Graphing Proportional Relationships 1

We will now review what it means when two variables are **in direct variation** or **in proportion**. The basic idea is that whenever one variable changes, the other varies (changes) proportionally or at the same rate.

Example 1. The wholesaler posted the following table for the price of potatoes:

weight (kg)	5	10	15	20	25	30
cost	$5.50	$11.00	$16.50	$22.00	$27.50	$33.00

Each pair of cost and weight forms a rate — and so does each pair of weight and cost. However, it is more common to look at the rate "cost over weight", such as $27.50/(25 kg), than vice versa.

If all of the rates in the table are equivalent, then the weight and the cost *are* proportional.

To check for that, we have several means. One is to calculate **the unit rate** (the rate for 1 kg) from each of these rates, and check whether you get the same unit rate.

In this case, that is so. The unit rate is $1.10/kg, no matter which rate from the table we'd use to calculate it.

One other way to check is, if one quantity doubles (or triples), will the other double (or triple) also?
This is especially useful for noticing if the quantities are *not* in direct variation.

Example 2. Here, when the weight doubles from 5 kg to 10 kg, the price also doubles. But what happens with the price when the weight doubles from 10 kg to 20 kg?

weight (kg)	5	10	15	20	25	30
cost	$6	$12	$18	$22	$26	$30

The price does not double! So, the quantities are not in proportion.
The seller is giving you some discount if you purchase higher quantities.

Also, if you calculate the unit rate from $6/(5 kg) and from $22/(20 kg), they are not equal. (Verify this.)

1. Are the quantities in a proportional relationship? If yes, list the unit rate.

a.

time (hr)	0	1	2	3	4	5
distance (km)	0	50	90	140	190	240

b.

time (hr)	0	1	2	3	4	5
distance (km)	0	45	90	135	180	225

c.

age (days)	0	1	2	3	4	5	6	7
height (in)	0	0	0	1	2	3	4	5

d.

length (m)	0	0.5	1	1.5	2	4	5	10
cost ($)	0	3	6	9	12	24	30	60

2. Now consider the tables of values in #1 as functions, where the variable listed on top is the independent variable. For the ones where the quantities were in proportion, calculate the rate of change.

   What is its relationship to the unit rate?

When two quantities are in a proportional relationship, or in direct variation (the two are synonyms):

(1) Each rate formed by the quantities is equivalent to any other rate of the quantities.

(2) The equation relating the two quantities is of the form $y = mx$, where y and x are the variables, and m is a constant. The constant m is called the **constant of proportionality** and is also the unit rate.

(3) When plotted, the graph is a straight line that goes through the origin.

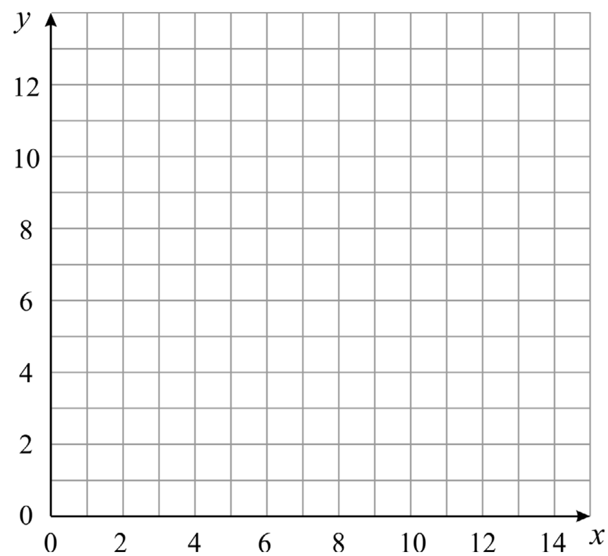
3. Choose an equation from below where the variables x and y are in direct variation (proportional):

$y = \frac{3}{x}$ $\quad\quad$ $y = 3x$

$xy = 3$ $\quad\quad$ $y = x^3$

Then graph that equation in the grid.

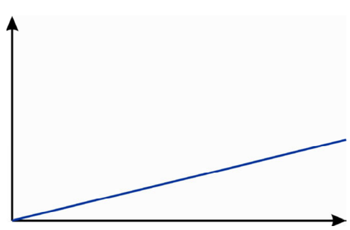
Hint: The point (0, 0) is always included in direct variation. All you need to do is plot one other point, and then draw a line through the origin and that point.



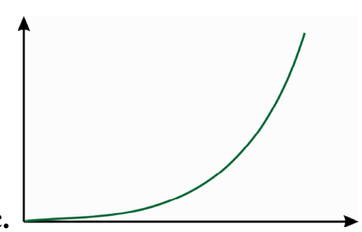
4. Choose the representations that show a proportional relationship.

a.

b.



c.



d.

x	0	1	2	3	4	5
y	15	17	19	21	23	25

e. $y = 2x + 9$

f. $y = (3/4)x$

g.

x	0	4	8	12	16	20
y	0	3	6	9	12	15

5. Two of the above representations are the exact same relationship. Which ones?