Hours of Daylight – Anchorage

Working with Sinusoidal Models

"When am I ever going to use this?"

Using the concepts in this worksheet, you will be able to create a mathematical model for the hours of daylight where you live.

The hours of daylight in a given part of the world changes from day to day. In the northern hemisphere, the Summer Solstice (around June 21) is the longest day of the year and the Winter Solstice (around December 21) is the shortest day of the year. The United States Naval Observatory allows website visitors to download the hours of daylight for any day of the year and any location in the world (Source: aa.usno.navy.mil).

The hours of daylight on the 21st day of the month for selected cities are shown in the tables below.

Hours of Daylight on 21st Day of the Month – Anchorage, Alaska											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
06:52	09:39	12:20	15:18	17:59	19:22	18:00	15:16	12:21	09:31	06:47	05:27

Hours of Daylight on 21st Day of the Month – Phoenix, Arizona											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
10:19	11:13	12:10	13:13	14:02	14:22	14:02	13:12	12:11	11:10	10:18	09:56

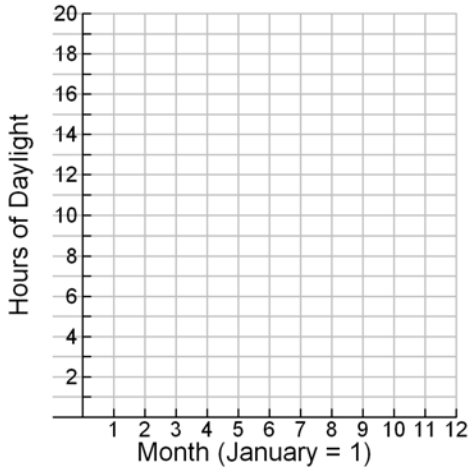
Hours of Daylight on 21st Day of the Month – Easter Island, Chile											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
13:34	12:52	12:07	11:18	10:41	10:26	10:40	11:17	12:05	12:52	13:34	13:51

Hours of Daylight on 21st Day of the Month – Perth, Australia											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
14:39	13:25	12:07	10:43	09:37	09:09	09:36	10:42	12:05	13:27	14:40	15:11

1. What is the significance of June 21 for each of the cities?

2. What is the significance of December 21 for each of the cities?

3. On the axes below, draw the scatter plot for the hours of daylight in Anchorage, Alaska. Be sure to convert the times from hour-minute format (e.g. 12:20) to hour format (e.g. 12.33).



4. Which type of sinusoidal function will best fit the data without requiring a horizontal shift: $\sin(m), -\sin(m), \cos(m)$, or $-\cos(m)$? Explain.

5. Find a sinusoidal model of the type identified in Exercise 4 that fits the data well.

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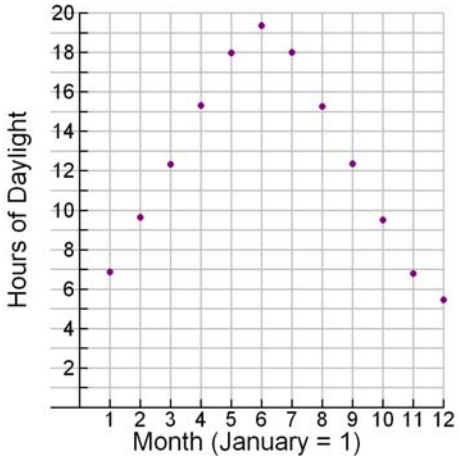
1. What is the significance of June 21 for each of the cities?

That day is the longest day of the year for Anchorage and Phoenix and is the shortest day of the year for Easter Island and Perth.

2. What is the significance of December 21 for each of the cities?

That day is the shortest day of the year for Anchorage and Phoenix and is the longest day of the year for Easter Island and Perth.

3. On the axes below, draw the scatter plot for the hours of daylight in Anchorage, Alaska. Be sure to convert the times from hour-minute format (e.g. 12:20) to hour format (e.g. 12.33).



4. Which type of sinusoidal function will best fit the data without requiring a horizontal shift: $\sin(m), -\sin(m), \cos(m)$, or $-\cos(m)$? Explain.

We first observe that month 12 (December) is the same as month 0 (December of the previous year). Consequently, we predict that the vertical intercept will be 5.45 hours as well. The data appears to start at its minimum value (0, 5.45), climb to it maximum value (6, 19.37), and then descend to its minimum value (12, 5.45). The function $h(m) = -\cos(m)$ exhibits this same basic behavior.

5. Find a sinusoidal model of the type identified in Exercise 4 that fits the data well.

The maximum value is 19.37 and the minimum value is 5.45. We use these values to find the amplitude and midline.

$$midline = \frac{max + min}{2} \qquad amplitude = \frac{max - min}{2}$$
$$= \frac{19.37 + 5.45}{2} \qquad = \frac{19.37 - 5.45}{2}$$
$$= 12.41 \qquad = 6.96$$

Since there are 12 months in a year, we expect that the period of the function is 12 months. We find B.

$$B = \frac{2\pi}{period}$$
$$= \frac{2\pi}{12}$$
$$= \frac{\pi}{6}$$
$$h(m) = -amplitude\left(\cos\left(B \cdot m\right)\right) + midline$$
$$= -6.96\cos\left(\frac{\pi}{6}m\right) + 12.41$$

The hours of daylight on the 21st day of month m are given by $h(m) = -6.96\cos\left(\frac{\pi}{6}m\right) + 12.41$.

Worksheet Title	Hours of Daylight – Anchorage: Working with Sinusoidal Models					*Filename:*	m5031
Keywords	Hours of daylight, sinusoidal model, modeling, Easter Island, Chile, Naval Observatory, period, amplitude						
NCTM Standard		Content Standards			Process Standards		
		Number and Operations		X	Problem Solving		
	X	Algebra			Reasoning and Proof		
		Geometry			Communication		
		Measurement		X	Connections		
	X	Data Analysis and Probability			Representations		
Grade Band		PreK – 2					
		3 – 5					
		6 – 8					
	X	9 – 12					
Data Type	Table						

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