Speed, Time, and Distance Problems

There are many jokes about algebra word problems where a train leaves a station at a certain hour. You can now solve these types of problems with your knowledge of systems of equations. One of the most effective ways to do so is to first build a chart.

Example 1. A train leaves a station at 9:00 AM and travels with a constant speed of 90 km/h. Another train leaves the same station 10 minutes later, traveling to the same direction at the speed of 100 km/h. At what time will the second train reach the first?

We will be using the formula $d = vt$ extensively in these problems. Let's build a chart. The goal is to have TWO, not three or more, variables present in the chart. The formula $d = vt$ has three variables, and since the speed, distance, and time can be different for each train, theoretically we could have six variables. However, invariably, the problem gives information for one or some of these variables, and something about the situation means that the distance or the time or the speed is the same for both trains.

To get started, we gather some information in the chart. The distance that train 1 and train 2 travel until they meet is the same, so that is why we use the same variable, d, for it.

	distance	*velocity*	*time*
Train 1	d	90 km/h	t_1
Train 2	d	100 km/h	t_2

The times (t_1 and t_2) are different, but we do know that they differ by 10 minutes, so, actually we will get by using only *one* variable for time, like this:

	distance	*velocity*	*time*
Train 1	d	90 km/h	t
Train 2	d	100 km/h	$t - 10$

The chart now contains only two variables. However, we have one more thing to change. The speed is in km/h, whereas the 10 has to do with minutes. For our equation to work, the time units need to be the same, so we will change the 10 to 1/6 (in hours).

	distance	*velocity*	*time*
Train 1	d	90 km/h	t
Train 2	d	100 km/h	$t - 1/6$

The equations always follow the same formula: $d = vt$, and we use that same formula for both Train 1 and Train 2. So, the two equations we get are:

$$\begin{cases} d = 90t \\ d = 100(t - 1/6) \end{cases}$$

The quickest way to solve this system is to set $90t$ equal to $100(t - 1/6)$ and solve for t.

1. Solve the system of equations from example 1 and answer the question: At what time will the second train reach the first? Is the answer surprising?

$$\begin{cases} d = 90t \\ d = 100(t - 1/6) \end{cases}$$

2. Your friend starts walking at a speed of 6 km/h from your home to his. Exactly 15 minutes later, you decide you want to join him so you take your bicycle and start after him, with a speed of 18 km/h. How far are you when you reach your friend?

	distance	***velocity***	***time***
Your friend			
You			

3. A tortoise and hare race a distance of 100 m. The hare gives the tortoise a 10-minute lead time. Then he quickly runs the 100 meters and wins the race. After the hare has finished, the tortoise takes an *additional* 6 minutes to reach the finish line. If the speed of the hare is 15 m/s, find the time the tortoise takes to finish the race and the tortoise's speed.

Hint: since the speed is in meters per second, and the distance is in meters, the time unit will be seconds.

	distance	***velocity***	***time***
Tortoise			
Hare			