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Foreword

Math Mammoth Grade 7 comprises a complete math curriculum for the seventh grade mathematics studies. This is a pre-algebra course, so students can continue to an algebra 1 curriculum after studying this.

The curriculum meets and actually exceeds the Common Core Standards (CCS) for grade 7. The two major areas where it exceeds those standards are linear equations (chapter 5) and the Pythagorean Theorem (chapter 9). Linear equations are covered in more depth than the CCS requires, and the Pythagorean Theorem belongs to grade 8 in the CCS. You can access a document detailing the alignment information either on the Math Mammoth website or in the download version of this curriculum.

The main areas of study in Math Mammoth Grade 7 are:

- The basics of algebra (expressions, equations, inequalities, graphing);
- Integers;
- Ratios, proportions, and percent;
- Geometry;
- Probability and statistics.

This book, 7-B, covers ratios and proportions (chapter 6), percent (chapter 7), geometry (chapter 8), the Pythagorean Theorem (chapter 9), probability (chapter 10), and statistics (chapter 11). The rest of the topics are covered in the 7-A worktext.

Some important points to keep in mind when using the curriculum:

- The two books (parts A and B) are like a “framework”, but you still have a lot of liberty in planning your student’s studies. The five chapters in part 7-A are best studied in the order presented. However, you can study the chapters on geometry, probability, and statistics at most any point during the year. The chapters on ratios & proportions and percent (in part 7-B) are best left until the student has learned to solve one-step equations (in chapter 3).

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. However, you can still use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously. This type of flexible use of the curriculum enables you to truly individualize the instruction for the student.

- Don’t automatically assign all the exercises. Use your judgment, trying to assign just enough for your student’s needs. You can use the skipped exercises later for review. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.
- For review, the curriculum includes a worksheet maker (Internet access required), mixed review lessons, additional cumulative review lessons, and the word problems continually require usage of past concepts. Please see more information about review (and other topics) in the FAQ at <https://www.mathmammoth.com/faq-lightblue.php>

I heartily recommend that you view the full user guide for your grade level, available at <https://www.mathmammoth.com/userguides/>

Lastly, you can find free videos matched to the curriculum at <https://www.mathmammoth.com/videos/>

I wish you success in teaching math!

Maria Miller, the author

Chapter 6: Ratios and Proportions

Introduction

Chapter 6 reviews the concept, which has already been presented in previous grades, of the ratio of two quantities. From this concept, we develop the related concepts of a rate (so much of one thing per so much of another thing) and a proportion (an equation of ratios). We also study how tables of equivalent ratios can help to solve problems with rates, and how cross-multiplying can help to solve problems with proportions.

The lesson *Unit Rates* defines the concept of the unit rate, shows how to calculate one, and gives practice at doing so, including practice with complex fractions. We also consider rates as two quantities that vary, graph the corresponding equation in the coordinate grid, and tie in the concept of unit rate with the concept of slope.

The concept of direct variation is introduced in the lesson *Proportional Relationships*. Writing and graphing equations gives a visual understanding of proportionality. In two following lessons, students also practice solving rate problems in different ways, using the various methods they have learned throughout the chapter.

The lessons *Scaling Figures, Floor Plans, and Maps* give useful applications and more practice to master the concepts of proportions.

Before the *Chapter Review* there is also an optional lesson, *Significant Digits*, that deals with the concept of the accuracy of a measurement and how it limits the accuracy of the solution. It is optional because significant digits is not a standard topic for seventh grade, yet the concept in it is quite important, especially in science.

Keep in mind that the specific lessons in the chapter can take several days to finish. They are not “daily lessons.” Instead, use the general guideline that seventh graders should finish about 12 pages a week in order to finish the curriculum in about 40 weeks. Also, I recommend not assigning all the exercises by default, but that you use your judgment, and strive to vary the number of assigned exercises according to the student’s needs.

Please see the user guide at <https://www.mathmammoth.com/userguides/> for more guidance on using and pacing the curriculum.

There are free videos matched to the curriculum at <https://www.mathmammoth.com/videos/> (choose 7th grade).

The Lessons in Chapter 6

	page	span
Ratios and Rates	12	3 pages
Solving Problems Using Equivalent Rates	15	2 pages
Solving Proportions: Cross Multiplying	17	6 pages
Why Cross-Multiplying Works	23	1 page
Unit Rates	24	5 pages
Proportional Relationships	29	6 pages
Graphing Proportional Relationships—More Practice	35	2 pages
More on Proportions	37	4 pages
Scaling Figures	41	6 pages
Floor Plans	47	4 pages
Maps	51	6 pages
Significant Digits	57	2 pages
Chapter 6 Mixed Review	59	3 pages
Chapter 6 Review	62	5 pages

Helpful Resources on the Internet

Ratios and Proportions—Video Lessons by Maria

A set of free videos about ratios, rates, and proportions—by the author.

<http://www.mathmammoth.com/videos/proportions.php>

RATIOS AND RATES

Ratio Pairs Matching Game

Match cards representing equivalent ratios.

Easy: <http://nrich.maths.org/4824> Challenge: <http://nrich.maths.org/4821>

All About Ratios - Quiz

An interactive five-question quiz about equivalent ratios presented with pictures

<http://math.rice.edu/~lanius/proportions/quiz1.html>

Rate Module from BrainiacCamp

A comprehensive interactive lesson on the concepts of ratio, rate, and constant speed (for 6th and 7th grades). Includes an animated lesson, a virtual manipulative, and questions and problems to solve.

<http://www.brainiaccamp.com/lessons/rates/>

Ratios Activity from BBC Bitesize

An animated tutorial about dividing in a given ratio and scale models with some quiz questions along the way.

<http://www.bbc.co.uk/education/guides/znnycdm/activity>

Ratios and Rates: Quick Quiz

Practice ratios and rates with this short interactive online quiz.

<http://wps.pearsoned.com.au/mfql1/0,7860,759792--759794,00.html>

Ratio Quiz from BBC Skillswise

A multiple-choice quiz about the concept of ratio. You can take the quiz online or download it as a PDF or doc file.

<http://www.bbc.co.uk/skillswise/quiz/ma19rati-e1and2-quiz>

Ratio Quiz from Syvum

A 10-question online quiz about ratios and problem solving.

http://www.syvum.com/cgi/online/mult.cgi/gmat/math_review/arithmetic_5.tdf?0

Unit Rates Involving Fractions

Practice computing rates associated with ratios of fractions or decimals in this interactive activity.

https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-ratio-proportion/cc-7th-rates/e/rate_problems_1

Ratio Word Problems

Reinforce your ratios skills with these interactive word problems.

https://www.khanacademy.org/math/pre-algebra/pre-algebra-ratios-rates/pre-algebra-ratio-word-problems/e/ratio_word_problems

Comparing Rates

Practice completing rate charts in this interactive online activity.

<https://www.khanacademy.org/math/pre-algebra/pre-algebra-ratios-rates/pre-algebra-rates/e/comparing-rates>

Free Ride

An interactive activity about bicycle gear ratios. Choose the front and back gears, which determines the gear ratio. Then choose a route, pedal forward, and make sure you land exactly on the five flags.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=178>

Exploring Rate, Ratio and Proportion (Video Interactive)

The video portion of this resource illustrates how these math concepts play a role in photography. The interactive component allows students to explore ratio equivalencies by enlarging and reducing images.

<http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.RATE>

Three-Term Ratios

Practice the equivalency of ratios by filling in the missing numbers in three-term ratios (for example, $2:7:5 = _ : 105 : _$) where the numbers represent the amounts of three colors in different photographs. Afterwards you get to assemble a puzzle from the nine photographs.

http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.RATE&lesson=html/object_interactives/3_term_ratio/use_it.html

[I=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.RATE&lesson=html/object_interactives/3_term_ratio/use_it.html](http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.RATE&lesson=html/object_interactives/3_term_ratio/use_it.html)

If the two links above don't work, use this link:

<http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB>

First choose Rate/Ratio/Proportion, and then either *Exploring Rate, Ratio, and Proportion* or *3-Term Ratios*.

PROPORTIONS

Ratios and Proportions

A tutorial with interactive practice exercises about ratios and proportions.

<https://www.wisc-online.com/learn/formal-science/mathematics/gem2004/ratios-and-proportions>

Solving Proportions Practice

In this interactive practice, you can choose to show a hint “vertically”, “horizontally”, or algebraically.

<http://www.xpmath.com/forums/arcade.php?do=play&gameid=97>

Solving Proportions

Practice solving basic proportions with this interactive exercise from Khan Academy.

https://www.khanacademy.org/exercise/proportions_1

Ratios, Rates, and Proportions Quiz

Use this multiple-choice self-check quiz to test your knowledge about ratios, rates, and proportions.

<http://www.phschool.com/webcodes10/index.cfm?wcprefix=ara&wcsuffix=0504&area=view>

Proportions: Short Quiz

Use this multiple-choice self-check quiz to test your knowledge about ratios, rates, and proportions.

<http://www.phschool.com/webcodes10/index.cfm?wcprefix=bja&wcsuffix=0602&area=view>

Challenge Board

Choose questions from the challenge board about rates, ratios, and proportions.

<http://www.quia.com/cb/158527.html>

<http://www.quia.com/cb/101022.html>

Write a Proportion to a Problem

Practice writing proportions to describe real-world situations in this interactive exercise.

https://www.khanacademy.org/math/pre-algebra/pre-algebra-ratios-rates/pre-algebra-write-and-solve-proportions/e/writing_proportions

Proportion Word Problems

Practice setting up and solving proportions to solve word problems in this interactive online activity.

<https://www.khanacademy.org/math/pre-algebra/pre-algebra-ratios-rates/pre-algebra-write-and-solve-proportions/e/constructing-proportions-to-solve-application-problems>

Rags to Riches—Proportions

Solve proportions and advance towards more and more difficult questions.

<http://www.quia.com/rr/35025.html>

Pennies to Heaven

This activity uses pennies to give students a context to investigate large numbers and measurements.

<https://www.illustrativemathematics.org/content-standards/6/RP/A/3/tasks/1291>

PROPORTIONAL RELATIONSHIPS

Gym Membership Plans

You are shown two situations and asked which one represents a proportional relationship. Click “Student view” to print the student worksheet. The solution is included further down the web page.

<https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/1983>

Proportional Relationships

A short video showing how ratios, tables, and graphs can help identify proportional relationships. A worksheet is also available.

<https://www.pbslearningmedia.org/resource/muen-math-rp-proportionalrelationships/proportional-relationships/>

Proportional Relationships

Practice telling whether or not the relationship between two quantities is proportional by reasoning about equivalent ratios.

<https://www.khanacademy.org/math/pre-algebra/pre-algebra-ratios-rates/pre-algebra-proportional-rel/e/analyzing-and-identifying-proportional-relationships>

Proportional Relationships: Graphs

Practice telling whether or not the relationship between two quantities is proportional by looking at a graph of the relationship.

<https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-ratio-proportion/cc-7th-graphs-proportions/e/analyzing-and-identifying-proportional-relationships-2>

Interpreting Graphs of Proportional Relationships

Practice reading and analyzing graphs of proportional relationships in this interactive online exercise.

<https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-ratio-proportion/cc-7th-equations-of-proportional-relationships/e/interpreting-graphs-of-proportional-relationships>

Writing Proportional Equations

Practice writing equations to describe proportional relationships in this interactive online activity.

<https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-ratio-proportion/cc-7th-equations-of-proportional-relationships/e/writing-proportional-equations>

SCALE DRAWINGS AND MAPS

Similar Triangles

A short lesson dealing with similar triangles and calculating the ratio of corresponding sides. At the bottom of the page, there is a set of practice questions.

<https://www.mathsisfun.com/geometry/triangles-similar.html>

Similar Shapes Exercises

Answer questions about the scale factors of lengths, areas, and volumes of similar shapes.

<http://www.transum.org/Maths/Activity/Similar/>

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Unit Rates

Remember that a rate is a ratio where the two terms have different units, such as 2 kg/\$0.45 and 600 km/5 h.

In a **unit rate**, the second term of the rate is one (of some unit).

For example, 55 mi/1 hr and \$4.95/1 lb are unit rates. The number “1” is nearly always omitted so those rates are usually written as 55 mi/hr and \$4.95/lb.

To convert a rate into an equivalent unit rate simply divide the numbers in the rate.

Example 1. Mark can ride his bike 35 km in $1\frac{1}{2}$ hours. What is the unit rate?

To find the unit rate, we use the principles of division by fractions to divide 35 km by $1\frac{1}{2}$ h. The units “km” and “hours” are divided, too, and become “km per hour” or “km/hour.”

$$\begin{aligned}\frac{35 \text{ km}}{1\frac{1}{2} \text{ h}} &= 35 \div \frac{3}{2} \text{ km/h} = 35 \times \frac{2}{3} \text{ km/h} \\ &= \frac{70}{3} \text{ km/h} = 23\frac{1}{3} \text{ km/h}.\end{aligned}$$

We could also use decimal division:
 $35 \text{ km} \div 1.5 \text{ h} = 23.333\dots \text{ km/h}.$

So the unit rate is $23\frac{1}{3}$ km per hour.

Example 2. A snail can slide through the mud 5 cm in 20 minutes. What is the unit rate?

Here, it is actually not clear whether we should give the unit rate as cm/min or cm/hr. Let’s do both.

(1) To get the unit rate in cm/min, we simply divide 5 cm \div 20 min. We get the fraction $5/20$. We also divide the units to get “cm/min.” So we get

$$5 \text{ cm} \div 20 \text{ min} = 5/20 \text{ cm/min} = 1/4 \text{ cm/min}$$

Or use decimals: $5 \text{ cm}/20 \text{ min} = 25/100 \text{ cm/min}$
 $= 0.25 \text{ cm/min}.$

(2) For centimeters per hour, we multiply both terms of the rate by 3 to get an equivalent rate of 15 cm in 60 minutes, which is 15 cm in 1 hour.

1. Find the unit rate.

a. \$125 for 5 packages

b. \$6 for 30 envelopes

c. \$1.37 for $\frac{1}{2}$ hour

d. $2\frac{1}{2}$ inches per 4 minutes

e. 24 m^2 per $\frac{3}{4}$ gallon

2. A person is walking $\frac{1}{2}$ mile every $\frac{1}{4}$ hour. Choose the correct fraction for the unit rate and simplify it.

$$\frac{\frac{1}{4}}{\frac{1}{2}} \text{ miles per hour} \quad \text{or} \quad \frac{\frac{1}{2}}{\frac{1}{4}} \text{ miles per hour}$$

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Chapter 7: Percent

Introduction

In this chapter we review the concept of percent as “per hundred” or as hundredth parts and how to convert between fractions, decimals, and percents. The lesson *Solving Basic Percentage Problems* is intended for review of sixth grade topics, focusing on finding a known percentage of a number (such as 21% of 56) or finding a percentage when you know the part and the total.

We take a little different perspective of these concepts in the lesson *Percent Equations*. Students write simple equations for situations where a price increases or decreases (discounts). This lesson also explains what a percent proportion is. Personally, I prefer *not* to use percent proportion but to write the percentage as a decimal and then write an equation. I feel that approach adapts better to solving complex problems than using percent proportion.

Here is a quick example to show the difference between the two methods. Let’s say an item is discounted by 22% and it now costs \$28. Then, the new price is 78% of the original. If we let p be the price of the item before the discount, we can write the percent proportion $\$28/p = 78/100$ and solve for p . If, we write the percentage 78% as the decimal 0.78, we get the equation $0.78p = \$28$. Personally, I consider percent proportion to be an optional topic, and the reason I have included it here is to make this curriculum fully meet the Common Core Standards for seventh grade.

The lesson *Circle Graphs* provides students a break from new concepts and allows them to apply the concept of percent in a somewhat familiar context. Next, we delve into the percentage of change. Students sometimes view the percentage of change as a totally different concept as compared to other percentage topics, but it is not that at all. To calculate the percentage of change, we still use the fundamental idea of *percentage = part/total*, only this time, the “part” is how much the quantity in question changes (the difference) and the “total” is the original quantity.

Tying in with percentage of change, students also learn to compare values using percentages, such as how many percent more or less one thing is than another. Once again, this is not really a new concept but is based on the familiar formula *percentage = part/total*. The percentage difference (or relative difference) is the fraction (*actual difference*)/(*reference value*).

Simple Interest is a lesson on the important topic of interest, using as a context both loans and savings accounts. Students learn to use the formula $I = prt$ in a great variety of problems and situations.

The text concludes with a review lesson of all of the concepts taught in the chapter.

You can find matching videos for topics in this chapter at <http://www.mathmammoth.com/videos/> (grade 7).

The Lessons in Chapter 7

	page	span
Review: Percent	72	3 pages
Solving Basic Percentage Problems	75	3 pages
Percent Equations	78	5 pages
Circle Graphs	83	2 pages
Percentage of Change	85	3 pages
Percentage of Change: Applications	88	4 pages
Comparing Values Using Percentages	92	4 pages
Simple Interest	96	6 pages
Chapter 7 Mixed Review	102	3 pages
Chapter 7 Review	105	2 pages

Helpful Resources on the Internet

Percent Videos by Maria

Videos on percent-related topics that match the lessons in this chapter.

<http://www.mathmammoth.com/videos/prealgebra/pre-algebra-videos.php#percent>

Percent Worksheets

Create an unlimited number of free customizable percent worksheets to print.

<http://www.homeschoolmath.net/worksheets/percent-decimal.php>

<http://www.homeschoolmath.net/worksheets/percent-of-number.php>

<http://www.homeschoolmath.net/worksheets/percentages-words.php>

PERCENTAGES, FRACTIONS, AND DECIMALS

Matching Fractions and Percent

Practice matching percentages with corresponding fractions in this timed matching game.

https://www.mathplayground.com/matching_fraction_percent.html

Decention Game

Build teams of three players by matching fractions, decimals, and percentages with the same value.

<https://www.mathplayground.com/Decention/index.html>

Fractions and Percent Matching Game

A simple matching game: match fractions and percentages.

http://www.mathplayground.com/matching_fraction_percent.html

What percentage is shaded?

Practice guessing what percentage of the pie chart has been shaded yellow in this interactive activity.

<http://www.interactivestuff.org/sums4fun/pietest.html>

Pie Chart and Questions

First, read a short illustrated lesson about pie charts. Then, click on the questions at the bottom of the page to practice.

<https://www.mathsisfun.com/data/pie-charts.html>

Flower Power

Grow flowers and harvest them to make money in this addictive order-'em-up game. Practice ordering decimals, fractions, and percentages.

<https://www.mangahigh.com/en/games/flowerpower>

Matching Fractions, Decimals, and Percentages

A simple matching memory game.

<http://nrich.maths.org/1249>

Doughnut Percents

This task involving the equivalence between fractions, percentages and decimals is one of a series of problems designed to develop students' team working skills.

<https://nrich.maths.org/6945>

Percent Goodies: Fraction-Decimal-Percent Conversions

Practice conversions between fractions, decimals and percents. There are three levels of difficulty and instant scoring for each. Note that fractions must be written in lowest terms.

<http://www.mathgoodies.com/games/conversions/>

BASIC PERCENTAGE CALCULATIONS

Penguin Waiter

A simple game where you calculate the correct tip to leave the waiter (levels “easy” and “medium”), the percentage that the given tip is (level “hard”), or the original bill (level “Super Brain”).

<https://www.funbrain.com/games/penguin-waiter>

Percent Jeopardy

An interactive jeopardy game where the questions have to do with a percentage of a quantity.

<http://www.quia.com/cb/42534.html>

Matching Percentage of a Number

Match cards that ask for a percentage of a number (such as 75% of 40) with the values. The game is fairly easy and can be completed using mental math.

<http://www.sheppardsoftware.com/mathgames/percentage/MatchingPercentNumber.htm>

Percent, Interest, Discount, and Sale Price Challenge Board

Score points by answering questions correctly. This game can be played by one or two players.

<https://www.quia.com/cb/55701.html>

The Percentage Game

This is a printable board game for 2-3 players that practices questions such as “20 percent of ___ is 18” or “___ is 40 percent of 45”.

<http://nzmaths.co.nz/resource/percentage-game>

Percentages

This page gives an illustrated explanation of the basic concept of percent. At the bottom of the page, there is also a series of practice questions.

<https://www.mathsisfun.com/percentage.html>

PERCENT OF CHANGE

Percent of Change Matching

Match five flashcards with given increases or decreases (such as “25 is decreased to 18”) with five percentages of increase/decrease.

<https://www.studystack.com/matching-182854>

Percent Shopping

Choose toys to purchase. In level 1, you find the sale price when the original price and percent discount are known. In level 2, you find the percent discount (percent of change) when the original price and the sale price are known.

http://www.mathplayground.com/percent_shopping.html

Rags to Riches: Percent Increase or Decrease

Answer simple questions about percent increase or decrease and see if you can win the grand prize in the game.

<http://www.quia.com/rr/230204.html>

Percentage Change 1

A self-marking quiz with 10 questions about percentage change. The link below goes to level 1 quiz, and at the bottom of that page you will find links to level 2, 3, 4, 5 and 6 quizzes.

http://www.transum.org/software/SW/Starter_of_the_day/Students/PercentageChange.asp

Percent of Change Quiz

Practice determining the percent of change in this interactive multiple-choice quiz.

<http://www.phschool.com/webcodes10/index.cfm?wcprefix=bja&wcsuffix=0607&area=view>

Percentage Increase and Decrease 4 in a Line

The web page provides a game board to print. Players take turns picking a number from the left column, and increase or decrease it by a percentage from the right column. They cover the answer on the grid with a counter. The first player to get four counters in a line wins.

<https://www.tes.co.uk/teaching-resource/percentage-increase-and-decrease-4-in-a-line-6256320>

Percentage of Increase Exercises

Find the percentage increase given the original and final values in this self-check quiz about percentage change.

http://www.transum.org/software/SW/Starter_of_the_day/Students/PercentageChange.asp?Level=3

Percentage of Decrease Exercises

Find the percentage decrease given the original and final values in this self-check quiz about percentage change.

http://www.transum.org/software/SW/Starter_of_the_day/Students/PercentageChange.asp?Level=4

Treasure Hunt - Percentage Increase and Decrease

The clues of this treasure hunt are printable percentage increase/decrease questions.

<https://www.tes.co.uk/teaching-resource/treasure-hunt--percentage-increase-and-decrease-6113809>

Percent Change Practice

Interactive flash cards with simple questions about percentage of change with three difficulty levels.

<http://www.thegreatmartinicompany.com/percent-percentage/percent-change.html>

Percent of Change Test

Multiple-choice questions about percentage of change to be solved without a calculator (mental math).

https://reviewgamezone.com/mc/candidate/test/?test_id=29446&title=Percent%20Change

Percent of Change Jeopardy

This is an online jeopardy game that provides you the game board, questions for percent increase, percent decrease, sales tax, discounts, and markups, the answers, and a scoreboard where you can enter the teams' points. However, it doesn't have a place to enter answers and requires someone to supervise the play and the teams' answers.

<http://www.superteachertools.us/jeopardyx/jeopardy-review-game.php?gamefile=2245685>

Percentage Difference

A short lesson about percentage difference, followed by practice questions at the bottom of the page.

<http://www.mathsisfun.com/percentage-difference.html>

INTEREST

Interest (An Introduction)

Read an introduction to interest, and then click on the questions at the bottom of the page to practice.

<https://www.mathsisfun.com/money/interest.html>

Quiz: Simple Interest

A quiz with five questions that ask for the interest earned, final balance, interest rate, or the principal.

<https://www.cliffsnotes.com/study-guides/algebra/algebra-ii/word-problems/quiz-simple-interest>

Simple Interest

Another quiz where you need to find the principal, the amount of time, interest earned, or the final amount in an account earning interest. Four out of nine questions have to do with terminology and the rest are math problems.

<http://www.proprofs.com/quiz-school/story.php?title=simple-interest>

Simple Interest Game

Answer questions about simple interest by clicking on the correct denominations in the cash register.

<http://www.math-play.com/Simple-Interest/Simple-Interest.html>

Simple Interest Quiz

Test your knowledge of simple interest with this interactive self-check quiz.

<https://www.proprofs.com/quiz-school/story.php?title=simple-interest>

Simple Interest Practice Problems

Practice using the formula for simple interest in this interactive online activity.

<http://www.transum.org/Maths/Activity/Interest/>

Calculating simple interest

This page includes several video tutorials plus a short three-question quiz on simple interest.

<https://www.sophia.org/concepts/calculating-simple-interest>

Compound interest

A simple introduction to compound interest with many examples.

<http://www.mathsisfun.com/money/compound-interest.html>

GENERAL

Percents Quiz

Review basic percent calculations with this short multiple-choice quiz.

<http://www.phschool.com/webcodes10/index.cfm?wcprefix=bj&wcsuffix=0605&area=view>

Percent Quiz

Practice determining what percentage one number is of another in this interactive online quiz.

<https://www.thatquiz.org/tq-3/?-j1c0-l7-mpnv600-p0>

Math at the Mall

Practice percentages while shopping at a virtual mall. Find the percentage of discount and the sales price, calculate the interest earned at the bank, compare health memberships at the gym and figure out how much to tip your waiter at the Happy Hamburger.

<http://www.mathplayground.com/mathatthemall2.html>

Percent Word Problems

Practice solving word problems involving percents in this interactive online activity.

https://www.khanacademy.org/math/algebra-basics/basic-alg-foundations/alg-basics-decimals/e/percentage_word_problems_1

Percents Quiz 2

Practice answering questions about percent in this multiple-choice online quiz.

<http://www.phschool.com/webcodes10/index.cfm?wcprefix=bj&wcsuffix=0606&area=view>

Percentages Multiple-Choice Test

Test your understanding of percentages with this self-check multiple choice quiz.

http://www.transum.org/Software/Pentransum/Topic_Test.asp?ID_Topic=28

Percentage Quiz

Reinforce your skills with this interactive multiple-choice quiz.

https://www.mathopolis.com/questions/q.php?id=877&site=1&ref=/percentage.html&qs=877_878_879_1301_1302_880_1303_1304

Practice on Significant Figures

A multiple-choice quiz that also reminds you of the rules for significant digits.

<https://web.archive.org/web/20161013062458/http://www.chemistrywithmsdana.org:80/wp-content/uploads/2012/07/SigFig.html>

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Percentage of Change

Percent(age of) change is a way to describe how much a price or some other quantity is increasing or decreasing (changing). Let's look at how to calculate the percentage a quantity is changing.

Example 1. A phone used to cost \$50. Now it has been discounted to \$45. What percentage was the discount?

Since this problem is asking for the *percentage*, we will use our basic formula $\frac{\text{part}}{\text{total}} = \text{percentage}$.

Because the change is relative to the *original* price, that original price becomes the "total" in our equation. The "part" is the actual amount by which the quantity changes, in this case \$5. So we get

$$\text{percentage} = \frac{\text{part}}{\text{total}} = \frac{\$5}{\$50} = 1/10 = 10\%$$

Essentially, we wrote **what fraction the \$5 discount is of the original \$50 price** and converted that fraction into a percentage.

In summary: To calculate the percent change, use the same basic formula that defines a percentage: *part/total*. Since the change is relative to the original price, the original price is the "total," and the change in price is the "part."

$$\text{percentage of change} = \frac{\text{part}}{\text{total}} = \frac{\text{difference}}{\text{original}}$$

1. Write an equation and calculate the percentage of change.

a. A toy construction set costs \$12. It is discounted and costs only \$8 now. What percentage is the discount?

$$\frac{\text{difference}}{\text{original}} =$$

b. A sewing kit costs \$20. It is discounted and costs only \$16 now. What percentage is the discount?

c. A bouquet of flowers used to cost \$15, but now it costs \$20. What is the percentage of increase?

d. The price of a stove was \$160. The price has increased, and now it costs \$200. What is the percentage of increase?

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Chapter 8: Geometry

Introduction

The main topics we study in this chapter are:

- various angle relationships
- drawing geometric figures, including basic geometric constructions
- pi and the area of a circle
- slicing 3-dimensional solids
- surface area and volume.

In the first lesson of the chapter, we examine various angle relationships: angles that are formed when several rays originate from the same starting point, vertical angles (formed when two lines intersect), and corresponding angles (formed when a line intersects two parallel lines). Then, the lesson *Angles in a Triangle* presents and proves the well-known result that the angles in a triangle sum to 180 degrees. With this knowledge, students are now able to solve various problems that involve unknown angles.

Next, students practice drawing geometric figures. Basic geometric constructions are done just like in ancient times: with only a compass and straightedge (a ruler without measurement units). These constructions help students to think about the main defining features of a figure. Personally I have always enjoyed geometric constructions because they are like little puzzles to solve.

Students also draw figures using a normal ruler and compass in the lesson *Drawing Problems*. They especially determine whether the given information defines a unique figure (triangle or a parallelogram).

Then we turn our attention to pi. Students first learn the definition of pi as a ratio of a circle's circumference to its diameter in the lesson *Circumference of a Circle*. Then they learn and practice how to calculate the area of a circle in a wide variety of word problems and applications. We also briefly study the proof for the formula for the area of a circle. I feel it is important that students encounter justifications for mathematical formulas and procedures and even read some proofs before high school. We don't want students to think that mathematics is only a bag of magic tricks or formulas to memorize that seemingly came out of nowhere. Proofs and logical thinking are foundations to mathematics and school mathematics should not be left without them.

After this, we slice three-dimensional solids with a plane, and learn that the result is always a two-dimensional shape. Students see that in a concrete way by slicing cubes and pyramids made of modeling clay. Some Internet links (provided in the lesson) will also help students to visualize what happens when a solid is cut with a plane.

In this chapter, students also solve a variety of problems concerning surface area and volume and practice converting between various units of area and volume. While these topics tend to involve lots of calculations and less possibilities for hands-on activities, they are very important in real life.

Consider mixing the lessons from this chapter with lessons from some other chapter, such as the chapter on probability. For example, the student could study topics from this chapter and from the probability chapter on alternate days, or study a little from each chapter each day. Such, somewhat spiral, usage of the curriculum can help prevent boredom, and also to help students retain the concepts better.

Recall also that I don't recommend automatically assigning all the problems. Use your judgment.

You can find matching videos for topics in this chapter at <http://www.mathmammoth.com/videos/> (grade 7).

The Lessons in Chapter 8

	page	span
Angle Relationships	113	5 pages
Angles in a Triangle	118	5 pages

Basic Geometric Constructions	123	6 pages
More Constructions	129	5 pages
Drawing Problems	134	7 pages
Circumference of a Circle	141	3 pages
Area of a Circle	144	3 pages
Proving the Formula for the Area of a Circle	147	2 pages
Area and Perimeter Problems	149	5 pages
Surface Area	154	4 pages
Conversions Between Customary Units of Area	158	3 pages
Conversions Between Metric Units of Area	161	3 pages
Slicing Three-Dimensional Shapes	164	7 pages
Volume of Prisms and Cylinders	171	4 pages
Chapter 8 Mixed Review	175	3 pages
Chapter 8 Review	178	8 pages

Helpful Resources on the Internet

ANGLE RELATIONSHIPS

Math Warehouse - Angles

Find interactive demonstrations, examples and practice problems concerning various types of angles.

<http://www.mathwarehouse.com/geometry/angle/complementary-angles.php>

<http://www.mathwarehouse.com/geometry/angle/supplementary-angles.php>

<http://www.mathwarehouse.com/geometry/angle/vertical-angles.php>

<http://www.mathwarehouse.com/geometry/triangles/>

Angles at a Point

Drag the points of the angles to see how the angle measurements of the model change.

http://www.transum.org/software/SW/Angle_Theorems/ShowOne.asp?T=1

Angles Around a Point

A short lesson showing that angles around a point add up to 360 degrees, followed by self-check questions.

<http://www.mathsisfun.com/angle360.html>

Angle Points Exercises

Apply the properties of angles at a point, angles at a point on a straight line, and vertically opposite angles in this interactive online exercise.

http://www.transum.org/software/SW/Starter_of_the_day/Students/AnglePoints.asp?Level=4

Complementary & Supplementary Angles from Maths Is Fun

Each page includes a clear explanation, an interactive exploration, and self-check interactive questions.

<http://www.mathsisfun.com/geometry/complementary-angles.html>

<http://www.mathsisfun.com/geometry/supplementary-angles.html>

Complementary & Supplementary Angles

Find missing angles in this interactive exercise.

https://www.khanacademy.org/math/geometry/hs-geo-foundations/hs-geo-angles/e/complementary_and_supplementary_angles

Working with Angles

Online lessons with interactive self-check questions from Absorb Mathematics course. The lessons cover measuring angles, the types of angles, angles on a straight line, and other angle topics.

<http://www.absorblearning.com/mathematics/demo/units/KCA003.html>

Identifying Supplementary, Complementary, and Vertical Angles

Practice telling whether two angles are supplementary, complementary, or vertical.

<https://www.khanacademy.org/math/basic-geo/basic-geo-angle/vert-comp-supp-angles/e/identifying-supplementary-complementary-vertical>

Finding Missing Angles

Use your knowledge of vertical, complementary, and supplementary angles to find missing angles in this exercise.

<https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-geometry/cc-7th-unknown-angle-algebra/e/find-missing-angles>

Angle Relationships Quiz

Test your knowledge about angle relationships with this interactive online quiz.

<https://quizlet.com/152463094/test>

Solving for Unknown Angles from Khan Academy

Use your knowledge of supplementary and complementary angles to solve questions of varying difficulty. Some questions involve writing and solving an equation.

<https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-geometry/cc-7th-angles/e/solving-for-unknown-angles>

Angle Sums

Examine the angles in a triangle, quadrilateral, pentagon, hexagon, heptagon or octagon. Can you find a relationship between the number of sides and the sum of the interior angles?

<http://illuminations.nctm.org/Activity.aspx?id=3546>

Angles in a Triangle Quiz

Find the size of the angle marked with a letter in each triangle.

http://www.transum.org/software/SW/Starter_of_the_day/Students/AnglesInTriangle/Quiz.asp

CONSTRUCTIONS

Geometric Construction

These lessons cover constructions for perpendicular lines, an equilateral triangle, angle bisection, parallel lines, and copying an angle. They include explanations, interactive animations, and self-check questions.

<http://www.absorblearning.com/mathematics/demo/units/KCA006.html>

Animated Geometric Constructions

Simple animations that show how to do basic geometric constructions.

<http://www.mathsisfun.com/geometry/constructions.html>

Constructing a Triangle with Three Known Sides

A short demonstration showing how to construct a triangle with three known sides using a compass and a ruler.

<http://www.mathsisfun.com/geometry/construct-ruler-compass-1.html>

Constructing Triangles

Practice constructing triangles in this interactive activity from Khan Academy.

<https://www.khanacademy.org/math/7th-engage-ny/engage-7th-module-6/7th-module-6-topic-b/e/constructing-triangles>

Triangle Inequality Theorem

Answer questions about the third side of a triangle when the lengths of two sides are given.

https://www.khanacademy.org/math/7th-engage-ny/engage-7th-module-6/7th-module-6-topic-b/e/triangle_inequality_theorem

PI AND THE CIRCUMFERENCE OF A CIRCLE

A Rolling Circle Illustrating Pi

Two animations where a circle with diameter 1 rolls on a number line one complete roll. Of course having rolled once around its circumference, it now lands at pi.

<http://www.mathwarehouse.com/animated-gifs/images/demonstration-of-Pi-in-math-animation.gif>

<http://i.imgur.com/dsCw0.gif>

Approximating Pi

How did Archimedes find the approximate value of pi? This interactive tool illustrates Archimedes' basic approach with inscribed or circumscribed polygons.

<http://www.pbs.org/wgbh/nova/physics/approximating-pi.html>

5 Trillion Digits of Pi

As of 2014, the world record for computing digits of pi was over 13 trillion digits.

<http://www.numberworld.org/digits/Pi/>

Radius, Diameter and Circumference

Practice finding the radius, diameter, or circumference of a circle in this interactive online activity.

https://www.khanacademy.org/math/geometry/hs-geo-foundations/hs-geo-area/e/radius_diameter_and_circumference

Circumference Quiz

Test your skills with this interactive self-check quiz.

<https://maisonetmath.com/circumference-quizzes/302-circumference-of-circles>

Amazing History of Pi

A short and simple introduction to the history of pi.

<https://web.archive.org/web/20170204081925/http://ualr.edu/lasmoller/pi.html>

AREA OF CIRCLE

Interactive Area of a Circle

Explore and discover the relationship between the area, radius, and graph of a circle. Just click and drag the points.

<http://www.mathwarehouse.com/geometry/circle/interactive-area.php>

Circle Tool from Illuminations

How do the area and circumference of a circle compare to its radius and diameter? Investigate these relationships in the Intro and Investigation sections and then hone your skills in the Problems section.

<http://illuminations.nctm.org/Activity.aspx?id=3547>

Area of a Circle and Its Formula

Seven practice problems concerning the area of the circle with full solutions.

<http://www.mathwarehouse.com/geometry/circle/area-of-circle.php>

Quiz: Area of a Circle

Practice finding the area of a circle in this interactive online quiz.

<http://www.phschool.com/webcodes10/index.cfm?wcprefix=bja&wcsuffix=1003&area=view>

Area and Perimeter Practice

A 10-question quiz that will let you practice finding the area and circumference of a circle.

<http://www.thatquiz.org/tq-4/?-j201g-la-p1ug>

Area and Circumference of a Circle - Test from BBC Bitesize

The level of difficulty increases as you proceed from the first to the last question in this 10-question quiz about the area and circumference of a circle.

<http://www.bbc.co.uk/bitesize/quiz/q90581037>

Compound Circles

Find the area and perimeter of a shape made from two semicircles in this interactive activity.

<https://flashmaths.co.uk/viewFlash.php?id=61>

Area and Circumference of a Circle

A 15-question multiple-choice quiz.

<http://www.proprofs.com/quiz-school/story.php?title=area-circumference--circle>

Area of a Circle

An interactive activity where you cut a circle into wedges in order to determine its area.

http://www.learner.org/courses/learningmath/measurement/session7/part_b/index.html

Area of Circles

Cut a circle into sectors and rearrange the sectors to form a figure close to a parallelogram in this interactive activity. By increasing the number of sectors, the figure gets closer and closer to a perfect parallelogram.

<https://www.geogebra.org/m/fyqAUV22>

Area of a Circle, How to Get the Formula

An animation that illustrates how we can find the area of a circle by drawing triangles into it. The area of the circle is then the limit of the sum of the areas of the interior triangles as the number of triangles goes to infinity.

<https://www.youtube.com/watch?v=YokKp3pwVFc>

AREA AND PERIMETER

Area Tool

Use this tool to determine how the length of the base and the height of a figure can be used to determine its area.

How are the area formulas for trapezoids, parallelograms, and triangles similar? How do they differ?

<http://illuminations.nctm.org/Activity.aspx?id=3567>

Free Worksheets for the Area of Triangles, Quadrilaterals, and Polygons

Generate customizable worksheets for the area of triangles, parallelograms, trapezoids, or polygons in the coordinate grid. Options include scaling, image size, workspace, border, and more.

http://www.homeschoolmath.net/worksheets/area_triangles_polygons.php

BBC Bitesize - Area

Brief revision (review) “bites,” including a few interactive questions, about the area of triangles, parallelograms, and compound shapes.

<http://www.bbc.co.uk/bitesize/ks3/maths/measures/area/revision/4/>

Geometry Area/Perimeter Quiz from ThatQuiz.org

Find the area and perimeter of rectangles, triangles, parallelograms, and trapezoids. You can modify the quiz parameters, for example to omit a shape, or to solve for an unknown side when perimeter/area is given.

<http://www.thatquiz.org/tq-4/?-j1i00f-lc-p0>

Find Areas of Trapezoids

Practice finding the areas of trapezoids in this interactive online activity from Khan Academy.

https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-geometry-topic/cc-6th-area/e/areas_of_trapezoids_rhombi_and_kites

Find Areas of Compound Shapes

Practice finding the areas of complex shapes that are composed of smaller shapes in this interactive online exercise.

<https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-geometry-topic/cc-6th-area/e/area-of-quadrilaterals-and-polygons>

Area of Composite Shapes

Find the areas of combined shapes made up of one or more simple polygons and circles.

http://www.transum.org/software/SW/Starter_of_the_day/Students/Areas_of_Composite_Shapes.asp?Level=4

Metric Area

Read a short, illustrated lesson about metric area. Then, click on the questions at the bottom of the page to practice.

<http://www.mathsisfun.com/measure/metric-area.html>

Metric Unit Conversions: Area

Use these printable worksheets for review or to brush up on metric conversion skills.

<http://www.dadsworksheets.com/worksheets/metric-si-unit-conversions-metric-si-area.html>

Volume of Prisms and Cylinders

<https://www.proprofs.com/quiz-school/story.php?title=MTA2MzA07UGR>

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Area of a Circle

The area of a circle is given by this formula: $A = \pi r^2$, where r is the radius of the circle.

Read the formula as: “Area equals pi r squared.” It means that you first multiply the radius by itself and then multiply the result by π .

You can remember the formula by thinking, “Pie are square.” Of course pies are usually round, not square! It is bad English, as well, but the purpose of this silly mnemonic is just to help you remember the formula.

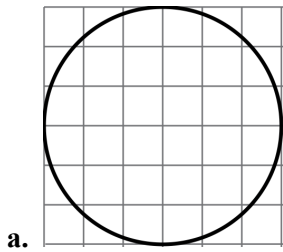
Example 1. The radius of a circle measures 8 cm. What is its area?

We use the formula: $A = \pi r^2 = \pi \cdot 8 \text{ cm} \cdot 8 \text{ cm} \approx 3.14 \cdot 64 \text{ cm}^2 = 200.96 \text{ cm}^2$, or about 200 cm^2 .

Remember to always give your answer for an area in square units, be it square inches, square centimeters, square meters, square feet, *etc.* If no measuring unit is given, use “square units.”

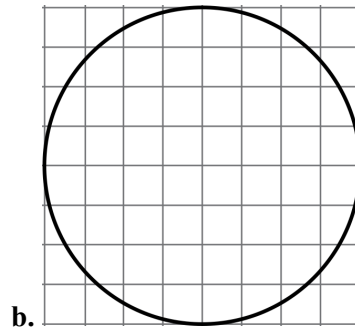
You can use a calculator for all the problems in this lesson.

1. Estimate the area of the circles by counting squares and parts of squares. After that, calculate the area to the nearest tenth of a square unit.



Estimation: _____ square units

Calculation: _____ square units



Estimation: _____ square units

Calculation: _____ square units

2. Find the areas of these circles.

a. A circle with a radius of 7.0 cm.

Round the answer to the nearest ten square centimeters (to 2 significant digits).

Area =

b. A circle with a radius of $10 \frac{1}{4}$ in.

Round the answer to the nearest square inch (to 3 significant digits).

Area =

c. A circle with a *diameter* of 75.0 cm.

Round the answer to the nearest ten square centimeters (to 3 significant digits).

Area =

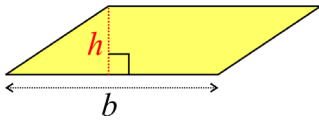
d. A circle with a radius of 17 ft 4 in.

Round the answer to the nearest thousand square inches (to 3 significant digits).

Area =

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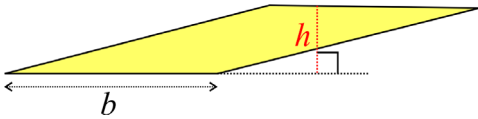
Area and Perimeter Problems



The area of a parallelogram is

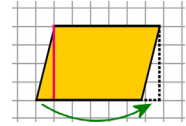
$$A = bh$$

where b is the base and h is the altitude (height).



The **altitude** of a parallelogram is a perpendicular line segment from the base, or the extension of the base, to the top. Thus, the altitude might not be inside the parallelogram.

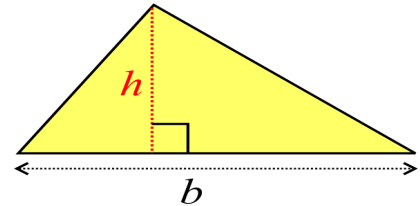
Recall that from any parallelogram we can cut off a triangular piece and move it to the other side to make it a rectangle. That is why the formula for the area of a parallelogram is so similar to the formula for the area of a rectangle.



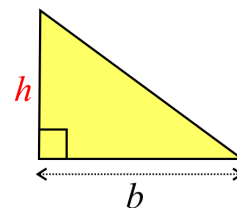
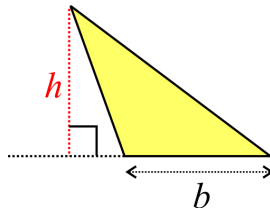
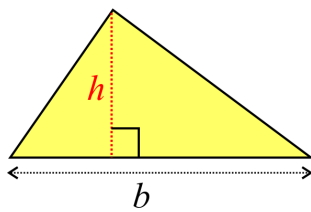
Since any triangle is half of its corresponding parallelogram, the area of a triangle is half the area of that parallelogram:

$$A = \frac{bh}{2}$$

where b is the base and h is the altitude of the triangle.



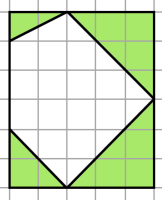
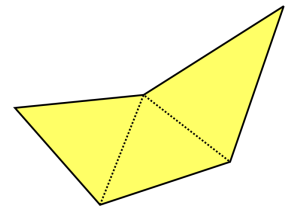
The **altitude** of a triangle is a line from one vertex to the opposite side that is perpendicular to that side. It can:



- (1) fall inside the triangle; (2) fall outside the triangle; (3) be one of the sides of a right triangle.

To calculate the area of a polygon:

- (1) Divide it into rectangles, triangles, and other simple shapes.
- (2) Calculate the area of each part separately.
- (3) Add the area of each of the parts.



Sometimes we can use another strategy to find the area of a polygon.

- (1) Draw a rectangle around the polygon.
- (2) Calculate the areas of the triangles and quadrilaterals that are outside the polygon but inside the rectangle.
- (3) Subtract those areas from the area of the entire rectangle.

You may use a calculator for all the problems in this lesson.

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Chapter 9: The Pythagorean Theorem

Introduction

This is a relatively short chapter focusing on the Pythagorean Theorem and its applications. The Pythagorean Theorem is actually not part of the Common Core Standards for seventh grade. The Common Core places it in eighth grade. However, I have included it in this curriculum because it is a traditional topic in pre-algebra. That way, Math Mammoth Grade 7 works as a full pre-algebra curriculum while fully meeting (and exceeding) the Common Core Standards for grade 7. If you are following the Common Core Standards strictly, you can safely omit this entire chapter, because your student(s) will encounter these topics in eighth grade.

First, students need to become familiar with square roots, so they can solve the equations that result from applying the Pythagorean Theorem. The first lesson of the chapter introduces taking a square root as the opposite operation to squaring a number. The lesson includes both applying a guess-and-check method and using a calculator to find the square root of a number.

Next, students learn how to solve simple equations that include taking a square root. This makes them fully ready to study the Pythagorean Theorem and apply it.

The Pythagorean Theorem is introduced in the lesson by that name. Students learn to verify that a triangle is a right triangle by checking if it fulfills the Pythagorean Theorem. They apply their knowledge about square roots and solving equations to solve for an unknown side in a right triangle when two of the sides are given.

Next, students solve a variety of geometric and real-life problems that require the Pythagorean Theorem. This theorem is extremely important in many practical situations. Students should show their work for these word problems to include the equation that results from applying the Pythagorean Theorem to the problem and its solution.

There are literally hundreds of proofs for the Pythagorean Theorem. In this chapter, we present one easy proof based on geometry (not algebra). As an exercise, students are asked to supply the steps of reasoning to another geometric proof of the theorem, and for those interested the lesson also provides an Internet link that has even more proofs of this theorem.

You can find matching videos for topics in this chapter at <http://www.mathmammoth.com/videos/> (grade 7).

The Lessons in Chapter 9

	page	span
Square Roots	189	<i>4 pages</i>
Equations That Involve Taking a Square Root	193	<i>5 pages</i>
The Pythagorean Theorem	198	<i>5 pages</i>
The Pythagorean Theorem: Applications	203	<i>7 pages</i>
A Proof of the Pythagorean Theorem	210	<i>1 page</i>
Chapter 9 Mixed Review	211	<i>3 pages</i>
Chapter 9 Review	214	<i>3 pages</i>

Helpful Resources on the Internet

The Pythagorean Theorem - Video Lessons by Maria

A set of free videos that teach the topics in this chapter - by the author herself.

<http://www.mathmammoth.com/videos/prealgebra/pre-algebra-videos.php#pythagorean>

SQUARE ROOTS

Squares and Square Roots

A fun lesson about squares and square roots with lots of visuals and little tips. It is followed by 10 interactive multiple-choice questions.

<http://www.mathsisfun.com/square-root.html>

The Roots of Life

Practice finding square roots of perfect squares and help the roots of a tree grow.

<http://www.hoodamath.com/games/therootsoflife.html>

Square Root Game

Match square roots of perfect squares with the answers. Includes several levels.

<http://www.math-play.com/square-root-game.html>

Approximating Square Roots

Practice finding the approximate value of square roots by thinking about perfect squares.

https://www.khanacademy.org/math/pre-algebra/pre-algebra-exponents-radicals/pre-algebra-square-roots/e/square_roots_2

Equations with Square Roots & Cube Roots

Test your knowledge of square roots and cube roots with this interactive online quiz.

<https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-numbers-operations/cc-8th-roots/e/equations-w-square-and-cube-roots>

Pyramid Math

Choose “SQRT” to find square roots of perfect squares. Drag the correct answer to the jar on the left. This game is pretty easy.

<http://www.mathnook.com/math/pyramidmath.html>

Rags to Riches Square Root Practice

Answer multiple-choice questions that increase in difficulty. The questions include finding a square root of perfect squares, determining the two nearest whole numbers to a given square root, and finding square roots of numbers that aren't perfect squares to one decimal digit.

<http://www.quia.com/rr/382994.html>

THE PYTHAGOREAN THEOREM

Pythagorean Theorem - Braining Camp

This learning module includes a lesson, an interactive manipulative, multiple-choice questions, real-life problems, and interactive open-response questions.

<https://www.brainiaccamp.com/content/pythagorean-theorem/>

Pythagoras' Theorem from Maths Is Fun

A very clear lesson about the Pythagorean Theorem and how to use it, followed by 10 interactive practice questions.

<http://www.mathsisfun.com/pythagoras.html>

Pythagorean Triplets

Move the two orange points in this activity to find Pythagorean Triplets, sets of three whole numbers that fulfill the Pythagorean Theorem.

<http://www.interactive-maths.com/pythagorean-triples-ggb.html>

The Pythagorean Theorem Quiz

A 10-question quiz that asks for the length of the third side of a right triangle when the two sides are given.

<http://www.thatquiz.org/tq-A/?-j10-la-p1ug>

Interactivate: Pythagorean Theorem

Interactive practice problems for calculating the third side of a right triangle when two sides are given.

<http://www.shodor.org/interactivate/activities/PythagoreanExplorer/>

Exploring the Pythagorean Theorem

This multimedia mathematics resource shows how the Pythagorean Theorem is an important math concept used in the structural design of buildings. Using an interactive component, students construct right triangles of various sizes to explore calculations of the Pythagorean Theorem.

<http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.SHAP&ID2=AB.MATH.JR.SHAP.PYTH>

Pythagorean Theorem Challenge

Review the Pythagorean Theorem in this interactive self-check quiz.

<https://www.khanacademy.org/math/basic-geo/basic-geometry-pythagorean-theorem/pythagorean-theorem-app/e/pythagorean-theorem-word-problems>

Pythagorean Theorem Test

Test your knowledge of the Pythagorean Theorem in this interactive online quiz.

<http://www.mathportal.org/math-tests/trigonometry-tests/tests-in-right-triangle-trigonometry.php?testNo=1&testName=Pythagorean-Theorem-Test>

Pythagoras in 3D

A challenge problem: can you find the longest dimension of a box?

<http://www.interactive-maths.com/pythagoras-in-3d-ggb.html>

PROOF

Proving the Pythagorean Theorem

See if you can figure out two more proofs of the Pythagorean theorem. Only the pictures are given to you. Tips and Solutions are available.

http://www.learner.org/courses/learningmath/geometry/session6/part_b/more.html

Annotated Animated Proof of the Pythagorean Theorem

Watch the animation to learn a proof of the Pythagorean Theorem.

<http://www.davis-inc.com/pythagor/proof2.html>

Many Proofs of the Pythagorean Theorem

A list of animated proofs.

http://www.takayaiwamoto.com/Pythagorean_Theorem/Pythagorean_Theorem.html

Pythagorean Theorem and its many proofs

A collection of 111 approaches to prove this theorem.

<http://cut-the-knot.com/pythagoras/>

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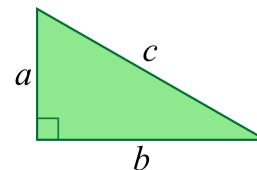
The Pythagorean Theorem

You will now learn a very famous mathematical result, the Pythagorean Theorem, which has to do with the lengths of the sides in a right triangle. First, we need to study some terminology.

In a right triangle, the two sides that are perpendicular to each other are called **legs**. The third side, which is always the longest, is called the **hypotenuse**.

In the image on the right, the sides a and b are the legs, and c is the hypotenuse.

Note: We don't use the terms "leg" and "hypotenuse" to refer to the sides of an acute or obtuse triangle — this terminology is restricted to *right* triangles.

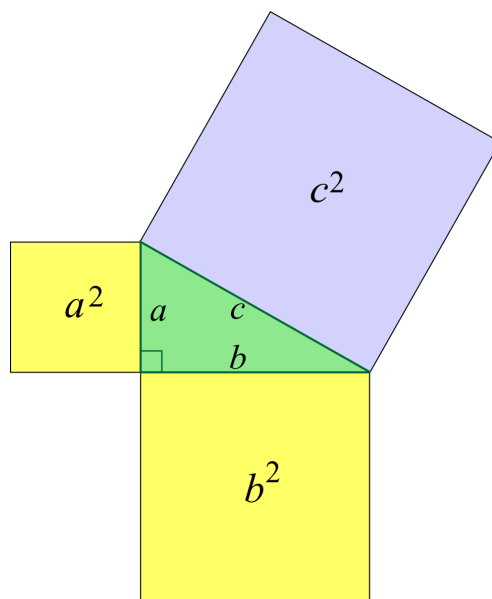


The Pythagorean Theorem states that **the sum of the squares of the legs equals the square of the hypotenuse**.

In symbols it looks much simpler:

$$a^2 + b^2 = c^2$$

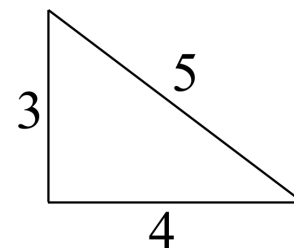
The picture shows squares drawn on the legs and on the hypotenuse of a right triangle. Verify visually that the total area of the two yellow squares drawn on the legs looks about equal to the area of the blue square on the hypotenuse.



We will prove this theorem in another lesson.

For now, let's get familiar with it and learn how to use it.

1. This is the famous 3-4-5 triangle: its sides measure 3, 4, and 5 units. It is a right triangle. Check that the Pythagorean Theorem holds for it by filling in the numbers below.



$$\square^2 + \square^2 \stackrel{?}{=} \square^2$$

$$\square + \square \stackrel{?}{=} \square$$

$$\square = \square$$

2. a. Check that the Pythagorean Theorem holds for a triangle with sides 6, 8, and 10 units long by filling in the numbers at the right.

$$\square^2 + \square^2 \stackrel{?}{=} \square^2$$

$$\square + \square \stackrel{?}{=} \square$$

$$\square = \square$$

- b. Use a compass and a ruler to draw a triangle with sides 6, 8, and 10 cm long. You can review the box, "A Triangle with Three Given Sides," on page 127. Measure its angles: did you get a right triangle?

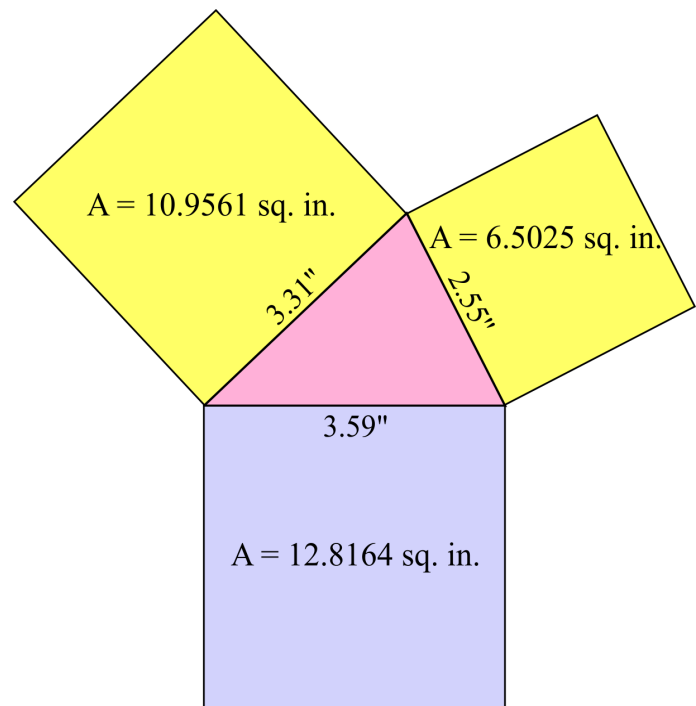
Example 1. This triangle is *not* a right triangle, so the Pythagorean Theorem does *not* hold:

$$2.55^2 + 3.31^2 \stackrel{?}{=} 3.58^2$$

$$6.5025 + 10.9561 \stackrel{?}{=} 12.8164$$

$$17.4586 > 12.8164$$

The sum of the areas of the squares drawn on the two shortest sides is more than the area of the square drawn on the longest side. As you can see, the triangle is acute.



Example 2. Is a triangle with sides 4 cm, 5 cm, and 7 cm a right triangle?

$$4^2 + 5^2 \stackrel{?}{=} 7^2$$

We check if 4, 5, and 7 fulfill the Pythagorean Theorem (on the right). They don't. In fact, $4^2 + 5^2 < 7^2$ and the triangle is obtuse. (You can check that by drawing it.)

$$16 + 25 \stackrel{?}{=} 49$$

$$41 < 49$$

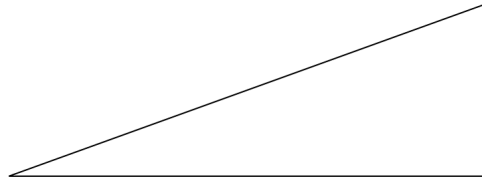
This triangle is obtuse.

3. For each set of lengths, determine whether they form a right triangle using the Pythagorean Theorem. Notice carefully which length is the hypotenuse.

a. 6, 9, 13

b. 12, 13, 5

4. **a.** Measure each side of this triangle to the nearest millimeter.



b. Verify that the sum of the areas of the squares on the legs is *very close* to the area of the square on the hypotenuse. I say “very close” because the process of measuring is always inexact, and therefore your calculations and results will probably not yield true equality, just something close.

5. For each set of lengths below, determine whether the lengths form an acute, right, or obtuse triangle—or *no* triangle. You can construct the triangles using a compass and a ruler and also use the Pythagorean theorem.

a. 9, 6, 4

b. 13, 11, 10

c. 12, 14, 28

d. 15, 20, 25

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Chapter 10: Probability

Introduction

Probability is new to Math Mammoth students, as the topic doesn't appear at all in grades 1-6. However, most students have an intuitive understanding of probability based on hearing the terms “probably” and “likely,” listening to weather forecasts, and so on.

In the past, probability wasn't taught until high school—for example, I personally encountered it for the first time in 12th grade. However, since probability is such a useful and easily accessible field of math, it was felt that it should be introduced sooner, so during the 1990s and 2000s it “crept” down the grade levels until many states required probability even in elementary school. The Common Core Standards include probability starting in 7th grade. I feel that is good timing because by 7th grade students have studied fractions, ratios, and proportions, so they have the tools they need to study probability. Moreover, they will need an understanding of the basic concepts of probability in order to understand the statistical concepts that they will study in middle school and high school.

In this chapter we start with the concept of simple (classic) probability, which is defined as the ratio of the number of favorable outcomes to the number of all possible outcomes. Students calculate probabilities that involve common experiments, to include flipping a coin, tossing a pair of dice, picking marbles, or spinning a spinner.

The lesson *Probability Problems from Statistics* introduces probability questions involving the phrase “at least,” which are often solved by finding the probability of the complement event. For example, it might be easier to count the number of students who got at most D+ on a test than to count the number of students who got at least C-.

In the next lesson, *Experimental Probability*, students conduct experiments, record the outcomes, and calculate both the theoretical and experimental probabilities of events, in order to compare the two. They will draw a card from a deck or roll a die hundreds of times. The download version of this curriculum includes spreadsheet files for some of the lengthier probability simulations. You can also access those simulations at the web page http://www.mathmammoth.com/lessons/probability_simulations.php

Next, we study compound events, which combine two or more individual simple events. Tossing a die twice or choosing first a girl then a boy from a group of people are compound events. Students calculate the probabilities of compound events by using the complete sample space (a list of all possible outcomes). They construct the sample space in several ways: by drawing a tree diagram, by making a table, or simply by using logical thinking to list all the possible outcomes.

The last major topic in this chapter is simulations. Students design simulations to find the probabilities of events. For example, we let heads represent “female” and tails represent “male,” so we can toss a coin to simulate the probability of choosing a person of either sex at random. Later in the lesson, students design simulations that use random numbers. They generate those numbers by using either the free tool at <http://www.random.org/integers> or a spreadsheet program on a computer.

In the last lesson of the chapter, *Probabilities of Compound Events*, we learn to calculate the probability of a compound event by *multiplying* the probabilities of the individual events (assuming the outcomes of the individual events are independent of each other). This topic exceeds the Common Core Standards for 7th grade and thus is optional. I have included it here because the idea studied in the lesson is very simple and I feel many students will enjoy it.

You can find matching videos for some of the topics in this chapter at http://www.mathmammoth.com/videos/probability/probability_lessons.php

The Lessons in Chapter 10

	page	span
Probability	222	3 pages
Probability Problems from Statistics	225	2 pages
Experimental Probability	227	3 pages
Count the Possibilities	230	6 pages
Using Simulations to Find Probabilities	236	6 pages
Probabilities of Compound Events	242	4 pages
Chapter 10 Mixed Review	246	3 pages
Chapter 10 Review	249	2 pages

Helpful Resources on the Internet

Probability Videos by Maria

These video lessons cover topics that have been chosen to complement the lessons in this chapter.

http://www.mathmammoth.com/videos/probability/probability_lessons.php

SIMPLE PROBABILITY

Probability Game with Coco

A multiple-choice online quiz on simple probability.

<http://www.math-play.com/Probability-Game.html>

Probability Circus

Choose the spinner that matches the probability in this interactive online activity.

http://www.hbschool.com/activity/probability_circus/

Simple Probability

Practice finding probabilities of events, such as rolling dice, drawing marbles out of a bag, and spinning spinners.

https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-probability-statistics/cc-7th-basic-prob/e/probability_1

Card Sharks Game

Use your knowledge of probability to bet on whether the next card is higher or lower than the last one.

<http://mrnussbaum.com/cardsharks/>

Mystery Spinners

In this activity, you need to find the hidden mystery spinner working from only one clue.

<http://www.scottle.edu.au/ec/viewing/L2384/index.html>

Simple Probability Quiz

Reinforce your probability skills with this interactive self-check quiz.

<http://www.phschool.com/webcodes10/index.cfm?wcprefix=ara&wcsuffix=1201>

EXPERIMENTAL PROBABILITY

Probability Tools

Play around with dice, coins, spinners, playing cards, counters and digit cards. In each type of game, you can choose from a variety of settings such as number of dice, number of trials, whether to display results as a frequency diagram or table.

<http://www.interactive-maths.com/probability-tools-flash.html>

Sample worksheet from

www.mathmammoth.com

Adjustable Spinner

Create a virtual spinner with variable-sized sectors to compare experimental results to theoretical probabilities. You can choose the sizes of the sectors, the number of sectors, and the number of trials.

<http://www.shodor.org/interactivate/activities/AdjustableSpinner/>

Interactive Customizable Spinners

Use a tool to build colored spinners. Then, test the spinner over a number of spins. Compare the actual results with the expected results.

<https://fuse.education.vic.gov.au/Resource/LandingPage?ObjectId=8eb446b6-bd1e-446a-848a-935dce8b0b70>

Dice Duels Tool

Explore the experimental probability distributions when you roll between two and five dice, and either add, subtract, or multiply the numbers. The tool graphs the results, and can do up to 9,999 rolls.

<https://fuse.education.vic.gov.au/Resource/LandingPage?ObjectId=f8341288-4733-4604-abf3-7c1d9de7fc4b>

Dice Roll

Choose the number of virtual dice to roll and how many times you want to roll them. The page shows both the actual results and expected (theoretical) probabilities, and the simulation works for a very large numbers of rolls.

<http://www.btwaters.com/probab/dice/dicemain3D.html>

Coin Flip

This virtual coin toss shows the results numerically and can generate at least 100,000 flips.

<http://www.btwaters.com/probab/flip/coinmainD.html>

Coin Toss Simulation

Another virtual coin toss. This one shows the results both using images of coins and numerically.

<http://syzygy.virtualave.net/multicointoss.htm>

Rocket Launch

A three-stage rocket is about to be launched. In order for a successful launch to occur, all three stages of the rocket must successfully pass their pre-takeoff tests. By default, each stage has a 50% chance of success, however, this can be altered by dragging the bar next to each stage. Observe how many tries it takes until there is a successful launch.

<http://mste.illinois.edu/activity/rocket/>

Find the Bias

A die (one cube of dice) has been weighted (loaded) to favor one of the six numbers. Roll the die to work out which is the favored face. Explore how many rolls are needed for you to be reasonably sure of a conclusion.

<https://fuse.education.vic.gov.au/Resource/LandingPage?ObjectId=0ec1fe96-7c91-4975-9863-766a1fe9c1c5>

Load One Dice (Biased dice)

Make a biased die by loading it to favor one of the six numbers. Then roll the loaded die, and compare the shape of theoretical data distributions with experimental results.

<https://fuse.education.vic.gov.au/Resource/LandingPage?ObjectId=b60d5135-608d-4fd6-8e65-a81fa1dc172a>

Racing Game with One Die

Explore how experimental probability relates to fair and unfair games with this two-car race. You choose which and how many numbers of the die make each of the cars move. Other options include the number of trials and the length of the race (in segments). The program calculates the percentage of wins for each car and draws a pie chart.

<http://www.shodor.org/interactivate/activities/RacingGameWithOneDie/>

COMPOUND EVENTS AND TREE DIAGRAMS

Tree Diagrams and Spinners Quiz

Practice reading tree diagrams in this interactive 10-question quiz.

<https://www.thatquiz.org/tq/practicetest?8x24p16y3o5t>

Lucky 16 Game

You place counters on the game board, and then they will be removed based on the sum of two dice that are rolled. Try to predict the best positions for the counters before the game starts.

<https://fuse.education.vic.gov.au/Resource/LandingPage?ObjectId=31ee684d-742e-4fe3-bb55-e8e6e61e0d6d>

Airport Subtraction Game

This game is based on rolling two dice and subtracting the results. Your task is to place your plane at the back of the queue on one of the runways. Try to predict which lane is most likely to clear quickly.

<https://fuse.education.vic.gov.au/Resource/LandingPage?ObjectId=0c1b063f-52ec-4564-ac6e-265ddcbdcada>

Quiz: Compound Probability

Test your knowledge of compound probability with this interactive self-check quiz.

<http://www.phschool.com/webcodes10/index.cfm?wcprefix=aba&wcsuffix=1205>

How could I send the check and not pay the bill?

What is the probability that Tessellation will put each of the three checks into the correct envelopes if she does it randomly? The page includes a hint and a complete solution (click “answer” at the bottom of page).

<http://figurethis.nctm.org/challenges/c69/challenge.htm>

Flippin’ Discs

In this interactive activity, you throw two discs. You win if they both show the same color. You can run the game 100 times and see the detailed results. Can you explain why you win approximately half the time? Explore the situation also with 3, 4, and even 5 discs. The solution is found in a link near the top left of the page.

<http://nrich.maths.org/4304>

“Data Analysis & Probability Games” from MathWire

A list of board and dice games to help to teach topics appropriate for beginners in probability.

<http://mathwire.com/games/datagames.html>

Cross the Bridge

This is a printable board game based on throwing two dice and the probabilities for the sum of the dice.

<http://www.mathsphere.co.uk/downloads/board-games/board-game-17-crossing-the-river.pdf>

At Least One...

The tree diagram and related discussion on this page guides students’ thinking to help them answer probability questions like, “What is the probability of getting at least one head by flipping a coin ten times?” A link near the top left of the page leads to the solution.

<http://nrich.maths.org/7286>

SIMULATIONS

Probability Simulations in Excel

These spreadsheet files match some of the lengthier probability simulations in this chapter.

http://www.mathmammoth.com/lessons/probability_simulations.php

Random Integer Generator

Choose how many numbers, how many columns, and the values of the integers and then click to generate.
<https://www.random.org/integers/>

Coin Toss Simulation

Another virtual coin toss. This one shows the results both using images of coins and numerically.
<http://syzygy.virtualave.net/multicointoss.htm>

Marbles

Run repeated experiments where you draw 1, 2, or 3 marbles from a set of blue, red, purple, and green marbles. The results shown include the frequencies of each possible outcome, the experimental probabilities, and the theoretical probabilities. This activity can be used not only to explore probabilities but also to perform simulations.
<http://www.shodor.org/interactivate/activities/Marbles/>

Interactivate: *Fire!!* and *Directable Fire!!*

In these two activities, you first set the probability that a fire will spread from tree to tree in a forest of 100 trees. Then you click the tree where the fire starts and watch it spread. In the Directable Fire activity, you can set the probabilities for each direction to be different. Repeat the activity several times to see that the amount of forest that burns varies (for any set probability of fire spreading).
<http://www.shodor.org/interactivate/activities/Fire/>
<http://www.shodor.org/interactivate/activities/DirectableFire/>

COMPOUND PROBABILITY

Probability Quiz

Test your knowledge about probability in this interactive self-check quiz.
<http://www.phschool.com/webcodes10/index.cfm?wcprefix=ama&wcsuffix=7454>

Probability of Compound Events Quiz

Practice finding the probability of compound events. Some problems deal with replacement and non-replacement.
<https://maisonetmath.com/probability/quizzes/409-probability-of-compound-events>

Probability Quiz

Reinforce your skills with this interactive multiple-choice quiz.
<http://www.phschool.com/webcodes10/index.cfm?wcprefix=aqa&wcsuffix=1005&area=view>

FOR FUN

Monty Hall Paradox

Try this interactive version of the famous Monty Hall problem. Behind which door is the car? If you choose the wrong one, you'll win a goat instead.
<http://www.math.ucsd.edu/~crypto/Monty/monty.html>

What Does Random Look Like?

This problem challenges our thinking about randomness. Make up a sequence of twenty Hs and Ts that *could* represent a sequence of heads and tails generated by a fair coin. Then use the animation to generate truly random sequences of 20 coin flips. Can you learn how to spot fakes?
<http://nrich.maths.org/7250>

Same Number

Imagine you are in a class of thirty students. The teacher asks everyone to secretly write down a whole number between 1 and 225. How likely is it for everyone's numbers to be different? The web page provides an interactive simulation so you can experiment with this problem. The following discussion also leads students to the classic birthday problem. The solution is found in a link near the top left of the page.
<http://nrich.maths.org/7221>

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Counting the Possibilities

A **sample space** is a list of all possible outcomes of an experiment.

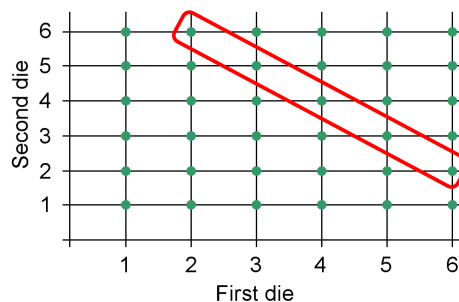
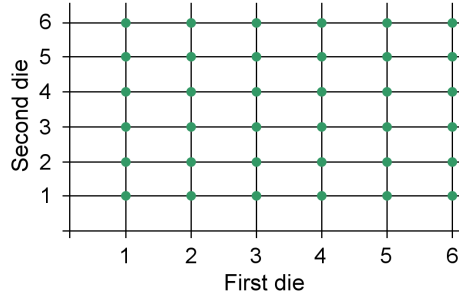
Example 1. We roll two dice. The sample space for this experiment is shown in the grid on the right. Each dot represents one outcome. For example, the point (1, 4) means that the first die shows 1 and the second die shows 4.

Notice that there are a total of $6 \cdot 6 = 36$ possible outcomes.

What is the probability of getting the sum of 8 when rolling two dice? The chart helps answer that question. First we find out and count how many outcomes give you the sum 8:

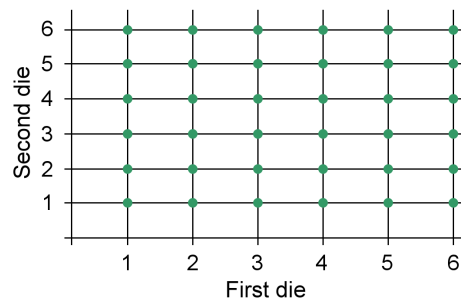
You could roll $2 + 6$, $3 + 5$, $4 + 4$, $5 + 3$, or $6 + 2$. Those number pairs are circled in the second graphic.

So there are five favorable outcomes out of 36 possible. Therefore, the probability of getting 8 as a sum is $5/36$.



1. **a.** How many outcomes are there for rolling the same number on both dice (such as (5, 5))?

b. What is the probability of rolling the same number on both dice?



2. **a.** What is the probability of rolling 5 on the first die and 6 on the second?

b. What is the probability of rolling 5 on one die and 6 on the other?

c. What is the probability of getting a sum of 7 when rolling two dice?

d. What is the probability of getting a sum of at least 6 when rolling two dice?

3. You roll a six-sided die twice. Find the probabilities.

a. $P(1; 5)$

c. $P(\text{even}; \text{odd})$

b. $P(2; 5 \text{ or } 6)$

d. $P(6; \text{not } 6)$

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Chapter 11: Statistics

Introduction

Math Mammoth Grade 7 ends with a study of statistics. The chapter begins with a review lesson to remind students how to make a boxplot and a histogram and how to calculate the mean absolute deviation—all topics that were studied in 6th grade.

The first focus area of the chapter is random sampling. Students learn that sampling methods vary and that random sampling is likely to produce an *unbiased sample*—a sample that represents the population well. In the lesson *Using Random Sampling*, students choose several random samples from a population of 100 geometric shapes, and they see first hand that random samples can vary—even a lot. Yet if the sample size is sufficiently large, or if we have several random samples, we can be relatively confident in concluding something about the population itself. Students will also practice making inferences about populations based on several random samples.

The second major topic is comparing two populations, either directly or by using samples from the populations. Students learn to use the overall distributions and the measures of center and variability to compare two sets of data in various ways. While some of the ways in which we compare the data are informal only, all of the concepts presented are fundamental to the use of statistics in various sciences. Students also do a project where they gather data on their own from two populations and compare them.

If you are using the curriculum as a traditional pre-algebra program (as a preparation for algebra studies), you can consider omitting this chapter.

The Lessons in Chapter 11

	page	span
Review of Data Analysis	255	5 pages
Random Sampling	260	4 pages
Using Random Sampling	264	7 pages
Comparing Two Populations	271	8 pages
Comparing Two Samples	279	6 pages
Chapter 11 Mixed Review	285	4 pages
Chapter 11 Review	289	3 pages

Helpful Resources on the Internet

GRAPHS AND PLOTS

Statistics videos by Maria

Videos on statistics topics that are helpful for the lessons in this chapter.

http://www.mathmammoth.com/videos/statistics/statistics_lessons.php

Statistics Interactive Activities from Shodor

A set of interactive tools for exploring and creating boxplots, histograms, dot plots, and stem-leaf plots. You can enter your own data or explore the examples.

<http://www.shodor.org/interactivate/activities/BoxPlot/>

<http://www.shodor.org/interactivate/activities/Histogram/>

<http://www.shodor.org/interactivate/activities/PlopIt/>

<http://www.shodor.org/interactivate/activities/StemAndLeafPlotter/>

Analyzing and Displaying Data Gizmos from ExploreLearning

Gizmos are interactive online tools with lesson plans that allow you to explore and learn about the topic in a virtual, dynamic environment. This page includes gizmos for box-and-whisker plots, histograms, stem-and-leaf plots, polling, and more. The gizmos work for 5 minutes for free. You can also sign up for a free trial account.

<https://www.explorelarning.com/index.cfm?method=cResource.dspResourceExplorer&browse=Math/Grade+6-8/Data+Analysis+and+Probability>

Box-and-Whisker Plots Quiz

Review box-and-whisker plots with this interactive self-check quiz.

<http://www.phschool.com/webcodes10/index.cfm?wcprefix=bja&wcsuffix=1203>

Create A Box and Whisker Chart

An online tool for creating a box-and-whisker plot from your own data. Includes lots of options.

<https://www.meta-chart.com/box-and-whisker>

Stem-and-Leaf Plots Quiz

An online multiple-choice quiz that is created randomly. Refresh the page (or press F5) to get another quiz.

<http://www.phschool.com/webcodes10/index.cfm?wcprefix=asa&wcsuffix=0905&area=view>

Make Your Own Stem-and-Leaf Plot

Enter values from your own data, and this web page creates your stem-and-leaf plot for you.

<http://www.mrnussbaum.com/graph/sl.htm>

Comparing Data Displays

Practice interpreting and comparing dot plots, histograms, and box plots in this interactive online activity.

<https://www.khanacademy.org/math/pre-algebra/pre-algebra-math-reasoning/pre-algebra-frequency-dot-plot/e/comparing-data-displays>

Double Box-and-Whisker Plots Practice

Interactive practice questions about double box-and-whisker plots. The site may require a login with your Google or Facebook account or a free registration.

<http://www.ck12.org/statistics/Double-Box-and-Whisker-Plots/asmtpractice/Double-Box-and-Whisker-Plots-Practice/>

Displaying Univariate Data Practice

Interactive practice questions about distributions and various types of plots.

<http://www.ck12.org/statistics/Displaying-Univariate-Data/asmtpractice/Displaying-Univariate-Data-Practice/>

STATISTICAL MEASURES

Mean, Median, Mode, and Range

A lesson that explains how to calculate the mean, median, and mode for a set of data given in different ways. It also has interactive exercises.

http://www.cimt.org.uk/projects/mepres/book8/bk8i5/bk8_5i2.htm

Measures of Center and Quartiles Quiz from ThatQuiz.org

An online quiz about the measures of center and quartiles in boxplots, stem-and-leaf plots, and dot plots.

<http://www.thatquiz.org/tq-5/?-jr0t0-l1-p0>

GCSE Bitesize Mean, Mode and Median

Lessons with simple explanations and examples.

<http://www.bbc.co.uk/schools/gcsebitesize/maths/statistics/measuresofaveragerev1.shtml>

Quiz: Boxplots and Stem-and-leaf Plots

Practice interpreting graphs with this interactive online quiz.

<https://www.thatquiz.org/tq/practicetest?pw4bt4kw18id6>

Mean Deviation

A simple explanation about what the mean absolute deviation is, how to find it, and what it means.

<http://www.mathsisfun.com/data/mean-deviation.html>

Mean Absolute Deviation

Several videos explaining how to calculate the mean absolute deviation of a data set.

<http://www.onlinemathlearning.com/measures-variability-7sp3.html>

Working with the Mean Absolute Deviation (MAD)

A tutorial and questions where you are asked to create line plots with a specified mean absolute deviation.

http://www.learner.org/courses/learningmath/data/session5/part_e/working.html

SAMPLING

Random and Biased Sampling

A comprehensive lesson to read that explains about unbiased types of sampling.

(If the link does not work, then copy and paste it into your browser.)

<http://www.ck12.org/na/Random-and-Biased-Sampling-in-a-Population---7.SP.1,2-1/lesson/user%3Ac2ZveDJA3N3ZWdvLm9yZw../Random-and-Biased-Sampling-in-a-Population---7.SP.1%252C2/>

Identify a Random Sample

What a sample and random sample. On the left of the web page are 3 other videos on (unbiased) and biased samples.

https://learnzillion.com/lesson_plans/5266-identify-a-random-sample

Analyze Numbers of Jubes

Test a machine that randomly packages three types of fruit jubes. Look at patterns in numbers of jube types.

<https://fuse.education.vic.gov.au/Resource/LandingPage?ObjectId=c8dad524-5a86-4cf3-8ff4-cfe81813e230>

Random or not? Analyse Runs of Jubes

Look at patterns in sequences of jube types, analyse the results of large samples, and much more.

<https://fuse.education.vic.gov.au/Resource/LandingPage?ObjectId=2a4c7dd8-2319-46b2-a4d9-797174223fec>

What Does Random Look Like?

Generate several sequences of twenty coin flips. Get a feel for what you would expect a random sequence to have.

<http://nrich.maths.org/7250>

Making Inferences about a Population by Analyzing Random Samples

This is a short instructional video about random sampling.

https://learnzillion.com/lesson_plans/6910-make-inferences-about-a-population-by-analyzing-random-samples

Random Sampling

Multiple-choice questions that test your understanding of the basics of random sampling.

<https://www.khanacademy.org/math/recreational-math/math-warmup/random-sample-warmup/e/random-sample-warmup>

Valid Claims

Practice figuring if we took a random sample and are we able to draw valid conclusions from the data.
<https://www.khanacademy.org/math/probability/statistical-studies/statistical-questions/e/valid-claims>

Making Inferences from Random Samples

Questions about what can reasonably be inferred, from a random sample, about an entire population
<https://www.khanacademy.org/quetzalcoatl/content-improvement/middle-school-content/e/making-inferences-from-random-samples>

Population and Sample (Random Sampling)

A detailed explanation about obtaining a random sample from a population. Includes practice problems.
https://www.learner.org/courses/learningmath/data/session1/part_d/index.html

Matchbox Machine: Take a Sample

take a sample of 100 matchboxes and make a boxplot to analyze the results, and much more.
<https://fuse.education.vic.gov.au/Resource/LandingPage?ObjectId=38397b5d-25b8-4523-a0cd-0fc627a8a241>

Fix the Matchbox Machine: Scoop Size and Speed

Check whether a machine is packing most matchboxes with an acceptable number of matches, look at boxplots made after taking samples of 100 matchboxes, and much more.
<https://fuse.education.vic.gov.au/Resource/LandingPage?ObjectId=4087511a-60a5-48ec-b866-998882cc83cb>

Polling: Neighborhood Gizmo

Conduct a phone poll of citizens in a small neighborhood to determine their response to a yes-or-no question. The gizmo works for 5 minutes for free. A free trial account is also available.
<http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=507>

Make inferences about a population by analyzing random samples

A video lesson that teaches how to make inferences about a population based on random samples.
https://learnzillion.com/lesson_plans/6910-make-inferences-about-a-population-by-analyzing-random-samples

Capture-Recapture Method

Learn how this method for estimating population size works. The second link is an interactive simulation.
<https://nrich.maths.org/9609>

http://www.cengage.com/biology/discipline_content/animations/capture_recapture.html

COMPARING TWO POPULATIONS

Comparing Populations - Khan Academy

Multiple-choice questions to practice comparing centers of distributions in terms of their spread.
<https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-probability-statistics/cc-7th-population-sampling/e/comparing-populations>

Grade 7 Mathematics Module 5, Topic D, Lesson 22

Students express the difference in sample means as a multiple of a measure of variability, and learn what a difference in sample means can show about the population means.
<https://www.engageny.org/resource/grade-7-mathematics-module-5-topic-d-lesson-22>

Grade 7 Mathematics Module 5, Topic D, Lesson 23

Students use data from random samples to draw informal inferences about the difference in population means.
<https://www.engageny.org/resource/grade-7-mathematics-module-5-topic-d-lesson-23>

FACTS AND FIGURES

GapMinder

Visualizing human development trends (poverty, health, gaps, income) This is an interactive, dynamic tool.
<https://www.gapminder.org/tools>

WorldOdometers

World statistics updated in real time. Useful for general educational purposes— for some stunning facts.
<http://www.worldometers.info>

Sample worksheet from
www.mathmammoth.com

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Comparing Two Populations

In this lesson, we will use measures of center and variability to compare two sets of data.

Example 1. The two dot plots on the right show the ages of two different groups of children. The top plot shows a group of children of ages 5 to 7, and the bottom plot a group of ages 3 to 5.

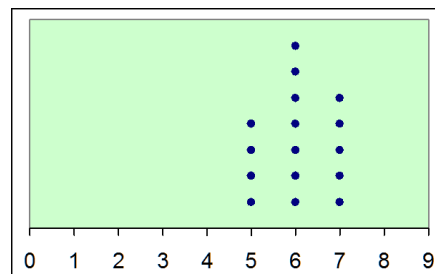
The center of the top plot is its median at 6 years old. The center of the bottom plot is its median at 4 years old. Each distribution is clustered about its median with little variability (spread). Although both groups include 5-year-old children, there is otherwise no overlap in ages.

Intuitively we notice that the ages of these two groups are distinctly different. In statistical terms, we would say that there is a **significant difference** in the ages of the two groups.

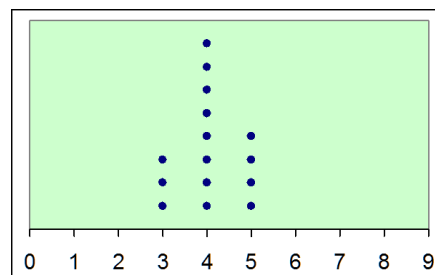
There is a way to quantify the significance of the difference numerically: **compare the difference in the measures of center to the measure of variability.**

The measures of center—the medians—are 6 years old and 4 years old, so the difference between them is 2 years. The measure of variability—the interquartile range or IQR—of the first group is 1.5 years and of the second, 1 year. Let's round to 1 year in the comparison.

The difference in the medians (2 years) is about twice the measure of variability (about 1 year). This means that the difference is indeed significant. If the difference in centers had been only a fraction ($1/2$ or less) of the measure of variability, then the difference would not have been significant.



median 6 years IQR 1.5 years



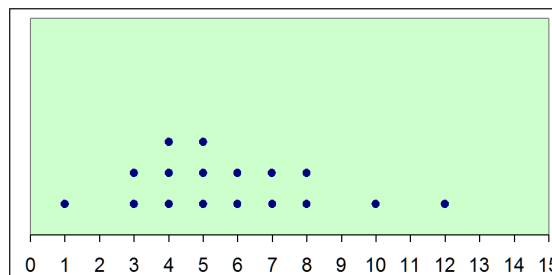
median 4 years IQR 1 year

Example 2. The two dot plots on the right again show the ages of two different groups of children.

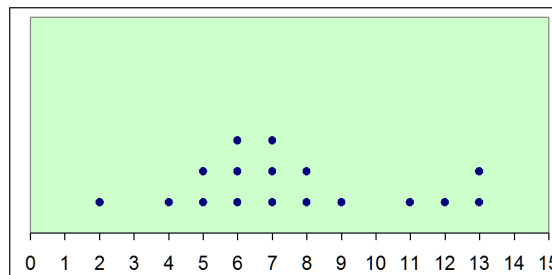
This time, we can see from the plots that there is great variability in the ages in both groups. The data is very spread out. The data sets also overlap a lot: the first group has children from 1 to 12 years old, and the second from 2 to 13 years old, which means the overlap is from 2 to 12 years old.

The difference in the medians is 2 years. However, the interquartile range is much bigger now (3.5 and 4.5 years; we can round it to 4 years). Therefore, the difference in the medians (2 years) is only about $1/2$ of the measure of variability.

This fact, along with the large overlap, helps us to see that the difference in the medians is not very significant. The ages of these groups of children are not greatly different.



median 5 years IQR 3.5 years



median 7 years IQR 4.5 years

Example 3. The two graphs show science test scores for two classes, 7-A and 7-B. Which class did better, generally speaking?

We can see the answer just by looking at the distributions: The bars in the graph for 7-B are more skewed towards the right than the bars in the graph for 7-A. So class B did better.

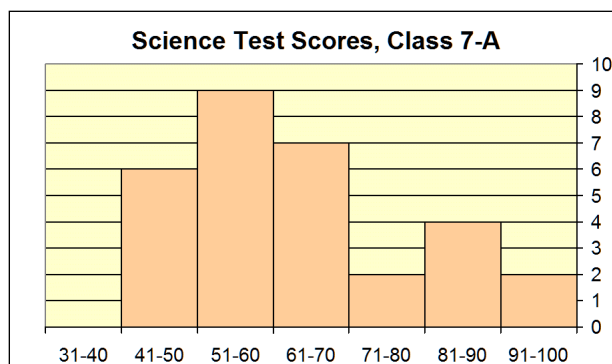
To find out *how much* better, we will compare the means of both data sets. The mean test score for class 7-A was 64.2 points and for class 7-B it was 74.8 points. The difference is 10.6 points.

Is that a significant difference?

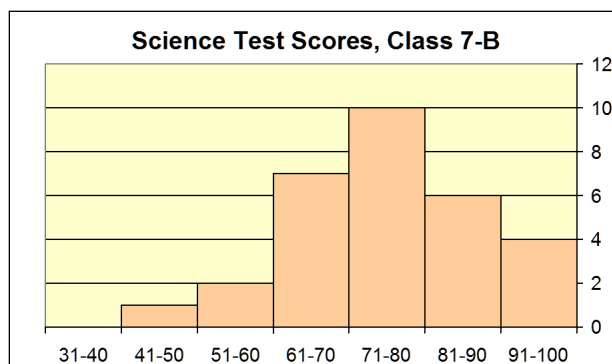
To check that, we compare the difference of 10.6 points to how variable or spread out the distributions are. The more variability there is in the two distributions, the bigger the difference in the means has to be for it to be significant.

We will use mean absolute deviation as a measure of variability. The mean absolute deviation of each data set is close to 11 points. So the difference in the two means (10.6 points) is about one time the measure of variability. That tells us that the difference *is* significant.

If the difference in the means had been, say, 0.3 times the measure of variability (only 3 points), then the difference wouldn't have been significant.

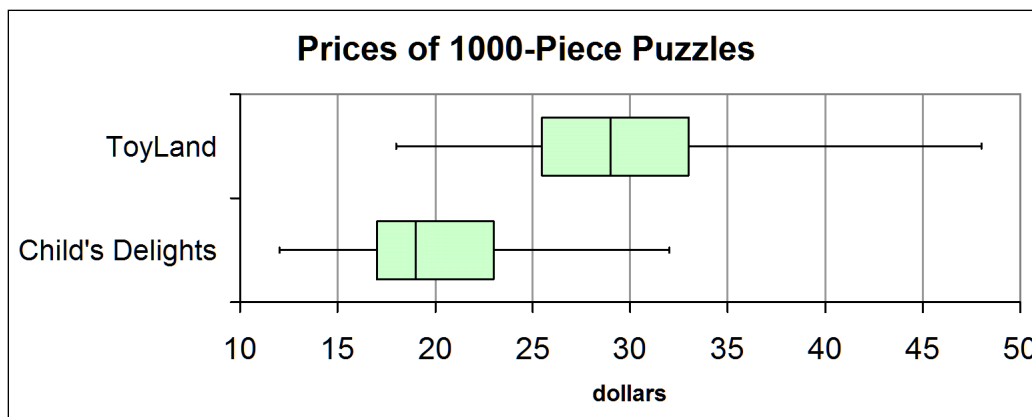


mean 64.2 MAD 11.8



mean 74.8 MAD 10.3

Example 4. The boxplots show the prices of 1000-piece puzzles in two stores, ToyLand and Child's Delights.

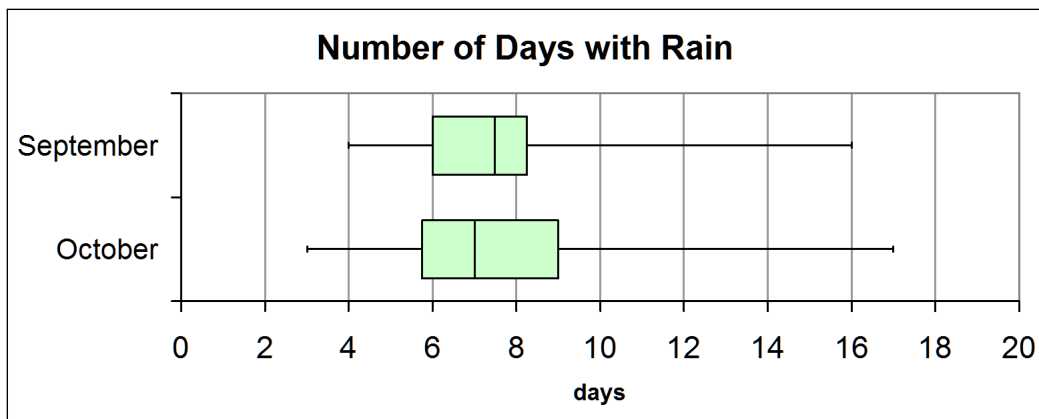


Boxplots make comparing sets of data very easy, since you can immediately see both the centers (medians) and the spread (interquartile range) of the data sets from the plot.

We can see from the medians that, overall, the puzzles in Child's Delights are cheaper. The prices in Toyland vary more, though, so you can find some cheap puzzles there, as well.

But while the ranges of the prices are quite different, the interquartile ranges (the lengths of the boxes) are similar: about 8 dollars for ToyLand and about 6 dollars for Child's Delights. The difference in the medians is about 10 dollars, which is about 1.5 times the interquartile range. That is a significant difference.

1. Jim studied the rain patterns in his home town. He made boxplots to show the number of days with rain in September and October from data collected over 20 years.



a. Describe the overlap of the two distributions.

b. Based on the medians, overall which month has more days with rain?

Which month has the greater variability in the number of days with rain?

c. Estimate from the plot the medians for October and September and their difference.

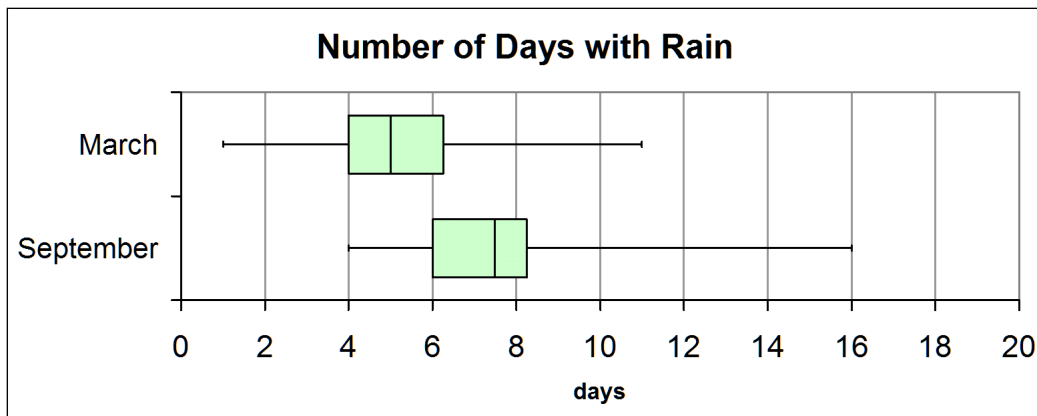
median (September) _____ median (October) _____ difference _____

d. Estimate from the plot the interquartile ranges for October and September.

IQR (September) _____ IQR (October) _____

e. Based on your answers to (c) and (d), is the difference in the medians significant?

2. Jim also made boxplots to compare the number of days with rain in March and September.



a. We can see that these two distributions overlap each other quite a bit, but not completely. Let's say that a certain month had 4 rainy days. Which is more likely, that the month was March or September?

b. Let's say that a certain month had 8 rainy days. Which is more likely, that the month was March or September?

c. Based on the plots, overall which month has more days with rain?

Which month has the greater variability in the number of days with rain?

d. Find the difference in the medians and the interquartile ranges.

e. Compare the difference in the medians to the variability of the data.

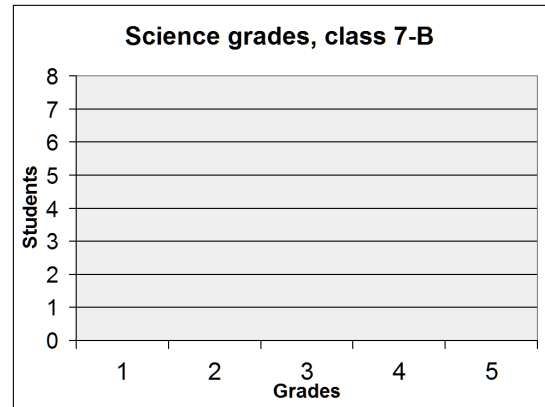
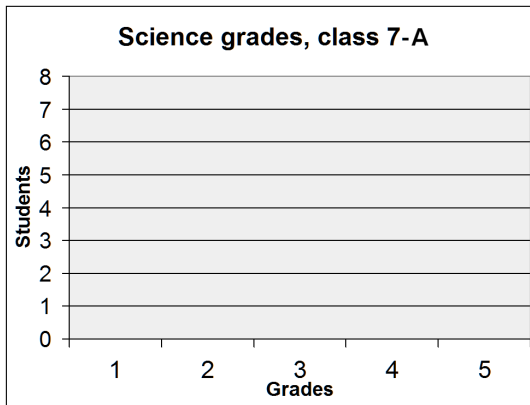
The difference in the medians is about _____ times the interquartile range.

Are there significantly more days with rain in September than in March?

3. The following are the science grades of two 7th grade classes. This school grades on a five-point system where 5 = A, 4 = B, 3 = C, 2 = D, and 1 = F. Make bar graphs from the data.

Class 7-A	
Grades	Students
1	5
2	8
3	7
4	5
5	2

Class 7-B	
Grades	Students
1	3
2	6
3	7
4	7
5	4



a. Compare the bar graphs visually.

Based on the graphs, did either class do better overall? If so, which one?

Did either class have more variability in the grades? If so, which one?

b. Now, calculate the mean of the grades for each class and the difference in the means.

Class A, mean: _____ Class B, mean: _____ Difference: _____

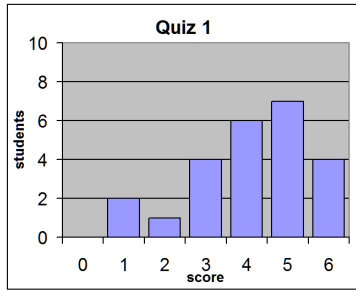


c. The mean absolute deviations of the data are 1.01 (class A) and 1.02 (class B).

The numbers are quite close. This means the variability is similar in both sets of data.

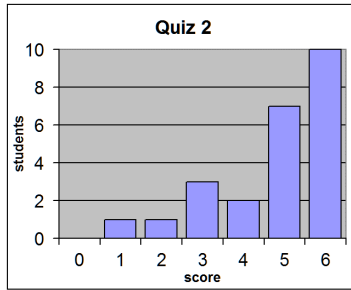
Compare the difference in the mean scores to the variability of the data, and use that to explain whether one of the classes did *significantly* better than the other.

4. Mrs. Ross gave her calculus class three quizzes. The bar graphs for the scores are below.



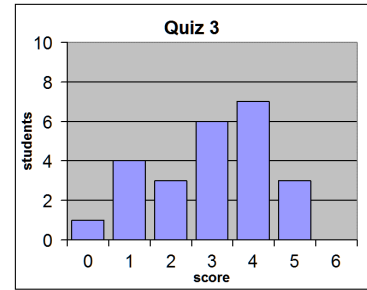
mean _____

MAD 1.14



mean _____

MAD 1.13



mean _____

MAD 1.22

- Look at the graphs. Mrs. Ross felt one of the quizzes turned out too easy (the students didn't!). Which one?
- In which quiz did the students fare the worst?
- The mean scores for the three quizzes were: 2.96, 4.13, and 4.79. Match each mean with the correct graph.
- Compare quiz 2 and quiz 3 now. What is the difference in the means for quiz 2 and quiz 3? _____
 This difference is about _____ times the mean absolute deviation of the data (1.13).
 Is the difference in the means significant?