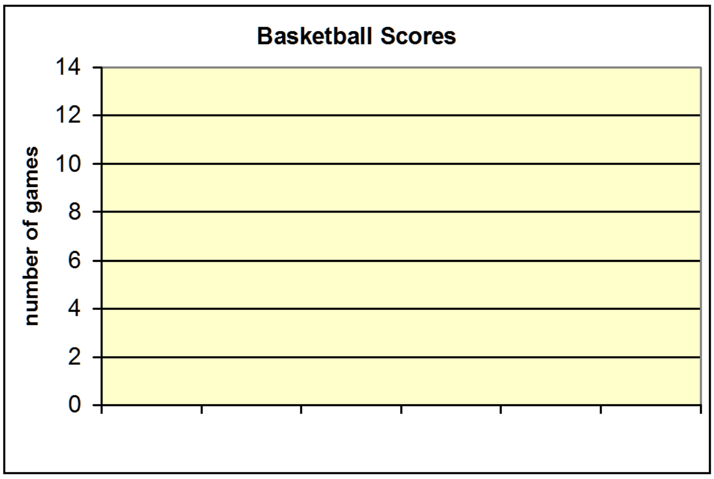
Height (cm)	Frequency



2. Make a histogram from this data, which lists all the scores a basketball team had in the games in one season. Make six bins.

60 62 68 71 72 72 73 74 74 74 75 75 76 77 77 77
78 78 78 79 79 81 81 82 83 83 85 86 88 90 92 95

Score	Frequency



3. **a.** As this distribution in problem 2 has its peak near the centre, you could use either mean or median as a measure of centre. This time, find the median and the interquartile range.

median ____________ interquartile range ____________

b. Describe the variation in the data. Is it very scattered (varied), somewhat so, or not very much so?

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

Stem-and-Leaf Plots

This lesson is optional.

A stem-and-leaf plot is made using the numbers in the data, and it looks a little bit like a histogram or a dot plot turned sideways.

In this plot, the tens digits of the individual numbers become the **stems**, and the ones digits become the **leaves**. For example, the second row 2 | 1 2 5 8 actually means 21, 22, 25, and 28. Notice how the leaves are listed in order from the smallest to the greatest.

Ages of the participants in the County Fair Karaoke Contest:

14 18 21 22 25 28 30 30
31 33 33 36 37 40 45 58

Stem	Leaf
1	4 8
2	1 2 5 8
3	0 0 1 3 3 6 7
4	0 5
5	8

4 | 5 means 45

Since stem-and-leaf plots show not only the *shape* of the distribution but also the individual *values*, they can be used to get a quick overview of the data. This distribution has a central peak and is somewhat skewed to the right.

You can also find the median fairly easily because you can follow the individual values from the smallest to the largest, and find the middle one.

Stem-and-leaf plots are most useful for numerical data sets that have 15 to 100 individual data items.

1. **a.** Complete the stem-and-leaf plot for this data:

   19 20 34 25 21 34 14 20 37 35 20 24 35 15 45 42 55

   (prices of hair dryers in three stores)

   b. What is the median?

Stem	Leaf
1	
2	
3	
4	
5	

5 | 4 means 54

2. **a.** Complete the stem-and-leaf plot for this data. This time, the stems are the first two digits of the numbers, and the leaves are the last digits.

   709 700 725 719 750 740 757 745 786 770 728 755

   (monthly rent, in dollars, for one-bedroom apartments in Houston, Texas)

   b. Find the median monthly rent.

   c. Find the interquartile range.

   d. Describe the spread of the distribution
   (is the data spread out a lot, a medium amount, a little, etc.)

Stem	Leaf
70	
71	
72	
73	
74	
75	
76	
77	
78	
79	

71 | 9 means 719

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

[This page is intentionally left blank.]

Sample worksheet from
https://www.mathmammoth.com

©2022 Taina Miller

Statistics Revision

You may use a calculator in all questions in this lesson.

1. Are these statistical questions or not? If not, change the question so that it becomes a statistical question.

   a. Which kind of books do the visitors of this library like the best?

   b. How many pages are in the book *How to Solve It* by G. Polya?

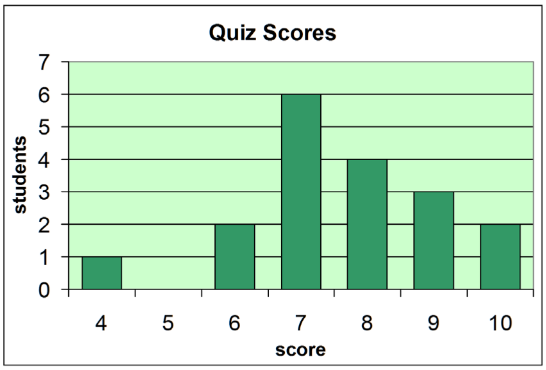
2. **a.** Find the mean, median and mode.
   Hint: recreate the list of the original data.

   Mean:

   Median:

   Mode:

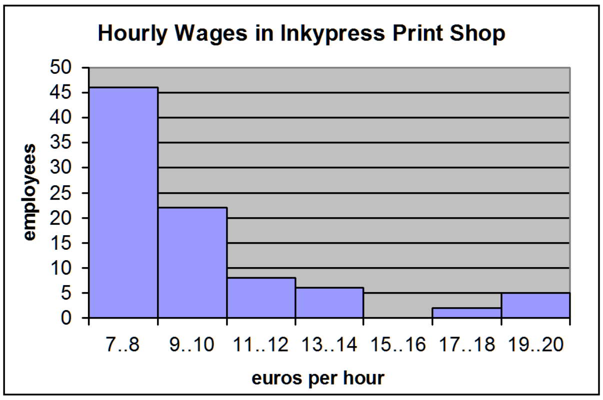
   b. Describe the shape and any striking features of the distribution.



3. This graph shows the hourly wages in euros per hour of the 89 employees in the Inkypress Print Shop.

   a. *About* what fraction of the people earn 7-8 euros/hour?

   b. Describe the shape and any striking features of the distribution.



   c. The mean is 9.66 euros/hour and the median is 8 euros/hour.
   Which is better in describing the majority's wages in this print shop?

   d. Which measure of variation should be used to describe this data, based on your answer to (c)?

Sample worksheet from https://www.mathmammoth.com

©2022 Taina Miller