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# Foreword

Math Mammoth Grade 7 comprises a complete math curriculum for the seventh grade mathematics studies. This is a pre-algebra course, so students can continue to an algebra 1 curriculum after studying this.

The curriculum meets and actually exceeds the Common Core Standards (CCS) for grade 7. The two major areas where it exceeds those standards are linear equations (chapter 5) and the Pythagorean Theorem (chapter 9). Linear equations are covered in more depth than the CCS requires, and the Pythagorean Theorem belongs to grade 8 in the CCS. You can access a document detailing the alignment information either on the Math Mammoth website or in the download version of this curriculum.

The main areas of study in Math Mammoth Grade 7 are:

- The basics of algebra (expressions, equations, inequalities, graphing);
- Integers;
- Ratios, proportions, and percent;
- Geometry;
- Probability and statistics.

This book, 7-A, covers the language of algebra (chapter 1), integers (chapter 2), one-step equations (chapter 3), rational numbers (chapter 4), and equations and inequalities (chapter 5). The rest of the topics are covered in the 7-B worktext.

Some important points to keep in mind when using the curriculum:

- The two books (parts A and B) are like a “framework”, but you still have a lot of liberty in planning your student’s studies. The five chapters in part 7-A are best studied in the order presented. However, you can study the chapters on geometry, probability, and statistics at most any point during the year. The chapters on ratios & proportions and percent (in part 7-B) are best left until the student has learned to solve one-step equations (in chapter 3).

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. However, you can still use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously. This type of flexible use of the curriculum enables you to truly individualize the instruction for the student.

- Don’t automatically assign all the exercises. Use your judgment, trying to assign just enough for your student’s needs. You can use the skipped exercises later for review. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.
- For review, the curriculum includes a worksheet maker (Internet access required), mixed review lessons, additional cumulative review lessons, and the word problems continually require usage of past concepts. Please see more information about review (and other topics) in the FAQ at <https://www.mathmammoth.com/faq-lightblue.php>

I heartily recommend that you view the full user guide for your grade level, available at <https://www.mathmammoth.com/userguides/>

Lastly, you can find free videos matched to the curriculum at <https://www.mathmammoth.com/videos/>

*I wish you success in teaching math!*

*Maria Miller, the author*

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# Chapter 1: The Language of Algebra

## Introduction

In the first chapter of *Math Mammoth Grade 7* we review all of the sixth grade algebra topics and also study some basic properties of the operations.

The main topics are the order of operations, expressions, and simplifying expressions in several different ways. The main principles are explained and practiced both with visual models and in abstract form, and the lessons contain varying practice problems that approach the concepts from various angles.

This chapter is like an introduction that lays a foundation for the rest of the year. For example, when we study integers in the next chapter, students will once again simplify expressions, just with negative numbers. Then when we study equations in chapters 3 and 5, students will again simplify expressions, use the distributive property, and solve equations.

Please note that it is not recommended to assign all the exercises by default. Use your judgment, and strive to vary the number of assigned exercises according to the student's needs. See the user guide at <https://www.mathmammoth.com/userguides/> for some further thoughts on using and pacing the curriculum.

You can find matching videos for topics in this chapter at <https://www.mathmammoth.com/videos/> (choose grade 7).

### The Lessons in Chapter 1

	page	span
The Order of Operations .....	11	4 pages
Expressions and Equations .....	15	3 pages
Properties of the Four Operations .....	18	4 pages
Simplifying Expressions .....	22	4 pages
Growing Patterns 1 .....	26	3 pages
The Distributive Property .....	29	5 pages
Chapter 1 Review .....	34	2 pages

### Helpful Resources on the Internet

#### ORDER OF OPERATIONS

##### Otter Rush

Practice exponents in this otter-themed math game.

[http://www.mathplayground.com/ASB\\_Otter\\_Rush.html](http://www.mathplayground.com/ASB_Otter_Rush.html)

##### Exponents Jeopardy Game

Practice evaluating exponents, equations with exponents, and exponents with fractional bases in this interactive Jeopardy-style game.

<http://www.math-play.com/Exponents-Jeopardy/exponents-jeopardy-math-game.html>

##### Choose A Math Operation

Choose the mathematical operation(s) so that the number sentence is true. Practice the role of zero and one in basic operations or operations with negative numbers. Helps develop number sense and logical thinking.

<http://www.homeschoolmath.net/operation-game.php>

### **Order of Operations Quiz**

A 10-question online quiz that includes two different operations and possibly parentheses in each question. You can also modify the quiz parameters yourself.

<http://www.thatquiz.org/tq-1/?-j8f-lk-p0>

### **The Order of Operations Millionaire**

Answer multiple-choice questions that have to do with the order of operations, and win a million. Can be played alone or in two teams.

<http://www.math-play.com/Order-of-Operations-Millionaire/order-of-operations-millionaire.html>

### **Exploring Order of Operations (Object Interactive)**

The program shows an expression, and you click on the correct operation (either +, -,  $\times$ ,  $\div$  or exponent) to be done first. The program then solves that operation, and you click on the *next* operation to be performed, *etc.*, until it is solved. Lastly, the resource includes a game where you click on the falling blocks in the sequence that the order of operations would dictate.

[http://www.learnalberta.ca/content/mejhm/html/object\\_interactives/order\\_of\\_operations/use\\_it.html](http://www.learnalberta.ca/content/mejhm/html/object_interactives/order_of_operations/use_it.html)

### **Make 24 Game**

Arrange the number cards, the operation symbols, and the parentheses, so that the expression will make 24.

[http://www.mathplayground.com/make\\_24.html](http://www.mathplayground.com/make_24.html)

### **Order of Operations Practice**

A simple online quiz of 10 questions. Uses parentheses and the four operations.

<http://www.onlinemathlearning.com/order-of-operations-practice.html>

## **WRITING EXPRESSIONS**

### **Algebraic Symbolism Matching Game**

Match each verbal statement with its algebraic expression.

<http://www.quia.com/mc/319817.html>

### **Who Wants to Be a Hundredaire?**

Try to work your way up to \$100 by responding correctly to multiple-choice questions about algebraic expressions.

<http://www.math-play.com/Algebraic-Expressions-Millionaire/algebraic-expressions-millionaire-game.html>

### **Equivalent Algebraic Expressions**

Practice determining whether or not two algebraic expressions are equivalent by manipulating the expressions. These problems require you to combine like terms and apply the distributive property.

<https://www.khanacademy.org/math/algebra-basics/alg-basics-algebraic-expressions/alg-basics-equivalent-expressions/e/equivalent-forms-of-expressions-1>

### **Visual Patterns**

Click on the pattern to see a larger image and the answer to step 43. Can you solve the equation?

<http://www.visualpatterns.org/>

### **Expressions and Variables Quiz**

Choose an equation to match the word problem or situation.

[http://www.softschools.com/quizzes/math/expressions\\_and\\_variables/quiz815.html](http://www.softschools.com/quizzes/math/expressions_and_variables/quiz815.html)

### **Translating Words to Algebraic Expressions**

Match the correct math expression with the corresponding English phrase, such as “7 less than a number”. You can do this activity either as a matching game or as a concentration game.

<https://www.quia.com/jg/1452190.html>

### **Rags to Riches - Verbal and Algebraic Expressions**

Translate between verbal and algebraic expressions in this quest for fame and fortune.

<http://www.quia.com/rr/520475.html>

### **Algebra Noodle**

Play a board game against the computer while modeling and solving simple equations and evaluating simple expressions. Choose level 2 (level 1 is too easy for 7th grade).

<http://www.free-training-tutorial.com/math-games/algebra-noodle.html>

### **Matching Algebraic Expressions with Word Phrases**

Five sets of word phrases to match with expressions.

<https://maisonetmath.com/algebra/algebra-quizzes/292-matching-algebraic-expressions-with-word-phrases>

## **PROPERTIES OF THE OPERATIONS**

### **Properties of Operations at Quizlet**

Includes explanations, online flashcards, and a test for the properties of operations (commutative, associate, distributive, inverse, and identity properties). The inverse and identity properties are not covered in this chapter of Math Mammoth but can be learned at the website. The identity property refers to the special numbers that do not change addition or multiplication results (0 and 1).

<http://quizlet.com/2799611/properties-of-operation-flash-cards/>

### **Commutative/Associative/Distributive Properties Matching Game**

Match the terms and expressions in the two columns.

[http://www.quia.com/cm/61114.html?AP\\_rand=1554068841](http://www.quia.com/cm/61114.html?AP_rand=1554068841)

### **Properties of Multiplication**

Simple online practice about the commutative, associative, distributive, and identity properties of multiplication.

<http://www.aaamath.com/pro74b-propertiesmult.html>

### **Properties of Multiplication**

Simple online practice about the commutative, associative, distributive, and identity properties of multiplication.

<http://www.aaamath.com/pro74ax2.htm>

### **Properties of the Operations Scatter Game**

Drag the corresponding items to each other to make them disappear.

<http://quizlet.com/763838/scatter>

### **Associative, Distributive and Commutative Properties**

Examples of the various properties followed by a simple self-test.

<http://www.mathwarehouse.com/properties/associative-distributive-and-commutative-properties.php>

## **SIMPLIFYING EXPRESSIONS**

### **Simplifying Algebraic Expressions Quiz**

An online quiz of 15 questions.

<http://www.quia.com/quiz/1200540.html>

### **Distributive Property Quiz**

Reinforce your skills with this interactive online quiz.

<https://www.thatquiz.org/tq/practicetest?13y7ctojuwrlbs>

## THE DISTRIBUTIVE PROPERTY

### Factor the Expressions Quiz

Factor expressions such as  $3x + 15$  into  $3(x + 5)$ .

<http://www.thatquiz.org/tq-0/?-jh00-l3-p0>

### Distributive Property Practice

Guided practice for applying the distributive property, such as writing  $-8(-7a + 10)$  as  $56a - 80$ .

<http://www.hstutorials.net/dialup/distributiveProp.htm>

### Distributive Property Game

Solve questions related to the usage of the distributive property amidst playing a game. Play either a bouncing balls game or free-kick soccer game, with the same questions.

[http://reviewgamezone.com/games3/bounce.php?test\\_id=22828&title=DISTRIBUTIVE%20PROPERTY](http://reviewgamezone.com/games3/bounce.php?test_id=22828&title=DISTRIBUTIVE%20PROPERTY)

[http://reviewgamezone.com/games3/freekick.php?test\\_id=22828&title=DISTRIBUTIVE%20PROPERTY](http://reviewgamezone.com/games3/freekick.php?test_id=22828&title=DISTRIBUTIVE%20PROPERTY)

## EVALUATE EXPRESSIONS

### Escape Planet

Choose the equation that matches the words.

[http://www.harcourtschool.com/activity/escape\\_planet\\_6/](http://www.harcourtschool.com/activity/escape_planet_6/)

### Evaluating Expressions Quiz

Includes ten multiple-choice questions.

<https://maisonetmath.com/algebra/algebra-quizzes/290-evaluating-expressions>

### Writing and Evaluating Expressions Quiz

This quiz has 12 multiple-choice questions and tests both evaluating and writing expressions.

<http://www.quibblo.com/quiz/aWAUlc6/Writing-Evaluating-Expressions>

## TERMINOLOGY

### Coefficients, Like Terms, and Constants

How to find and name the coefficients, like terms, and constants in expressions.

<http://mathcentral.uregina.ca/qq/database/qq.09.07/h/maddie1.html>

### Identifying Variable Parts and Coefficients of Terms

After the explanations, you can generate exercises by pushing the button that says “new problem.” The script shows you a multiplication expression, such as  $-(3e)(3z)m$ , and you need to identify its coefficient and variable part, effectively by first simplifying it.

[http://www.onemathematicalcat.org/algebra\\_book/online\\_problems/id\\_var\\_part\\_coeff.htm#exercises](http://www.onemathematicalcat.org/algebra_book/online_problems/id_var_part_coeff.htm#exercises)

### Identifying Parts of Algebraic Expressions

Answer questions about variables, expressions, equations, and inequalities in this multiple-choice test.

[https://reviewgamezone.com/mc/candidate/test/?test\\_id=23122&title=Identifying%20Parts%20of%20Algebraic%20Expressions](https://reviewgamezone.com/mc/candidate/test/?test_id=23122&title=Identifying%20Parts%20of%20Algebraic%20Expressions)

### Algebra - Basic Definitions

Clear definitions with illustrations of basic algebra terminology, including term, coefficient, constant, and expression.

<http://www.mathsisfun.com/algebra/definitions.html>

## GENERAL

### **Balance Beam Activity**

A virtual balance that poses puzzles where the student must think algebraically to find the weights of various figures. Includes three levels.

<http://mste.illinois.edu/users/pavel/java/balance/index.html>

### **Nine Digits Puzzles**

Practice your reasoning skills with these interactive online puzzles.

[https://www.transum.org/software/SW/Starter\\_of\\_the\\_day/students/hot/NineDigits.asp?Level=2](https://www.transum.org/software/SW/Starter_of_the_day/students/hot/NineDigits.asp?Level=2)

### **Algebra Puzzle**

Find the value of each of the three objects presented in the puzzle. The numbers given represent the sum of the objects in each row or column.

[http://www.mathplayground.com/algebra\\_puzzle.html](http://www.mathplayground.com/algebra_puzzle.html)

### **Algebraic Expressions - Online Assessment**

During this online quiz you must simplify expressions, combine like terms, use the distributive property, express word problems as algebraic expressions and recognize when expressions are equivalent. Each incorrect response will allow you to view a video explanation for that problem.

<https://maisonetmath.com/algebra/algebra-quizzes/287-algebraic-expressions-assessment>



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# Simplifying Expressions

**Example.** Simplify  $2x \cdot 4 \cdot 5x$ .

Notice, this expression contains only multiplications (because  $2x$  and  $5x$  are also multiplications).

Since we can multiply in any order, we can write this expression as  $2 \cdot 4 \cdot 5 \cdot x \cdot x$ .

Now we multiply 2, 4, and 5 to get 40. What is left to do? The part  $x \cdot x$ , which is written as  $x^2$ .

So,  $2x \cdot 4 \cdot 5x = 40x^2$ .

Note: The equals sign used in  $2x \cdot 4 \cdot 5x = 40x^2$  signifies that the two expressions are equal no matter what value  $x$  has. That equals sign does not signify an equation that needs to be solved.


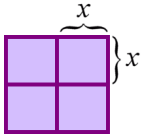
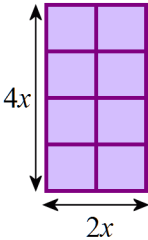
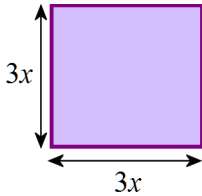
Similarly, we can simplify the expression  $x + x$  and write  $2x$  instead. That whole process is usually written as  $x + x = 2x$ .

Again, the equals sign there does not indicate an equation to solve, but just the fact that the two expressions are equal. In fact, if you think of it as an equation, *any* number  $x$  satisfies it! (Try it!)

1. Simplify the expressions.

a. $p + 8 + p + p$	b. $p \cdot 8 \cdot p \cdot p \cdot p$	c. $2p + 4p$
d. $2p \cdot 4p$	e. $5x \cdot 2x \cdot x$	f. $y \cdot 2y \cdot 3 \cdot 2y \cdot y$

2. Write an expression for the area and perimeter of each rectangle.

<p>a. </p> <p>area =</p> <p>perimeter =</p>	<p>b. </p> <p>area =</p> <p>perimeter =</p>
<p>c. </p> <p>area =</p> <p>perimeter =</p>	<p>d. </p> <p>area =</p> <p>perimeter =</p>

3. a. Sketch a rectangle with sides  $2b$  and  $7b$  long.

b. What is its area?

c. What is its perimeter?

4. a. The perimeter of a rectangle is  $24s$ .  
Sketch one such rectangle.

What is its area?

*Hint: there are many possible answers.*

b. Find the area and perimeter of your rectangle  
in (a) if  $s$  has the value 3 cm.

5. a. Which expression below is for an area of a rectangle? Which one is for a perimeter?

$$4a + 4b \quad 2a \cdot 2b$$

b. Sketch the rectangle.

6. a. Find the value of the expressions  $3p$  and  $p + 3$  for different values of  $p$ .

Value of $p$	$3p$	$p + 3$
0		
0.5		
1		
1.5		
2		
2.5		
3		
3.5		
4		

b. Now, look at the table. Can you tell which is larger,  $3p$  or  $p + 3$ ?

Some review! In algebra, a **term** is an expression that consists of numbers and/or variables that are multiplied together. A single number or variable is also a term.

- Examples.**
- $2xy$  is a term, because it only contains multiplications, a number, and variables.
  - $(5/7)z^3$  is a term. Remember, the exponent is a shorthand for repeated multiplication.
  - Addition and subtraction separate the individual terms in an expression from each other. For example, the expression  $2x^2 - 6y^3 + 7xy + 15$  has four terms, separated by the plus and minus signs.
  - $s + t$  is *not* a term, because it contains addition. Instead, it is a sum of *two* terms,  $s$  and  $t$ .

The number by which a variable or variables are multiplied is called a **coefficient**.

- Examples.**
- The term  $0.9ab$  has the coefficient 0.9.
  - The coefficient of the term  $m^2$  is 1, because you can write  $m^2$  as  $1 \cdot m^2$ .

If the term is a single number, such as  $7/8$ , we call it a **constant**.

**Example.** The expression  $1.5a + b^2 + 6/7$  has three terms:  $1.5a$ ,  $b^2$ , and  $6/7$ . The last term,  $6/7$ , is a constant.

7. Fill in the table.

Expression	the terms in it	coefficient(s)	Constants
$(5/6)s$			
$w^3$			
$0.6x + 2.4y$			
$x + 3y + 7$			
$p \cdot 101$			
$x^5y^2 + 8$			

The two terms in the expression  $2x^2 + 5x^2$  are **like terms**: they have the same variable part ( $x^2$ ). Because of that, we can add the two terms to simplify the expression. To do that, simply add the coefficients 2 and 5 and use the same variable part:  $2x^2 + 5x^2 = 7x^2$ . It is like adding 2 apples and 5 apples.

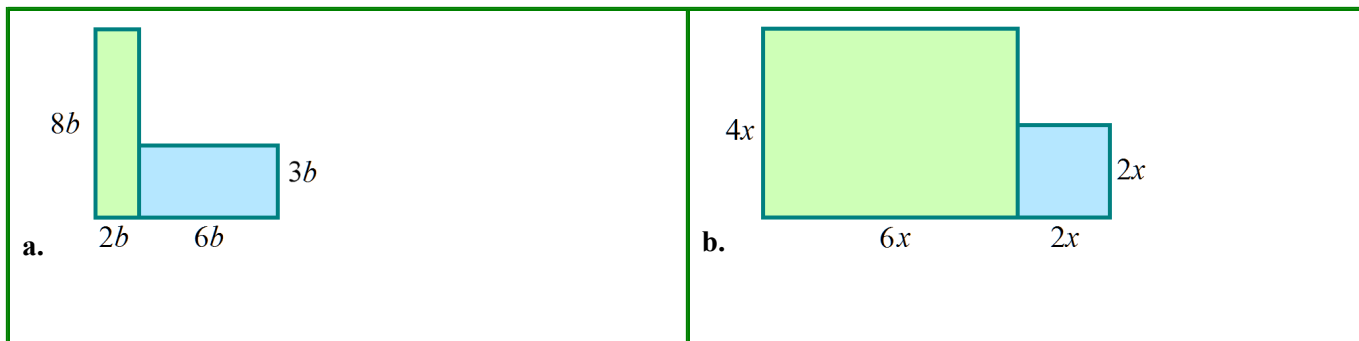
However, you cannot add (or simplify)  $2x + 7y$ . That would be like adding 2 apples and 7 oranges.

**Example.** Simplify  $6x - x - 2x + 9x$ . The terms are like terms, so we simply add or subtract the coefficients:  $6 - 1 - 2 + 9 = 12$  and tag the variable part  $x$  to it. The expression simplifies to  $12x$ .

8. Simplify the expressions.

a. $5p + 8p - p$	b. $p^2 + 8p^2 + 3p^2$	c. $12a^2 - 8a^2 - 3a^2$
------------------	------------------------	--------------------------

9. Write an expression for the total area.



In the following problems, write an expression for part (a), and then for part (b) write an equation and solve it. Don't skip writing the equation, even if you can solve the problem without it, because we are practicing writing equations! You don't have to use algebra to solve the equations—you can solve them in your head or by guessing and checking.

10. **a.** The length of a rectangle is 4 meters and its width is  $w$ .  
What is its perimeter? Write an expression.

**b.** Let's say the perimeter has to be 22 meters. How wide is the rectangle then?  
Write an equation for this situation, using your expression from (a).

Remember, you do not have to use algebra to solve the equation—you can solve it in your head or by "guess and check." But do write the equation.

11. **a.** Linda borrows six books from the library each week, and her mom borrows two.  
How many books, in total, do both of them borrow in  $w$  weeks? Write an expression.

**b.** How many weeks will it take them to have borrowed 216 books? Write an equation.

12. **a.** Alice buys  $y$  containers of mints for \$6 apiece. A fixed shipping cost of \$5 is added to her order. What is her total cost? Write an expression.

**b.** Alice's total bill was \$155. How many containers of mints did she buy?  
Write an equation.

### Puzzle Corner

**a.** What is the total value, in cents, if Ashley has  $n$  dimes and  $m$  quarters?  
Write an expression.

**b.** The total value of Ashley's coins is 495 cents. How many dimes and quarters can she have?  
*Hint: make a table to organize the possibilities.*

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# Chapter 2: Integers

## Introduction

This chapter deals with integers, which are signed (positive or negative) whole numbers. We begin with a review of addition and subtraction of integers from 6th grade. Then we study in detail multiplication and division of integers and conclude with negative fractions and the order of operations.

The first lesson reviews the concepts of integers, absolute value, the opposite of an integer, and simple inequalities on a number line. The next lessons present the addition and subtraction of integers through two visual models: first as movements on a number line, and then using positive and negative counters. These lessons also endeavor to connect the addition and subtraction of integers with various situations from real life.

The lesson *Subtraction of Integers* includes this important principle: Any subtraction can be converted into an addition (of the number with the opposite sign) and vice versa. This principle allows us to calculate not only subtractions such as  $5 - (-7)$  but also problems that contain both addition and subtraction. These mixed problems become simple sums after the subtractions have been converted into additions. Converting subtractions into additions or vice versa is also important when simplifying expressions. For example,  $5 + (-x)$  can be simplified to  $5 - x$ .

Next, we study the distance between two integers. This can be found by taking the absolute value of their difference: the distance between  $x$  and  $y$  is  $|x - y|$ . Students learn to use this formula to find distances between integers, and they also compare the result the formula gives to the answer they get by logical thinking.

The lesson *Multiplying Integers* not only teaches the mechanics of how to multiply integers but also gives both intuitive understanding and formal justification for the shortcut, “a negative times a negative makes a positive.” This formal justification using the distributive property introduces and illustrates the type of careful and precise reasoning that mathematicians use in proofs.

The next lesson, on the division of integers, leads naturally into the topic of negative fractions in the following lesson. The final topic is a review of the order of operations: we perform several operations at a time with integers.

You can find matching videos for the topics in this chapter at <http://www.mathmammoth.com/videos/prealgebra/pre-algebra-videos.php#integers>

### The Lessons in Chapter 2

	page	span
Integers .....	41	4 pages
Addition and Subtraction on the Number Line .....	45	4 pages
Addition of Integers .....	49	3 pages
Subtraction of Integers .....	52	4 pages
Adding and Subtracting Many Integers .....	56	2 pages
Distance and More Practice .....	58	4 pages
Multiplying Integers .....	62	5 pages
Dividing Integers .....	67	2 pages
Negative Fractions .....	69	4 pages
The Order of Operations .....	73	2 pages
Chapter 2 Mixed Review .....	75	2 pages
Chapter 2 Review .....	77	3 pages

## Helpful Resources on the Internet

### Integers Video Lessons by Maria

A set of free videos that teach the topics in this book — by the author.

<http://www.mathmammoth.com/videos/prealgebra/pre-algebra-videos.php#integers>

### Free Downloadable Integer Fact Sheets

[http://www.homeschoolmath.net/download/Add\\_Subtract\\_Integers\\_Fact\\_Sheet.pdf](http://www.homeschoolmath.net/download/Add_Subtract_Integers_Fact_Sheet.pdf)

[http://www.homeschoolmath.net/download/Multiply\\_Divide\\_Integers\\_Fact\\_Sheet.pdf](http://www.homeschoolmath.net/download/Multiply_Divide_Integers_Fact_Sheet.pdf)

## ORDERING INTEGERS

### Number Balls

Click the balls in the ascending order of numbers.

<http://www.sheppardsoftware.com/mathgames/numberballs/numberballsAS2.htm>

### Plot Inequalities Quiz

Practice plotting simple inequalities, such as  $x < -7$ , on a number line in this 10-question interactive quiz.

<https://www.thatquiz.org/tq-o/?-j18-l3-p0>

### Compare Integers Quiz

A 10-question online quiz where you compare two integers. You can also modify the quiz parameters to include sums, differences, products, and quotients, which makes it more challenging.

<http://www.thatquiz.org/tq-8/?-j11-l1-p0>

### Diamond Drop

Drag integers to the empty spaces in comparison sentences (such as  $\_\_ < \_\_$ ,  $\_\_ = \_\_$ , and  $\_\_ > \_\_$ ) as they fall from the top of the screen. (The link does not work when clicked from the PDF file; please copy and paste it into your browser window.)

[http://oame.on.ca/CLIPS/swfPlayer.html?swfURL=lib/CL005\\_IntegersRepresentCompareOrder/CL005\\_C02\\_A05\\_C\\_DiamondDrop/CL005\\_C02\\_A05\\_C\\_DiamondDrop.swf](http://oame.on.ca/CLIPS/swfPlayer.html?swfURL=lib/CL005_IntegersRepresentCompareOrder/CL005_C02_A05_C_DiamondDrop/CL005_C02_A05_C_DiamondDrop.swf)

## ABSOLUTE VALUE AND OPPOSITES

### Number Opposites Challenge

Practice solving challenging problems finding the opposites of numbers.

<https://www.khanacademy.org/math/arithmetic-home/negative-numbers/number-opposites/e/opposites-on-the-number-line>

### Number Balls - Absolute Value

Click the balls in the ascending order of numbers.

<http://www.onlinemathlearning.com/absolute-value-game.html>

### Absolute Value Boxes

You are shown expressions with absolute value (such as  $|-11|$ ,  $-|8|$  and  $|3|$ ). Choose the one that has either the greatest or the least value.

<http://www.xpmath.com/forums/arcade.php?do=play&gameid=96>

### Absolute Value to Find Distance

Practice taking the absolute value of the difference of two numbers to find the distance between those numbers. Apply this principle to solve word problems.

<https://www.khanacademy.org/math/arithmetic-home/negative-numbers/abs-value/e/absolute-value-to-find-distance>

### Absolute Value Quiz

Find the absolute value of each integer or sum.

[http://www.softschools.com/quizzes/math/absolute\\_value/quiz1035.html](http://www.softschools.com/quizzes/math/absolute_value/quiz1035.html)



### **Absolute Value Exercises**

Click on “new problem” to get randomly generated practice-problems that practice absolute value.

[http://www.onemathematicalcat.org/algebra\\_book/online\\_problems/intro\\_abs\\_val.htm#exercises](http://www.onemathematicalcat.org/algebra_book/online_problems/intro_abs_val.htm#exercises)

### **Absolute Value Quiz - harder**

This quiz includes mixed operations and absolute value.

<http://www.softschools.com/testing/math/theme2.html>

## **ADDITION AND SUBTRACTION**

### **Number-Line Jump Maker**

Practice making different-sized jumps on a customizable number line with this interactive tool.

<http://www.ictgames.com/numberlineJumpMaker/>

### **Number-Line Addition**

Click on the addition sentence on the fruit that matches the number line. Choose level 3.

[http://www.sheppardsoftware.com/mathgames/integers/FS\\_NumberLine\\_integer.htm](http://www.sheppardsoftware.com/mathgames/integers/FS_NumberLine_integer.htm)

### **Number-Line Integer Subtraction**

Click on the subtraction sentence on the fruit that matches the number line. Choose level 3.

[http://www.sheppardsoftware.com/mathgames/integers/FS\\_NumberLine\\_int\\_minus.htm](http://www.sheppardsoftware.com/mathgames/integers/FS_NumberLine_int_minus.htm)

### **Add Negative Numbers on a Number Line**

Practice matching number line diagrams to addition expressions involving negative numbers.

<https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-negative-numbers-add-and-subtract/cc-7th-add-sub-neg-number-line/e/adding-negative-numbers-on-the-number-line>

### **Math Lines Integers**

Combine positive and negative balls to make the target number.

[http://www.mathplayground.com/math\\_lines\\_integers.html](http://www.mathplayground.com/math_lines_integers.html)

### **Spider Match**

Choose pairs of numbers that add to the given integer. Can be played as a multi-player game or against the computer.

<http://www.arcademics.com/games/spider-match/spider-match.html>

### **Subtracting Integers Activity**

Model subtraction problems by dragging individual + or – signs off the board. You may need to add neutral pairs (a positive-negative pair) to the working space to solve the problem.

<https://web.archive.org/web/20160207140841/http://mathstar.lacoe.edu/newmedia/integers/subtract/activities/activities.html>

### **Interpreting Negative Number Statements**

Practice matching addition and subtraction equations to real-world scenarios in this interactive exercise.

<https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-negative-numbers-add-and-subtract/cc-7th-add-sub-word-problems-w-negatives/e/negative-number-addition-and-subtraction--interpretation-problems>

### **Add Integers Quiz**

Practice adding integers in this 10-question online quiz.

<https://www.thatquiz.org/tq-1/?-j4101-lk-p0>

### **Understanding Subtraction as Adding the Opposite**

Practice your integer skills with this interactive quiz.

<https://www.khanacademy.org/math/arithmetic/arith-review-negative-numbers/arith-review-sub-neg-intro/e/understand-subtraction-as-adding-the-opposite>

### **Subtracting Negative Integers**

Practice subtracting positive and negative single-digit numbers in this interactive online activity.

[https://www.khanacademy.org/math/algebra-basics/basic-alg-foundations/alg-basics-negative-numbers/e/adding\\_and\\_subtracting\\_negative\\_numbers](https://www.khanacademy.org/math/algebra-basics/basic-alg-foundations/alg-basics-negative-numbers/e/adding_and_subtracting_negative_numbers)

### **Adding & Subtracting Several Negative Numbers**

Practice solving addition and subtraction problems with several integers.

<https://www.khanacademy.org/math/arithmetic/arith-review-negative-numbers/arith-review-add-and-sub-integers/e/integer-addition-and-subtraction-2>

### **Find the missing integers**

Fill in the missing integer in addition equations such as  $-23 + \underline{\quad} = -8$ .

[http://www.aaamath.com/g8\\_65\\_x3.htm](http://www.aaamath.com/g8_65_x3.htm)

### **Find the Missing Number**

Practice finding the missing value in an addition or subtraction equation involving negative numbers.

<https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-negative-numbers-add-and-subtract/cc-7th-add-and-sub-integers/e/integer-addition-and-subtraction>

### **Space Coupe to the Rescue**

By choosing a positive or negative number, the player controls the vertical position of a spaceship. If the spaceship reaches the same vertical position as a virus pod, the pod is destroyed.

<https://web.archive.org/web/20120428115125/http://pbskids.org:80/cyberchase/math-games/space-coupe-rescue/>

### **Red and Black Triplematch Game for Adding Integers**

This is a fun card game with 2-5 people to practice adding integers.

<http://mathmamawrites.blogspot.com/2010/07/black-and-red-triplematch-card-game-for.html>

### **Different Ways to Play “Integer War” Card Game**

This page explains various ways to play a common card game that is used to review integers.

<http://www.mathfilefoldergames.com/integer-war/>

### **Combine Like Terms with Negative Coefficients**

Simplify algebraic expressions by combining like terms. Coefficients on some terms are negative.

[https://www.khanacademy.org/math/algebra/introduction-to-algebra/alg1-manipulating-expressions/e/combining\\_like\\_terms\\_1](https://www.khanacademy.org/math/algebra/introduction-to-algebra/alg1-manipulating-expressions/e/combining_like_terms_1)

## **MULTIPLICATION AND DIVISION**

### **Integers Multiplication Blocks**

Click on numbers whose product is equal to the target number.

<http://www.xpmath.com/forums/arcade.php?do=play&gameid=58>

### **Integers Multiplication -144 to 144**

Solve as many integer multiplication problems as you can within one minute with these online flashcards.

<http://www.xpmath.com/forums/arcade.php?do=play&gameid=17>

### **Integers Division -81 to 81**

Solve as many integer division problems as you can within one minute with these online flashcards.

<http://www.xpmath.com/forums/arcade.php?do=play&gameid=16>

### **Fruit Splat Multiplication of Integers**

Click on the fruit that has the correct answer to the integer multiplication problem.

[http://www.sheppardsoftware.com/mathgames/integers/FS\\_Integer\\_multiplication.htm](http://www.sheppardsoftware.com/mathgames/integers/FS_Integer_multiplication.htm)

### **Find the Missing Number Quiz**

Practice integer multiplication with missing numbers in this interactive 10-question quiz.

<https://www.thatquiz.org/tq-1/?-j114-la-p0>

## **ALL OPERATIONS / GENERAL**

### **Rational Numbers on the Number Line**

Practice placing positive and negative fractions and decimals on the number line in this interactive online activity.

[https://www.khanacademy.org/math/6th-engage-ny/engage-6th-module-3/6th-module-3-topic-a/e/fractions\\_on\\_the\\_number\\_line\\_3](https://www.khanacademy.org/math/6th-engage-ny/engage-6th-module-3/6th-module-3-topic-a/e/fractions_on_the_number_line_3)

### **Integer Operations Quiz**

Practice several operations with integers in this 10-question interactive quiz.

<https://www.thatquiz.org/tq-1/?-jh8f-la-p0>

### **Multiplying & Dividing Negative Numbers Word Problems**

Use this set of interactive word problems to reinforce your knowledge of integers.

<https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-negative-numbers-multiply-and-divide/cc-7th-mult-div-neg-word-problems/e/negative-number-word-problems-1>

### **Order of Operations with Negative Numbers**

Practice evaluating expressions using the order of operations in this interactive online activity.

[https://www.khanacademy.org/math/algebra-basics/basic-alg-foundations/alg-basics-order-of-operations/e/order\\_of\\_operations](https://www.khanacademy.org/math/algebra-basics/basic-alg-foundations/alg-basics-order-of-operations/e/order_of_operations)

### **Integers Quiz**

Test your integer skills in this interactive online quiz.

<https://www.thatquiz.org/tq/practicetest?10w790opx3kmd>

### **Numerate Game with Integers**

In this game, two players take turns forming equations using the available tiles.

<http://www.transum.org/Maths/Game/Numerate/Default.asp?Level=2>

### **Integers Jeopardy**

A jeopardy game where the questions involve adding, subtracting, multiplying, and dividing integers.

<http://www.math-play.com/Integers-Jeopardy/Integers-Jeopardy.html>

### **Solve For Unknown Variable - Integer Review**

Find the value of an unknown variable in a given addition or subtraction equation with integers.

<https://maisonetmath.com/algebra/algebra-quizzes/294-solve-for-unknown-variable-integer-review>

### **Fruit Shoot Game: Mixed Integer Operations**

Practice all four operations with integers while shooting fruits. You can choose the difficulty level and the speed.

[http://www.sheppardsoftware.com/mathgames/fruitshoot/FS\\_Mixed\\_Integers.htm](http://www.sheppardsoftware.com/mathgames/fruitshoot/FS_Mixed_Integers.htm)

### **Student CLIPS in Mathematics**

Activities, video clips, and games for middle-school math topics, including integers.

<http://oame.on.ca/CLIPS/>

### **The History of Negative Numbers**

While they seem natural to us now, in the past negative numbers have spurred controversy and have been called “fictitious” and other names.

[http://nrich.maths.org/public/viewer.php?obj\\_id=5961](http://nrich.maths.org/public/viewer.php?obj_id=5961)

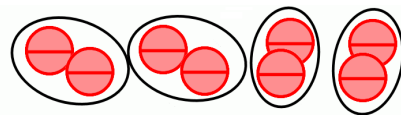
[http://www.classzone.com/books/algebra\\_1/page\\_build.cfm?content=links\\_app3\\_ch2&ch=2](http://www.classzone.com/books/algebra_1/page_build.cfm?content=links_app3_ch2&ch=2)

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# Dividing Integers

## Divide a negative number by a positive

The image illustrates  $(-8) \div 4$ , or eight negatives divided into four groups. We can see the answer is  $-2$ .



Any time a negative integer is divided by a positive integer, we can illustrate it as so many negative counters divided evenly into groups. The answer will be negative.

## Divide a positive integer by a negative. For example, $24 \div (-8) = ?$

Remember, multiplication is the opposite operation to division. Let's write the answer to  $24 \div (-8)$  as  $s$ . Then from that we can write a multiplication:

$$24 \div (-8) = s \Rightarrow (-8)s = 24$$

(You could use an empty line instead of  $s$ , if the variable  $s$  confuses you.)

The only number that fulfills the equation  $(-8)s = 24$  is  $s = -3$ . Therefore,  $24 \div (-8) = -3$ .

Similarly, each time you divide a positive integer by a negative integer, the answer is negative.

## Divide a negative integer by a negative. For example, $(-24) \div (-8) = ?$

Again, let's denote the answer to  $-24 \div (-8)$  with  $y$ , and then write a multiplication sentence.

$$-24 \div (-8) = y \Rightarrow (-8)y = -24$$

The only number that fulfills the equation  $(-8)y = -24$  is  $y = 3$ . Therefore,  $-24 \div (-8) = 3$ .

Similarly, each time you divide a negative integer by a negative integer, the answer is positive.

**Summary.** The symbols below show whether you get a positive or negative answer when you multiply or divide integers. Notice that the rules for multiplication and division are the same!

### Multiplication

$$\oplus \cdot \ominus = \ominus$$

$$\ominus \cdot \oplus = \ominus$$

$$\ominus \cdot \ominus = \oplus$$

$$\oplus \cdot \oplus = \oplus$$

### Examples

$$4 \cdot (-5) = -20$$

$$-4 \cdot 5 = -20$$

$$-4 \cdot (-5) = 20$$

$$4 \cdot 5 = 20$$

### Division

$$\oplus \div \ominus = \ominus$$

$$\ominus \div \oplus = \ominus$$

$$\ominus \div \ominus = \oplus$$

$$\oplus \div \oplus = \oplus$$

### Examples

$$20 \div (-5) = -4$$

$$-20 \div 5 = -4$$

$$-20 \div (-5) = 4$$

$$20 \div 5 = 4$$

Here is a shortcut for *multiplication* and *division* (NOT for addition or subtraction):

- If both numbers have the same sign (both are positive *or* negative), the answer is positive.
- If the numbers have different signs, the answer is negative.

1. Divide.

a.  $-50 \div (-5) = \underline{\hspace{2cm}}$

$-12 \div 2 = \underline{\hspace{2cm}}$

b.  $(-8) \div (-1) = \underline{\hspace{2cm}}$

$14 \div (-2) = \underline{\hspace{2cm}}$

c.  $81 \div (-9) = \underline{\hspace{2cm}}$

$-100 \div (-10) = \underline{\hspace{2cm}}$

2. Multiply. Then use the same numbers to write an equivalent division equation.

<b>a.</b> $-5 \cdot (-5) = \underline{\hspace{2cm}}$ $\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$	<b>b.</b> $9 \cdot (-6) = \underline{\hspace{2cm}}$ $\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$	<b>c.</b> $-80 \cdot 8 = \underline{\hspace{2cm}}$ $\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
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3. Four people shared a debt of \$280 equally. How much did each owe? Write an integer division.

4. In a math game, you get a negative point for every wrong answer and a positive point for every correct answer. Additionally, if you answer in 1 second, your negative points from the past get slashed in half!

Angie had accumulated 14 negative and 25 positive points in the game. Then she answered a question correctly in 1 second. Write an equation for her current "point balance."

5. Complete the patterns.

<b>a.</b>	<b>b.</b>	<b>c.</b>
$12 \div 4 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div (-7) = -3$	$60 \div \underline{\hspace{2cm}} = 2$
$8 \div 4 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div (-7) = -2$	$40 \div \underline{\hspace{2cm}} = 2$
$4 \div 4 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div (-7) = -1$	$20 \div \underline{\hspace{2cm}} = 2$
$0 \div 4 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div (-7) = 0$	$-20 \div \underline{\hspace{2cm}} = 2$
$(-4) \div 4 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div (-7) = 1$	$-40 \div \underline{\hspace{2cm}} = 2$
$(-8) \div 4 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div (-7) = 2$	$-60 \div \underline{\hspace{2cm}} = 2$
$(-12) \div 4 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div (-7) = 3$	$-80 \div \underline{\hspace{2cm}} = 2$
$(-16) \div 4 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div (-7) = 4$	$-100 \div \underline{\hspace{2cm}} = 2$

6. Here's a funny riddle. Solve the math problems to uncover the answer.

- |   |  |  |
|---|--|--|
| <b>E</b> $\underline{\hspace{2cm}} \div (-8) = 2$   | <b>N</b> $-12 \cdot (-5) = \underline{\hspace{2cm}}$ | <b>E</b> $(-144) \div 12 = \underline{\hspace{2cm}}$ |
| <b>E</b> $3 \cdot (-12) = \underline{\hspace{2cm}}$ | <b>H</b> $\underline{\hspace{2cm}} \div 12 = -5$     | <b>T</b> $-4 \cdot (-9) = \underline{\hspace{2cm}}$  |
| <b>N</b> $-15 \div \underline{\hspace{2cm}} = -5$   | <b>E</b> $\underline{\hspace{2cm}} \cdot (-6) = 0$   | <b>V</b> $-45 \div \underline{\hspace{2cm}} = 5$     |
| <b>G</b> $-1 \cdot (-9) = \underline{\hspace{2cm}}$ | <b>I</b> $-27 \div 9 = \underline{\hspace{2cm}}$     | <b>I</b> $-7 \cdot \underline{\hspace{2cm}} = -84$   |
| <b>S</b> $-48 \div 6 = \underline{\hspace{2cm}}$    | <b>N</b> $3 \cdot \underline{\hspace{2cm}} = -24$    |  |

Why is six afraid of seven? Because....

-8	-12	-9	-36	60	0	12	9	-60	36	3	-3	-8	-16

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# Chapter 3: One-Step Equations

## Introduction

The goal of this chapter is to solve one-step equations that involve integers. The first lesson reviews the concept of an equation and how to model equations using a pan balance (scale). Recall that the basic principle for solving equations is that, when you perform the same operation on both sides of an equation, the two sides remain equal.

The chapter presents two alternatives for keeping track of the operations to be performed on an equation. The one method, writing the operation under each side of the equation, is common in the United States. The other method, writing the operation in the right margin, is common in Finland. Either is adequate, and the choice is just a matter of the personal preference of the teacher.

The introduction to solving equations is followed by a lesson on addition and subtraction equations and another on multiplication and division equations. All the equations are easily solved in only one step of calculations. The twofold goal is to make the student proficient in manipulating negative integers and also to lay a foundation for handling more involved equations in Chapter 5.

In the next lesson, students write equations to solve simple word problems. Even though they could solve most of these problems without using the equations, the purpose of the lesson is to make the student proficient in writing simple equations before moving on to more complex equations from more difficult word problems.

The last topic, in the lesson *Constant Speed*, is solving problems with distance ( $d$ ), rate or velocity ( $v$ ), and time ( $t$ ). Students use the equivalent formulas  $d = vt$  and  $v = d/t$  to solve problems involving constant or average speed. They learn an easy way to remember the formula  $v = d/t$  from the unit for speed that they already know, “miles per hour.”

As a reminder, it is not recommended that you assign all the exercises by default. Use your judgment, and strive to vary the number of assigned exercises according to the student’s needs. Some students might only need half or even less of the available exercises, in order to understand the concepts.

### The Lessons in Chapter 3

	page	span
Solving Equations .....	82	7 pages
Addition and Subtraction Equations .....	89	4 pages
Multiplication and Division Equations .....	93	4 pages
Word Problems .....	97	3 pages
Constant Speed .....	100	7 pages
Chapter 3 Mixed Review .....	107	3 pages
Chapter 3 Review .....	110	2 pages

### Helpful Resources on the Internet

#### The Simplest Equations - Video Lessons by Maria

A set of free videos that teach the topics in this chapter - by the author.

<http://www.mathmammoth.com/videos/prealgebra/pre-algebra-videos.php#equations>

#### Stable Scales Quiz

In each picture, the scales are balanced. Can you find the weight of the items on the scales?

[http://www.transum.org/software/SW/Starter\\_of\\_the\\_day/Students/Stable\\_Scales\\_Quiz.asp](http://www.transum.org/software/SW/Starter_of_the_day/Students/Stable_Scales_Quiz.asp)



### **Model Algebra Equations**

Model an equation on a balance using algebra tiles (tiles with numbers or the unknown  $x$ ). Then, solve the equation by placing  $-1$  tile on top of  $+1$  tile or vice versa. Includes one-step and two-step equations.

<http://www.mathplayground.com/AlgebraEquations.html>

### **Balance When Adding and Subtracting**

Click on the buttons above the scales to add or subtract until you can figure out the value of  $x$  in the equation.

<http://www.mathsisfun.com/algebra/add-subtract-balance.html>

### **One-Step Equations Quizzes**

Practice one-step equations in these timed quizzes.

<http://crctlessons.com/One-Step-Equations/one-step-equations.html>

<http://crctlessons.com/One-Step-Equation-Test/one-step-equation-test.html>

### **Modeling with One-Step Equations**

Practice writing basic equations to model real-world situations in this interactive activity from Khan Academy.

<https://www.khanacademy.org/math/pre-algebra/pre-algebra-equations-expressions/pre-algebra-equation-word-problems/e/equations-in-one-variable-1>

### **Exploring Equations E-Lab**

Choose which operation to do to both sides of an equation in order to solve one-step equations.

<http://www.harcourtschool.com/activity/elab2004/gr6/12.html>

### **Algebra Four**

A connect four game with equations. For this level, choose difficulty “Level 1” and “One-Step Problems”.

<http://www.shodor.org/interactivate/activities/AlgebraFour/>

### **Function Machine**

Investigate the relationship between input and output values with this interactive online activity.

<https://www.mathplayground.com/functionmachine.html>

### **One-Step Equation Game**

Choose the correct root for the given equation (multiple-choice), and then you get to attempt to shoot a basket.

<http://www.math-play.com/One-Step-Equation-Game.html>

### **Arithmagons**

Find the numbers that are represented by question marks in this interactive puzzle.

[http://www.transum.org/Software/SW/Starter\\_of\\_the\\_day/starter\\_August20.ASP](http://www.transum.org/Software/SW/Starter_of_the_day/starter_August20.ASP)

### **Cars**

Use clues to help you find the total cost of four cars in this fun brainteaser.

[http://www.transum.org/Software/SW/Starter\\_of\\_the\\_day/starter\\_July16.ASP](http://www.transum.org/Software/SW/Starter_of_the_day/starter_July16.ASP)

### **Distance, Speed, and Time from BBC Bitesize**

Instruction, worked out exercises, and an interactive quiz relating to constant speed, time, and distance. A triangle with letters D, S, and T helps students remember the formulas for distance, speed, and time.

[http://www.bbc.co.uk/bitesize/standard/maths\\_i/numbers/dst/revision/1/](http://www.bbc.co.uk/bitesize/standard/maths_i/numbers/dst/revision/1/)

### **Absorb Advanced Physics - Speed**

An online tutorial that teaches the concept of average speed with the help of interactive simulations and exercises.

<http://www.absorblearning.com/advancedphysics/demo/units/010101.html#Describingmotion>

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# Multiplication and Division Equations

<p>Do you remember <b>how to show simplification</b>? Just cross out the numbers and write the new numerator above the fraction and the new denominator below it.</p> <p>Notice that the number you divide by (the 5 in the fraction at the right) isn't indicated in any way!</p>	$\frac{\cancel{35}^7}{\cancel{55}_{11}} = \frac{7}{11}$
<p>We can simplify expressions involving variables in exactly the same way.</p> <p>In the examples on the right, we cross out the <i>same number</i> from the numerator and the denominator. That is based on the fact that a number divided by itself is 1. We could write a little "1" beside each number that is crossed out, but that is usually omitted.</p>	$\frac{\cancel{2}x}{\cancel{2}} = x \quad \frac{\cancel{5}s}{\cancel{5}} = s$ $\frac{\cancel{4}x}{\cancel{4}} = x$
<p>In this example, we simplify the fraction <math>3/6</math> into <math>1/2</math> the usual way.</p>	$\frac{\cancel{3}x}{\cancel{6}_2} = \frac{1}{2}x \quad \text{or} \quad \frac{x}{2}$
<p><b>Notice:</b> We divide both the numerator and the denominator by 8, but <u>this leaves -1 in the denominator</u>. Therefore, the whole expression simplifies to <math>-z</math> instead of <math>z</math>.</p>	$\frac{\cancel{8}z}{-\cancel{8}} = \frac{z}{-1} = -z$

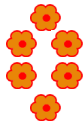
1. Simplify.

a. $\frac{8x}{8}$	b. $\frac{8x}{2}$	c. $\frac{2x}{8}$
d. $\frac{-6x}{-6}$	e. $\frac{-6x}{6}$	f. $\frac{6x}{-6}$
g. $\frac{6w}{2}$	h. $\frac{6w}{w}$	i. $\frac{6w}{-2}$


2. Draw the fourth and fifth steps of the pattern and answer the questions.



Step 1



Step 2



Step 3

a. How would you describe the growth of this pattern?

b. How many flowers will there be in step 39?

c. In step  $n$ ?

Now you should be ready to use multiplication and division to solve simple equations.

**Example 1.** Solve  $-2x = 68$ .

The unknown is being multiplied by  $-2$ . To isolate it, we need to divide both sides by  $-2$ . (See the solution on the right.)

We get  $x = -34$ . Lastly we check the solution by substituting  $-34$  in the place of  $x$  in the original equation:

$$\begin{aligned} -2(-34) &\stackrel{?}{=} 68 \\ 68 &= 68 \quad \text{It checks.} \end{aligned}$$

$$-2x = 68 \quad \text{This is the original equation.}$$

$$\frac{-2x}{-2} = \frac{68}{-2} \quad \text{We divide both sides by } -2.$$

$$\frac{\cancel{-2}x}{\cancel{-2}} = \frac{68}{-2} \quad \text{Now it is time to simplify. We cross out the } -2 \text{ factors on the left side. On the right side, we do the division.}$$

$$x = -34 \quad \text{This is the final answer.}$$

Note: Most people combine the first 3 steps into one when writing the solution. Here they are written out for clarity.

3. Solve. Check your solutions.

a.  $5x = -45$

b.  $-3y = -21$

c.  $-4 = 4s$

d.  $72 = -6y$

4. Solve. Simplify the one side first.

a.  $-5q = -40 - 5$

b.  $2 \cdot 36 = -6y$

c.  $3x = -4 + 3 + (-2)$

d.  $5 \cdot (-4) = -10z$

**Example 2.** Solve  $\frac{x}{-6} = -5$ .

Here the unknown is divided by  $-6$ . To undo that division, we need to *multiply* both sides by  $-6$ . (See the solution on the right.)

We get  $x = 30$ . Lastly we check the solution:

$$\frac{30}{-6} \stackrel{?}{=} -5$$

$$-5 = -5 \quad \checkmark$$

$$\frac{x}{-6} = -5$$

This is the original equation.

$$\frac{x}{-6} \cdot (-6) = -5 \cdot (-6)$$

We multiply both sides by  $-6$ .

$$\frac{x}{\cancel{-6}} \cdot \cancel{(-6)} = 30$$

Now it is time to simplify. We cross out the  $-6$  factors on the left side, and multiply on the right.

$$x = 30$$

This is the final answer.

When writing the solution, most people would combine steps 2 and 3. Here both are written out for clarity.

5. Solve. Check your solutions.

a.  $\frac{x}{2} = -45$

b.  $\frac{s}{-7} = -11$

c.  $\frac{c}{-7} = 4$

d.  $\frac{a}{-13} = -9 + (-11)$

6. Write an equation for each situation. Then solve it. Do not write the answer only, as the main purpose of this exercise is to practice writing equations.

- a. A submarine was located at a depth of 500 ft.  
There was a shark swimming at  $\frac{1}{6}$  of that depth.  
At what depth is the shark?

- b. Three towns divided highway repair costs equally.  
Each town ended up paying \$21,200.  
How much did the repairs cost in total?

**Example 3.** Solve  $-\frac{1}{5}x = 2$ . Here the unknown is multiplied by a negative fraction, but do not panic!

You see, you can *also* write this equation as  $\frac{x}{-5} = 2$ , where the unknown is simply divided by negative 5.

So what should we do in order to isolate  $x$ ?

That is correct! Multiplying by  $-5$  will isolate  $x$ . In the boxes below, this equation is solved in two slightly different ways, though both are doing essentially the same thing: multiplying both sides by  $-5$ .

**Multiplying a fraction by its reciprocal:**

$$-\frac{1}{5}x = 2 \quad | \cdot (-5)$$

$$(-5) \cdot \left(-\frac{1}{5}\right)x = (-5) \cdot 2 \quad \text{Note that } -5 \text{ times } -1/5 \text{ is } 1.$$

$$1x = -10$$

$$x = -10$$

**Canceling a common factor:**

$$-\frac{1}{5}x = 2 \quad \text{rewrite the equation}$$

$$\frac{x}{-5} = 2 \quad | \cdot (-5)$$

$$\frac{x}{\cancel{-5}} \cdot (\cancel{-5}) = 2 \cdot (-5)$$

$$x = -10$$

Lastly we check the solution by substituting  $-10$  in place of  $x$  in the original equation:

$$-\frac{1}{5}(-10) \stackrel{?}{=} 2$$

$$2 = 2 \quad \text{It checks.}$$

7. Solve. Check your solutions.

a.  $\frac{1}{3}x = -15$

b.  $-\frac{1}{6}x = -20$

c.  $-\frac{1}{4}x = 18$

d.  $-2 = -\frac{1}{9}x$

e.  $-21 = \frac{1}{8}x$

f.  $\frac{1}{12}x = -7 + 5$

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# Chapter 4: Rational Numbers

## Introduction

In this chapter we study *rational* numbers, which are numbers that can be written as a *ratio* of two integers. All fractions and whole numbers are rational numbers, and so are percents and decimals (except non-ending non-repeating decimals). Obviously, students already know a lot about rational numbers and how to calculate with them. Our focus in this chapter is to extend that knowledge to negative fractions and negative decimals.

The first lesson presents the definition of a rational number, how to convert rational numbers back and forth between their fractional and decimal forms, and a bit about repeating decimals (most fractions become repeating decimals when written as decimals). The next lesson deals with adding and subtracting rational numbers, with an emphasis on adding and subtracting negative fractions and decimals.

The next two lessons are about multiplying and dividing rational numbers. The first of the two focuses on basic multiplication and division with negative fractions and decimals. The second of the two compares multiplying and dividing in decimal notation to multiplying and dividing in fraction notation. Students come to realize that, though the calculations—and even the answers—may look very different, the answers are equal. The lesson also presents problems that mix decimals, fractions, and percents, and deals with real-life contexts for the problems and the importance of pre-estimating what a reasonable answer would be.

The lesson *Many Operations with Rational Numbers* reviews the order of operations and applies it to fraction and decimal problems with more than one operation. It also presents a simple method to solve complex fractions, which are fractions that contain another fraction, either in the numerator, in the denominator, or in both.

After a lesson on scientific notation, the instructional portion of the chapter concludes with two lessons on solving simple equations that involve fractions and decimals.

You can find matching videos for some of the topics in this chapter at <http://www.mathmammoth.com/videos/prealgebra/pre-algebra-videos.php#rational>

### The Lessons in Chapter 4

	page	span
Rational Numbers .....	116	8 pages
Adding and Subtracting Rational Numbers .....	124	6 pages
Multiply and Divide Rational Numbers 1 .....	130	4 pages
Multiply and Divide Rational Numbers 2 .....	134	7 pages
Many Operations with Rational Numbers .....	141	4 pages
Scientific Notation .....	145	3 pages
Equations with Fractions .....	148	5 pages
Equations with Decimals .....	153	3 pages
Chapter 4 Mixed Review .....	156	2 pages
Chapter 4 Review .....	158	5 pages



## Helpful Resources on the Internet

### Rational Numbers - Video Lessons by Maria

A set of free videos that teach the topics in this book - by the author.

<http://www.mathmammoth.com/videos/prealgebra/pre-algebra-videos.php#rational>

## RATIONAL NUMBERS

### Compare Positive and Negative Decimals

Choose the correct inequality or equals sign from the dropdown box between each set of two numbers.

[http://www.transum.org/software/SW/Starter\\_of\\_the\\_day/Students/Inequalities.asp?Level=2](http://www.transum.org/software/SW/Starter_of_the_day/Students/Inequalities.asp?Level=2)

### Compare Rational Numbers

Practice comparing decimals, percents, fractions, and mixed numbers in this interactive exercise.

<https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-fractions-decimals/cc-7th-add-sub-rational-numbers/e/comparing-rational-numbers>

### Recurring Decimals and Fractions

Grade or No Grade where you answer multiple-choice questions about repeating decimals.

<https://sites.google.com/a/revisemaths.org.uk/revise/number-files/recdefrac-gong.swf?attredirects=0>

### Terminating vs. Repeating Decimals Game

A card game that practices repeating and terminating decimals. Several fun twists to score extra points! This game costs \$1 (per download).

<http://www.teacherspayteachers.com/Product/Terminating-VS-Repeating-Decimals-Game-425199>

### Terminating and Repeating Decimals Worksheet

A 10-question online quiz about repeating decimals.

<http://worksheets.tutorvista.com/terminating-and-repeating-decimals-worksheet.html>

### Converting Repeating Decimals to Fractions

A lesson that explains the method for writing repeating decimals as fractions.

<http://www.basic-mathematics.com/converting-repeating-decimals-to-fractions.html>

### Classifying Numbers

Drag the given numbers to the correct sets. This chapter of Math Mammoth does not teach about square roots and irrational numbers, but you can probably do these activities, if you note that most square roots are irrational, and that the set of whole numbers is  $\{0, 1, 2, 3, 4, \dots\}$ .

[http://www.softschools.com/math/classifying\\_numbers/](http://www.softschools.com/math/classifying_numbers/)

[http://www.softschools.com/math/classifying\\_numbers/real\\_rational\\_integer\\_whole\\_natural\\_irrational\\_number\\_table/](http://www.softschools.com/math/classifying_numbers/real_rational_integer_whole_natural_irrational_number_table/)

### Number System Muncher

“Munch” or select all the numbers from the grid that are in the specified set. Again, this chapter of Math Mammoth does not teach about square roots and irrational numbers, but you can probably play the game, if you note the following: Finding a square root is the opposite operation of squaring. For example,  $\sqrt{25} = 5$  because  $5^2 = 25$ . Therefore,  $\sqrt{25}$  is actually a natural number (5). However, most square roots, such as  $\sqrt{5}$  and  $\sqrt{13}$  are irrational.

<https://web.archive.org/web/20160323053948/http://staff.argyll.epsb.ca/jreed/math9/strand1/munchers.htm>

## ADD AND SUBTRACT RATIONAL NUMBERS

### Adding and Subtracting Rational Numbers Test

A 15-question test about adding, subtracting, and comparing rational numbers.

<http://teachers.henrico.k12.va.us/math/hcpsalgebra1/Documents/examviewweb/ev2-2.htm>

### **Adding and Subtracting Rational Numbers Worksheets**

Generate a worksheet for adding and subtracting negative fractions and decimals.

[http://www.math-aids.com/Algebra/Algebra\\_1/Basics/Add\\_Sub\\_Rational.html](http://www.math-aids.com/Algebra/Algebra_1/Basics/Add_Sub_Rational.html)

### **Add Decimals Quiz**

Reinforce your decimal addition skills with this 10-question online quiz.

<https://www.thatquiz.org/tq-3/?-j1i1-lk-p0>

### **Add Fractions Quiz**

Add the fractions, and express the answer as a simple fraction in lowest terms in this 10-question quiz.

<https://www.thatquiz.org/tq-3/?-j1gh-la-p0>

### **Adding and Subtracting Rational Numbers Quiz**

Practice addition and subtraction of rational numbers with this interactive quiz.

[http://www.softschools.com/quizzes/math/adding\\_and\\_subtracting\\_rational\\_numbers/quiz3284.html](http://www.softschools.com/quizzes/math/adding_and_subtracting_rational_numbers/quiz3284.html)

### **Adding and Subtracting Negative Fractions**

Practice adding and subtracting positive and negative fractions with this interactive online activity.

[https://www.khanacademy.org/math/algebra-basics/basic-alg-foundations/alg-basics-fractions/e/adding\\_and\\_subtracting\\_fractions](https://www.khanacademy.org/math/algebra-basics/basic-alg-foundations/alg-basics-fractions/e/adding_and_subtracting_fractions)

### **Adding and Subtracting Rational Numbers**

Practice adding and subtracting negative fractions, decimals, and percents with this interactive online exercise.

[https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-fractions-decimals/cc-7th-add-sub-rational-numbers/e/adding\\_and\\_subtracting\\_rational\\_numbers](https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-fractions-decimals/cc-7th-add-sub-rational-numbers/e/adding_and_subtracting_rational_numbers)

### **Add and Subtract Fractions Quiz**

Add and subtract negative fractions and solve simple equations. Refresh the page to get a different set of questions.

<http://www.phschool.com/webcodes10/index.cfm?fuseaction=home.gotoWebCode&wcprefix=asa&wcsuffix=0204>

## **MULTIPLY AND DIVIDE RATIONAL NUMBERS**

### **Multiply and Divide Fractions Quizzes**

Multiple-choice quizzes of five questions. Refresh the page to get a different set of questions.

<http://www.phschool.com/webcodes10/index.cfm?fuseaction=home.gotoWebCode&wcprefix=asa&wcsuffix=0205>

### **Fractions Quiz**

Multiple-choice quiz of five questions. Refresh the page to get a different set of questions.

<http://www.phschool.com/webcodes10/index.cfm?wcprefix=asa&wcsuffix=0205&area=view>

### **Multiply and Divide Rational Numbers Quiz**

A multiple-choice quiz of five questions.

[http://www.softschools.com/quizzes/math/multiply\\_rational\\_numbers/quiz3285.html](http://www.softschools.com/quizzes/math/multiply_rational_numbers/quiz3285.html)

### **Multiply and Divide Fractions**

Practice multiplying and dividing fractions with positive numbers with this interactive exercise.

[http://www.onemathematicalcat.org/algebra\\_book/online\\_problems/md\\_fractions.htm#exercises](http://www.onemathematicalcat.org/algebra_book/online_problems/md_fractions.htm#exercises)

### **Divide Positive and Negative Fractions**

Practice dividing fractions. The fractions in these problems may be positive or negative.

[https://www.khanacademy.org/math/algebra-basics/basic-alg-foundations/alg-basics-fractions/e/dividing\\_fractions\\_2](https://www.khanacademy.org/math/algebra-basics/basic-alg-foundations/alg-basics-fractions/e/dividing_fractions_2)

### **Multiply Decimals by Fractions - Short Quiz**

Practice multiplying decimals by fractions with this multiple-choice quiz.

<https://www.sophia.org/concepts/multiplying-decimals-by-fractions>

### **Divide Fractions by Decimals - Short Quiz**

Practice dividing fractions by decimals with this multiple-choice quiz.

<https://www.sophia.org/concepts/dividing-fractions-by-decimals>

### **Multiplying and Dividing Negative Numbers Word Problems**

Practice matching situations to multiplication and division expressions and equations.

<https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-negative-numbers-multiply-and-divide/cc-7th-mult-div-neg-word-problems/e/negative-number-word-problems-1>

### **Fractions Quiz**

Practice negative fractions with this interactive online quiz.

<http://www.phschool.com/webcodes10/index.cfm?wcprefix=asa&wcsuffix=0204&area=view>

### **Simplify Complex Fractions**

Practice simplifying complex fractions with this interactive online activity from Khan Academy.

<https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-negative-numbers-multiply-and-divide/cc-7th-mult-div-fractions-2/e/complex-fractions>

### **Complex Fractions Quiz**

Practice simplifying complex fractions with this interactive quiz.

<http://www.saddleback.edu/faculty/LPerez/Algebra2go/prealgebra/fractions/compFrac.html>

## **SCIENTIFIC NOTATION**

### **Scientific Notation**

Interactive practice where you write the given number in scientific notation.

<http://www.xpmath.com/forums/arcade.php?do=play&gameid=21>

### **Scientific Notation Quiz**

Write numbers in scientific notation, and vice versa. You can also modify the quiz parameters.

<http://www.thatquiz.org/tq-c/?-j820-l6-p0>

### **Scientific Notation Quizzes**

Short, multiple-choice quizzes on scientific notation.

<http://www.phschool.com/webcodes10/index.cfm?wcprefix=asa&wcsuffix=0208&area=view>

## **EQUATIONS / GENERAL**

### **Equations Quizzes**

Five-question quizzes on simple equations with decimals or fractions. Refresh the page to get different questions.

<http://www.phschool.com/webcodes10/index.cfm?fuseaction=home.gotoWebCode&wcprefix=ara&wcsuffix=0404>

OR

<http://www.phschool.com/webcodes10/index.cfm?fuseaction=home.gotoWebCode&wcprefix=bja&wcsuffix=0508>

### **Fraction Four Game**

Choose “algebra” as the question type to solve equations that involve fractions in this connect-the-four game.

<http://www.shodor.org/interactivate/activities/FractionFour/>

### **One-Step Equations with Fractions**

This algebra 1 worksheet will produce one step problems containing fractions.

[http://www.math-aids.com/Algebra/Algebra\\_1/Equations/One\\_Step\\_Fractions.html](http://www.math-aids.com/Algebra/Algebra_1/Equations/One_Step_Fractions.html)

### **One-Step Equations: Fractions and Decimals**

Practice solving equations in one step by multiplying or dividing a number from both sides.

<https://www.khanacademy.org/math/algebra/one-variable-linear-equations/alg1-one-step-mult-div-equations/e/one-step-mult-div-equations-2>

### **7th Grade Numbers and Operations Jeopardy Game**

The questions in this game range from absolute value to different operations with rational numbers.

<http://www.math-play.com/7th-Grade-Numbers-and-Operations-Jeopardy/7th-Grade-Numbers-and-Operations-Jeopardy.html>

**Sample worksheet from**

[www.mathmammoth.com](http://www.mathmammoth.com)

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# Multiply and Divide Rational Numbers 1

In real life we often combine **fractions, decimals, ratios, and percents** — rational numbers in different forms — in the same situation. You need to be able to easily calculate with them in their different forms.

In this lesson, we will concentrate on multiplying and dividing *decimals* and *fractions* because percentages are usually rewritten as decimals and ratios as fractions before calculating with them.

## To multiply decimals

**Shortcut:** First multiply as if there were no decimal points. Then put the decimal point in the answer so that the number of decimal digits in the answer is the SUM of the number of the decimal digits in all the factors.

**Example 1.** Solve  $-0.2 \cdot 0.09$ .

Multiply  $2 \cdot 9 = 18$ . The answer will have three decimals *and* be negative (Why?), so the answer is  $-0.018$ .

## Multiply fractions and mixed numbers

1. Change any mixed numbers to fractions.
2. Multiply using the shortcut (multiply the numerators; multiply the denominators).

**Example 2.**  $-\frac{4}{5} \cdot \left(-5\frac{1}{8}\right)$

$$= -\frac{4}{5} \cdot \left(-\frac{41}{8}\right)$$

A negative times a negative makes a positive, so we can drop the minus signs in the next step.

$$= \frac{4 \cdot 41}{5 \cdot 8} = \frac{1 \cdot 41}{5 \cdot 2} = \frac{41}{10} = 4\frac{1}{10}$$

1. Write the rational numbers in their four forms.

ratio	fraction	decimal	percent	ratio	fraction	decimal	percent
a. 2:5	$= \frac{2}{5}$	$= 0.4$	$= 40\%$	d.	$= \frac{7}{20}$	$=$	$=$
b. 3:4	$=$	$=$	$=$	e.	$=$	$=$	$= 55\%$
c. 4:25	$=$	$=$	$=$	f.	$=$	$= 0.85$	$=$

2. Multiply these in your head.

a. $0.1 \cdot 6.5$	b. $-0.08 \cdot 0.006$	c. $-0.09 \cdot 0.02$
d. $-0.2 \cdot (-1.6)$	e. $-0.8 \cdot 1.1 \cdot (-0.02)$	f. $0.8^2$
g. $(-0.5)^2$	h. $(-0.2)^3$	i. $(-0.1)^5$

### 3. Multiply

<b>a.</b> $-\frac{1}{7} \cdot \left(-\frac{3}{8}\right)$	<b>b.</b> $\frac{1}{5} \cdot \left(-2\frac{1}{2}\right)$	<b>c.</b> $-\frac{2}{9} \cdot \frac{5}{6} \cdot \frac{3}{10}$
<b>d.</b> $-3\frac{1}{4} \cdot \frac{5}{2}$	<b>e.</b> $\frac{7}{18} \cdot \left(-\frac{12}{27}\right)$	<b>f.</b> $\frac{8}{7} \cdot \left(-\frac{3}{10}\right) \cdot 1\frac{1}{2}$

### 4. Multiply using the regular multiplication algorithm (write one number under the other).

<b>a.</b> $12.5 \cdot 2.5$	<b>b.</b> $-0.088 \cdot 0.16$
<b>c.</b> $-9.08 \cdot (-0.006)$	<b>d.</b> $24 \cdot (-0.0087)$

### To divide decimals

1. If the divisor has no decimal digits, you can divide using long division “as is.”
2. If the divisor does have decimal digits, multiply *both* the dividend and the divisor by the same number (usually a power of ten) to make the divisor into a whole number. Now with that whole number divisor, performing the long division has become straightforward.

**Example 3.** Solve  $6 \div 0.5$  without a calculator.

Since 0.5 fits into 6 exactly twelve times, the answer is 12. So mental math was sufficient in this case.

**Example 4.** Solve  $-92.91 \div 0.004$  without a calculator.

It may be easier to write the problem using a fraction line:

$$\frac{-92.91}{0.004} = \frac{-929.1}{0.04} = \frac{-9291}{0.4} = \frac{-92910}{4}$$

Notice how we multiply both the dividend and the divisor repeatedly by 10 until the divisor becomes a whole number (4). (You could, of course, simply multiply them both by 1,000 to start with.) Then we use long division.

The long division gives us the absolute value of the final answer, but we still need to apply the correct sign. So  $-92.91 \div 0.004 = -23,227.5$ .

Does this make sense? Yes. The answer has a very large absolute value because 0.004 is a very tiny number, and thus it “fits” into 92.91 multitudes of times.

$$\begin{array}{r} 23227.5 \\ 4 \overline{) 92910.0} \\ \underline{-8} \phantom{0} \\ 12 \phantom{0} \\ \underline{-12} \phantom{0} \\ 09 \phantom{0} \\ \underline{-8} \phantom{0} \\ 11 \phantom{0} \\ \underline{-8} \phantom{0} \\ 30 \phantom{0} \\ \underline{-28} \phantom{0} \\ 20 \phantom{0} \\ \underline{-20} \\ 0 \end{array}$$

5. Divide using mental math.

a. $-0.88 \div 4$	b. $8.1 \div 9$	c. $72 \div 10000$
d. $-1.6 \div (-0.2)$	e. $8 \div 0.1$	f. $0.8 \div (-0.04)$

6. Multiply both the dividend and the divisor by the same number so that you get a divisor that is a *whole number*. Then divide using long division. If necessary, round your answer to three decimal digits.

a. $27.6 \div 0.3$	b. $2.088 \div 0.06$
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**To divide fractions and mixed numbers**

1. Change any mixed numbers to fractions.
2. Divide using the shortcut. (Change the division into a multiplication by the reciprocal of the divisor.)

**Example 5.**  $\frac{4}{5} \div \left(-2\frac{1}{2}\right)$

$$= \frac{4}{5} \div \left(-\frac{5}{2}\right)$$

$$= \frac{4}{5} \cdot \left(-\frac{2}{5}\right) = -\frac{8}{25}$$

The answer makes sense, because  $2\frac{1}{2}$  does not fit into  $\frac{4}{5}$ , not even half-way.

7. Divide.

**a.**  $-\frac{2}{9} \div \frac{6}{7}$

**b.**  $\frac{9}{8} \div \left(-1\frac{1}{2}\right)$

**c.**  $-10 \div \frac{5}{6}$

**d.**  $-\frac{1}{9} \div \left(-\frac{1}{3}\right)$

**e.**  $10\frac{1}{5} \div \left(-2\frac{1}{3}\right)$

**f.**  $10 \div \frac{1}{6}$



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# Chapter 5: Equations and Inequalities

## Introduction

In this chapter we delve deeper into our study of equations. Now, the equations require two or more steps to solve and may contain parentheses. The variable may appear on both sides of the equation. Students will also write equations to solve simple word problems.

There is also another lesson on patterns of growth, which may seem to be simply a fascinating topic, but in reality presents the fundamentals of a very important concept in algebra—that of linear functions (although they are not mentioned by that name)—and complements the study of lines in the subsequent lessons.

After the section about equations, the text briefly presents the basics of inequalities and how to graph them on a number line. Students apply the principles for solving equations to solve simple inequalities and word problems that involve inequalities.

The last major topic is graphing. Students begin the section by learning to graph linear equations and continue on to the concept of slope, which in informal terms is a measure of the inclination of a line. More formally, slope can be defined as the ratio of the change in  $y$ -values to the change in  $x$ -values. The final lesson applies graphing to the previously-studied concepts of speed, time, and distance through graphs of the equation  $d = vt$  in the coordinate plane.

You might consider mixing the lessons from this chapter with lessons from some chapter from part B (such as chapter 6 or 8). For example, the student could study topics from this chapter and from the geometry chapter on alternate days, or study a little from each chapter each day. Such, somewhat spiral, usage of the curriculum can help prevent boredom, and also to help students retain the concepts better.

As a reminder, you will find free videos covering many topics of this chapter of the curriculum at <https://www.mathmammoth.com/videos/> (choose 7th grade).

### The Lessons in Chapter 5

	page	span
Two-Step Equations .....	168	5 pages
Two-Step Equations: Practice .....	173	4 pages
Growing Patterns 2 .....	177	4 pages
A Variable on Both Sides .....	181	6 pages
Some Problem Solving .....	187	3 pages
Using the Distributive Property .....	190	6 pages
Word Problems .....	196	3 pages
Inequalities .....	199	5 pages
Word Problems and Inequalities .....	204	2 pages
Graphing .....	206	4 pages
An Introduction to Slope .....	210	5 pages
Speed, Time and Distance .....	215	5 pages
Chapter 5 Mixed Review .....	220	3 pages
Chapter 5 Review .....	223	6 pages

## Helpful Resources on the Internet

### Pre-algebra Videos

A set of videos matching this pre-algebra course, made by the author.

<http://www.mathmammoth.com/videos/prealgebra/pre-algebra-videos.php>

### TWO-STEP EQUATIONS

#### Model Algebraic Equations with a Scale

Model and solve algebraic equations using a pan balance and tiles. Choose “2-Step Equations” for this level.

<http://www.mathplayground.com/AlgebraEquations.html>

#### Two-Step Equations Game

Choose the correct root for the given equation (multiple-choice), and then you get to attempt to shoot a basket. The game can be played alone or with another student.

<http://www.math-play.com/Two-Step-Equations-Game.html>

#### Solving Two-Step Equations

Type the answer to two-step-equations such as  $-4y + 9 = 29$ , and the computer checks it. If you choose “Practice Mode,” it is not timed.

<http://www.xpmath.com/forums/arcade.php?do=play&gameid=64>

#### Two-Step Equations

Practice solving equations that take two steps to solve in this interactive exercise from Khan Academy.

[https://www.khanacademy.org/math/algebra/one-variable-linear-equations/alg1-two-steps-equations-intro/e/linear\\_equations\\_2](https://www.khanacademy.org/math/algebra/one-variable-linear-equations/alg1-two-steps-equations-intro/e/linear_equations_2)

#### Two-Step Equations Word Problems

Practice writing equations to model and solve real-world situations in this interactive exercise.

<https://www.khanacademy.org/math/algebra-basics/alg-basics-linear-equations-and-inequalities/alg-basics-two-steps-equations-intro/e/linear-equation-world-problems-2>

#### Visual Patterns

Hundreds of growing patterns. The site provides the answer to how many elements are in step 43 of the pattern.

<http://www.visualpatterns.org/>

### SIMPLIFYING EXPRESSIONS

#### Factor the Expressions Quiz

Factor expressions. For example,  $-4x + 16$  factors into  $-4(x - 4)$ .

<http://www.thatquiz.org/tq-0/?-jh00-14-p0>

#### Simplifying Algebraic Expressions Practice Problems

Practice simplifying expressions such as  $4(2p - 1) - (p + 5)$  with these 10 questions. Answer key included.

<http://www.algebra-class.com/algebraic-expressions.html>

#### Distributive Property with Negative Numbers

Use the distributive property to remove the parentheses in this interactive exercise. Click to see an example.

<http://www.hstutorials.net/dialup/distributiveProp.htm>

#### Simplifying Algebraic Expressions (1)

Eight practice problems that you can check yourself about combining like terms and using the distributive property.

[http://www.algebra-lab.org/lessons/lesson.aspx?file=Algebra\\_BasicOpsSimplifying.xml](http://www.algebra-lab.org/lessons/lesson.aspx?file=Algebra_BasicOpsSimplifying.xml)

#### Simplifying Algebraic Expressions (2)

Check your knowledge of algebraic expressions with this interactive multiple-choice quiz.

<http://www.phschool.com/webcodes10/index.cfm?wcprefix=asa&wcsuffix=0602>

## MORE EQUATIONS

### Balance When Adding and Subtracting Game

An interactive balance where you add or subtract  $x$ 's and 1's until you leave  $x$  alone on one side.

<http://www.mathsisfun.com/algebra/add-subtract-balance.html>

### Solve Equations Quiz

A 10-question online quiz where you need to solve equations with an unknown on both sides.

<http://www.thatquiz.org/tq-0/?-j102-l4-p0>

### Equations Level 3 Online Exercise

Practice solving equations with an unknown on both sides in this self-check online exercise.

[http://www.transum.org/software/SW/Starter\\_of\\_the\\_day/Students/Equations.asp?Level=3](http://www.transum.org/software/SW/Starter_of_the_day/Students/Equations.asp?Level=3)

### Missing Lengths

Try to figure out the value of the letters used to represent the missing numbers.

[http://www.transum.org/software/SW/Starter\\_of\\_the\\_day/Students/Missing\\_Lengths.asp](http://www.transum.org/software/SW/Starter_of_the_day/Students/Missing_Lengths.asp)

### Equations Level 4 Online Exercise

Practice solving equations which include brackets in this self-check online exercise.

[http://www.transum.org/software/SW/Starter\\_of\\_the\\_day/Students/Equations.asp?Level=4](http://www.transum.org/software/SW/Starter_of_the_day/Students/Equations.asp?Level=4)

### Equations Level 5 Online Exercise

This exercise includes more complex equations requiring multiple steps to find the solution.

[http://www.transum.org/software/SW/Starter\\_of\\_the\\_day/Students/Equations.asp?Level=5](http://www.transum.org/software/SW/Starter_of_the_day/Students/Equations.asp?Level=5)

### Solving Equations Quizzes

Here are some short online quizzes that you can check yourself.

<http://www.phschool.com/webcodes10/index.cfm?wcprefix=ara&wcsuffix=0406&area=view>

<http://www.phschool.com/webcodes10/index.cfm?wcprefix=bja&wcsuffix=0701>

### Rags to Riches Equations

Choose the correct root to a linear equation.

<http://www.quia.com/rr/4096.html>

### Algebra Four Game

To practice the types of equations we study in this chapter, choose "Level 1," and tick the boxes "Variable on both sides," "Distributive Property," and "Two-Step Problems" (don't check "Quadratic Equations").

<http://www.shodor.org/interactivate/activities/AlgebraFour/>

### Solve Equations Exercises

Click "new problem" (down the page) to get a randomly generated equation to solve. This exercise includes an optional graph which the student can use as a visual aid.

[http://www.onemathematicalcat.org/algebra\\_book/online\\_problems/solve\\_lin\\_int.htm#exercises](http://www.onemathematicalcat.org/algebra_book/online_problems/solve_lin_int.htm#exercises)

### Equation Word Problems Quiz

Solve word problems which involve equations and inequalities in this multiple-choice online quiz.

<http://www.phschool.com/webcodes10/index.cfm?wcprefix=bja&wcsuffix=0704>

### Whimsical Windows - Equation Game

Write an equation for the relationship between  $x$  and  $y$  based on a table of  $x$  and  $y$  values. Will you discover the long lost black unicorn stallion?

<http://mrnussbaum.com/whimsical-windows/>

## INEQUALITIES

### Inequality

A six-question quiz on simple inequalities and their graphs.

<http://www.mathopolis.com/questions/q.php?id=12194>

### Equations and Inequalities

Test your skills with this multiple-choice quiz.

<http://www.phschool.com/webcodes10/index.cfm?wcprefix=ara&wcsuffix=0407&area=view>

### Plot Simple Inequalities

Practice plotting simple inequalities on a number line in this 10-question interactive quiz.

<https://www.thatquiz.org/tq-o/?-j18-11-p0>

### Match Inequalities and Their Plots

Match the statements with the corresponding diagrams in this interactive online activity.

[http://www.transum.org/software/SW/Starter\\_of\\_the\\_day/Students/InequalitiesB.asp?Level=5](http://www.transum.org/software/SW/Starter_of_the_day/Students/InequalitiesB.asp?Level=5)

### Solve Simple Inequalities

For each inequality, find the range of values for  $x$  which makes the statement true. An example is given.

[http://www.transum.org/software/SW/Starter\\_of\\_the\\_day/Students/InequalitiesC.asp?Level=6](http://www.transum.org/software/SW/Starter_of_the_day/Students/InequalitiesC.asp?Level=6)

### Two-Step Inequality Word Problems

Practice constructing, interpreting, and solving linear inequalities that model real-world situations.

<https://www.khanacademy.org/math/algebra/one-variable-linear-inequalities/alg1-two-step-inequalities/e/interpreting-solving-linear-inequalities>

## SPEED, TIME, AND DISTANCE

### Understanding Distance, Speed, and Time

An interactive simulation of two runners. You set their starting points and speeds, and observe their positions as the tool runs the simulation. It graphs the position of both runners in relation to time.

<http://illuminations.nctm.org/Activity.aspx?id=6378>

### Representing Motion

A tutorial an interactive quiz with various questions about speed, time, and distance.

[http://www.bbc.co.uk/schools/gcsebitesize/science/add\\_aqa\\_pre\\_2011/forces/represmotionrev1.shtml](http://www.bbc.co.uk/schools/gcsebitesize/science/add_aqa_pre_2011/forces/represmotionrev1.shtml)

### Absorb Advanced Physics - Speed

An online tutorial that teaches the concept of average speed with the help of interactive simulations and exercises.

<http://www.absorblearning.com/advancedphysics/demo/units/010101.html#Describingmotion>

### Distance-Time Graphs

An illustrated tutorial about distance-time graphs. Multiple-choice questions are included.

<http://www.absorblearning.com/advancedphysics/demo/units/010103.html>

### Distance-Time Graph

Click the play button to see a distance-time graph for a vehicle which moves, stops, and then changes direction.

[http://www.bbc.co.uk/schools/gcsebitesize/science/add\\_aqa\\_pre\\_2011/forces/represmotionrev5.shtml](http://www.bbc.co.uk/schools/gcsebitesize/science/add_aqa_pre_2011/forces/represmotionrev5.shtml)

### Distance Versus Time Graph Puzzles

Try to move the stick man along a number line in such a way as to illustrate the graph that is shown.

<http://davidwees.com/graphgame/>

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# A Variable on Both Sides

**Example 1.** Solve  $2x + 8 = -5x$ .

Notice that the unknown appears on both sides of the equation. To isolate it, we need to

- either subtract  $2x$  from both sides—because that makes  $2x$  disappear from the left side
- or add  $5x$  to both sides—because that makes  $-5x$  disappear from the right side.

$$\begin{array}{r|l}
 2x + 8 = -5x & + 5x \\
 2x + 8 + 5x = 0 & \text{(now add } 2x \text{ and } 5x \text{ on the left side)} \\
 7x + 8 = 0 & - 8 \\
 7x = -8 & \div 7 \\
 x = -8/7 & 
 \end{array}$$

Check:

$$\begin{array}{l}
 2 \cdot (-8/7) + 8 \stackrel{?}{=} -5 \cdot (-8/7) \\
 -16/7 + 8 \stackrel{?}{=} 40/7 \\
 -2 \frac{2}{7} + 8 = 5 \frac{5}{7} \quad \checkmark
 \end{array}$$

**Example 2.** Solve  $10 - 2s = 4s + 9$ .

To isolate  $s$ , we need to

- either add  $2s$  to both sides
- or subtract  $4s$  from both sides.

The choice is yours. Personally, I like to keep the unknown on the left side and eliminate it from the right.

$$\begin{array}{r|l}
 10 - 2s = 4s + 9 & - 4s \\
 10 - 2s - 4s = 9 & \text{(now simplify } -2s - 4s \text{ on the left side)} \\
 10 - 6s = 9 & - 10 \\
 -6s = -1 & \div (-6) \\
 s = 1/6 & 
 \end{array}$$

Check:

$$\begin{array}{l}
 10 - 2 \cdot (1/6) \stackrel{?}{=} 4 \cdot (1/6) + 9 \\
 10 - 2/6 \stackrel{?}{=} 4/6 + 9 \\
 9 \frac{4}{6} = 9 \frac{4}{6} \quad \checkmark
 \end{array}$$

1. Solve. Check your solutions (as always!).

a.  $3x + 2 = 2x - 7$

b.  $9y - 2 = 7y + 5$

2. Solve. Check your solutions (as always!).

a.  $11 - 2q = 7 - 5q$

b.  $6z - 5 = 9 - 2z$

c.  $8x - 12 = -1 - 3x$

d.  $-2y - 6 = 20 + 6y$

e.  $6w - 6.5 = 2w - 1$

f.  $5g - 5 = -20 - 2g$



## Combining like terms

Remember, in algebra, a *term* is an expression that consists of numbers, fractions, and/or variables that are multiplied. This means that the expression  $-2y + 7 + 8y$  has three terms, separated by the plus signs.

In the expression  $-2y + 7 + 8y$ , the terms  $-2y$  and  $8y$  are called **like terms** because they have the same variable part (in this case a single  $y$ ). We can **combine** (add or subtract) like terms.

To do that, it helps to organize the terms in the expression in alphabetical order according to the variable part and write the constant terms last. We get  $-2y + 8y + 7$  ( $8y - 2y + 7$  is correct, too).

Next, we add  $-2y + 8y$  and get  $6y$ . So the expression  $-2y + 7 + 8y$  simplifies to  $6y + 7$ .

**Example 3.** Simplify  $6y - 8 - 9y + 2 - 7y$ .

First, we organize the expression so that the terms with  $y$  are written first, followed by the constant terms.

For that purpose, we **view each operation symbol** (+ or -) **in front of the term as the sign of each term**.

In a sense, you can imagine each plus or minus symbol as being “glued” to the term that follows it. Of course the first term,  $6y$ , gets a “+” sign.

$$\textcircled{+6y} \textcircled{-8} \textcircled{-9y} \textcircled{+2} \textcircled{-7y}$$

After reordering the terms, the expression becomes  $6y - 9y - 7y - 8 + 2$ .

Now we need to combine the like terms  $6y$ ,  $-9y$ , and  $-7y$ . We do that by finding the sum of their coefficients 6,  $-9$ , and  $-7$ . Since  $6 - 9 - 7 = -10$ , we know that  $6y - 9y - 7y = -10y$ .

Similarly, we combine the two constant terms:  $-8 + 2 = -6$ .

Our expression therefore simplifies to  $-10y - 6$ .

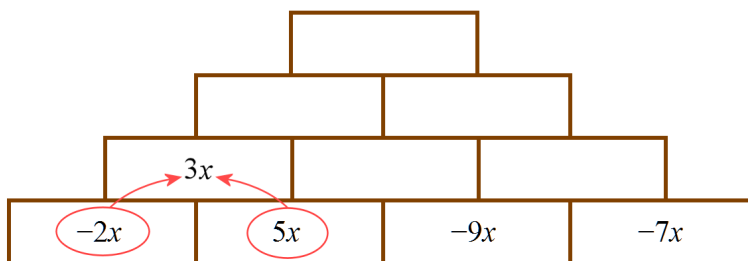
### Why can we do it this way?

Because subtracting a term is the same as adding its opposite. In symbols,

$$\begin{aligned} & 6y \quad -8 \quad -9y + 2 \quad -7y \\ = & 6y + (-8) + (-9y) + 2 + (-7y). \end{aligned}$$

In other words, the expression  $6y - 8 - 9y + 2 - 7y$  is the SUM of the terms  $6y$ ,  $-8$ ,  $-9y$ ,  $2$ , and  $-7y$ .

3. Fill in the pyramid! Add each pair of terms in neighboring blocks and write its sum in the block above it.



4. Organize the expressions so that the variable terms are written first, followed by constant terms.

a.  $6 + 2x - 3x - 7 + 11$

b.  $-s - 12 + 15s + 9 - 7s$

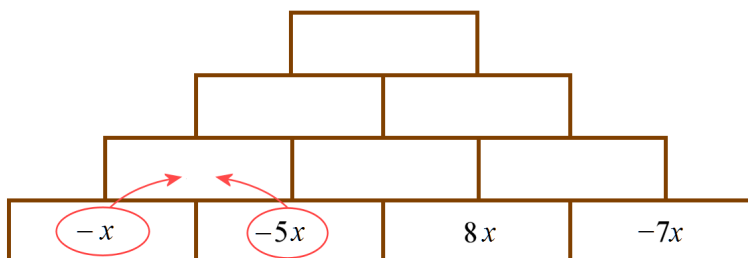
c.  $-8 + 5t - 2 - 6t$

5. Simplify the expressions in the previous exercise.

6. Simplify.

<p>a. <math>5x - 8 - 7x + 1</math></p>	<p>b. <math>-6a - 15a + 9a + 7a</math></p>
<p>c. <math>-8 + 7c - 11c + 8 - c</math></p>	<p>d. <math>10 - 5x - 8x - 9 + x</math></p>

7. Fill in the pyramid! Add each pair of terms in neighboring blocks and write its sum in the block above it.



8. Find what is missing from the sums.

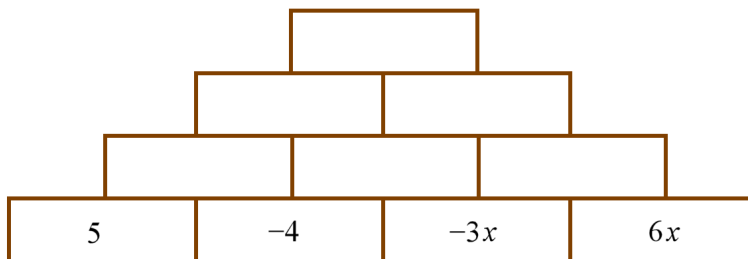
a.  $8x + 2 + \underline{\hspace{2cm}} = 5x + 8$

b.  $5b - 2 + \underline{\hspace{2cm}} = 2b + 7$

c.  $-2z + \underline{\hspace{2cm}} = 1 - 5z$

d.  $-4f + 3 + \underline{\hspace{2cm}} = -f - 1$

9. Fill in the pyramid! Add each pair of terms in neighboring blocks and write its sum in the block above it.



10. Simplify.

<p>a. <math>0.5y + 1.2y - 0.6y</math></p>	<p>b. <math>-1.6v - 1 - v</math></p>	<p>c. <math>-0.8k + 3 + 0.9k</math></p>
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11. A challenge! Solve the equation  $(-1/2)x - 6 + 8x + 7 - x = 0$ .

**Example 4.** One or both sides of an equation may have several terms with the unknown. In that case, we need to combine the like terms (simplify) before continuing with the actual solution.

$$3x + 7 - 5x = 6x + 1 - 5x$$

$$-2x + 7 = x + 1$$

$$-3x + 7 = 1$$

$$-3x = -6$$

$$3x = 6$$

$$x = 2$$

On the left side, combine  $3x$  and  $-5x$ .  
On the right side, combine  $6x$  and  $-5x$ .

$$-x$$

$$-7$$

$$\cdot (-1)$$

$$\div 3$$

12. Solve. Check your solutions.

**a.**  $6x + 3x + 1 = 9x - 2x - 7$

**b.**  $16y - 4y - 3 = -4y - y$

**c.**  $-26x + 12x = -18x + 8x - 6$

**d.**  $-9h + 4h + 7 = -2 + 5h + 9h + 8h$

13. Solve. Check your solutions.

<p><b>a.</b> <math>2x - 4 - 7x = -8x + 5 + 2x</math></p>	<p><b>b.</b> <math>-6 - 4z - 3z = 5z + 8 - z</math></p>
<p><b>c.</b> <math>8 - 2m + 5m - 8m = 20 - m + 5m - 2m</math></p>	<p><b>d.</b> <math>-x - x + 2x = 5 - 5x + 9x</math></p>
<p><b>e.</b> <math>-q + 2q - 5q - 6q = 20 - 7 - 9 + q</math></p>	<p><b>f.</b> <math>9 - s + 7 - 9s = 2 - 2s - 11</math></p>