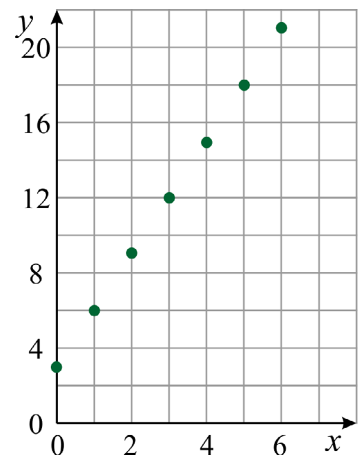
Linear Functions and the Rate of Change 1

If the graph of a function consists of points that fall on a single line, it is a **linear function**.

We will define a linear function in a different manner later, but for now, this is sufficient, so let's look at some examples.

Example 1. The input and output values in the table below define a function. Notice the patterns: the x-values increase by ones, and the y-values increase by 3s.

Input (x)	0	1	2	3	4	5	6
Output (y)	3	6	9	12	15	18	21



The graph shows that the points fall on a line. This is a linear function.

The **rate of change** of a function is the rate at which the output values change as compared to the change in the input values.

We calculate it as the ratio of $\dfrac{\text{change in output values}}{\text{change in input values}}$.

In the context of this graph, **rate of change** = $\dfrac{\text{difference in } y\text{-values}}{\text{difference in } x\text{-values}}$.

In this case, each time the x-values increase by 1, the y-values increase by 3. **The rate of change is 3/1 = 3**.

Example 2. The price of bananas is a function of their weight. What is the rate of change?

Weight in kg (input)	0	2	5	10	12	15
Price in $ (output)	0	5	12.50	25	30	37.50

Check how much the output (price) changes for a certain change in the input (the weight). For example, when the weight increases from 0 to 2 kg, the price increases from $0 to $5, or by $5. This happens also when the weight increases from 10 to 12 kg: the price increases $5 (from $25 to $30).

$$\text{Rate of change} = \frac{\$5}{2\text{ kg}} = \$2.50/\text{kg}$$

Note that if the independent and dependent variables have units, **we include the units in the rate of change**.

This rate of change tells us that for each one-kilogram increase in weight, the price increases by $2.50.

1. **a.** Calculate the rate of change in example 2, using the increase in weight from 5 to 10 kg, and the corresponding increase in price. Do you get the same rate of change as calculated in the example?

b. Do the same using the input values 10 kg and 15 kg.

2. What is the rate of change? Don't forget the units!

a.

Input (*t*)	2 hrs	3 hrs	4 hrs	5 hrs	6 hrs	7 hrs
Output (*d*)	$30	$45	$60	$75	$90	$105

b.

Input (*t*)	2 L	4 L	6 L	8 L
Output (*d*)	2.8 kg	5.6 kg	8.4 kg	11.2 kg

3. If a linear function contains the points (4, 15) and (9, 18), what is the rate of change?

4. A train travels at a constant speed, traveling 40 km in 20 minutes. Function D gives the distance (*d*) in kilometers that the train has traveled in *t* hours.

a. Fill in the output values.

***t* (hours)**	0 hrs	1 hr	2 hrs	3 hrs	4 hrs	5 hrs	6 hrs
***d* (km)**							

b. What is the rate of change? Use hours and kilometers.

5. Mr. Stevenson, a gardener, is being paid a base salary of $300 per week for taking basic care of the grounds at a college, plus $20 per hour for certain special tasks. We can model his weekly earnings (E) with the function $E = 300 + 20t$ where *t* is the number of hours he works at the special tasks.

a. How much does he get paid if he works five hours at the special tasks in a week?

b. How many hours would he need to work at the special tasks to earn $480 in a week?

c. What is the rate of change of this function?

6. Function D has the rate of change of (7 meters)/(20 minutes), and at 0 minutes, the output value is 0.5 meters.

a. Fill in the table.

Input (*t*)	0 min	10 min	20 min	30 min	40 min	50 min	60 min
Output (*d*)	0.5 m						

b. What could this depict?

7. The price of potatoes increases by $10 each time the weight increases by 5 kg.
How do the the rate of change and unit price compare in this situation?