

Grade 4-A Worktext International Version

ddition, subtraction patterns and graphs

L arge numbers and place value





Sample worksheet from www.mayimammora.comia Miller \mathbf{L}

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EDITION 1/2016

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Foreword

Math Mammoth International Version Grade 4-A and *Grade 4-B* worktexts comprise a complete maths curriculum for the fourth grade mathematics studies.

This curriculum is essentially the same as the *Math Mammoth Grade 4* sold in the United States (US version), only customised for international use. The US version is aligned to the "Common Core" Standards, so it may not be properly aligned to the fourth grade standards in your country. However, you can probably find material for any missing topics in the neighbouring grades of Math Mammoth.

The International version of Math Mammoth differs from the US version in these aspects:

- The curriculum teaches metric measurement units, not customary (imperial) units.
- The spelling conforms to British international standards.
- The format is adapted for printing on the setting of A4.

In 4th grade, students focus on multi-digit multiplication and division, learning to use bigger numbers, solving multi-step word problems that involve several operations, and they get started in studying fractions and decimals. This is accompanied by studies in geometry and measuring.

The year starts out with a revision of addition and subtraction, patterns and graphs. We illustrate word problems with bar diagrams and study finding missing addends, which teaches algebraic thinking. Children also learn addition and subtraction terminology, the order of operations and statistical graphs.

Next come large numbers—up to millions, and the concept of place value. At first the student revises thousands and some mental maths with them. Next are presented numbers up to one million, calculations with them, the concept of place value and comparing. In the end of the chapter we find out more about millions and an introduction to multiples of 10, 100 and 1 000.

The third chapter is all about multiplication. After briefly revising the concept and the times tables, the focus is on learning multi-digit multiplication (multiplication algorithm). The children also learn why it works when they multiply in parts. We also study the order of operations again, touch on proportional reasoning and do more money and change related word problems.

In part B, we first study division. The focus is on learning long division and using division in word problems. In geometry, we first revise area and perimeter, and then concentrate on the topic of angles. Students measure and draw angles, solve simple angle problems and classify triangles according to their angles. They also study parallel and perpendicular lines.

Fractions and decimals are presented last in the school year. These two chapters practise only some of the basic operations with fractions and decimals. The focus is still on conceptual understanding and on building a good foundation towards 5th grade maths, where fractions and decimals will be in focus.

When you use these books as your only or main mathematics curriculum, they can be like a "framework", but you do have some liberty in organising the study schedule. Chapters 1, 2 and 3 should be studied in that order, but you can be flexible with chapters 4 (Time and Measuring) and 6 (Geometry), and schedule them somewhat earlier or later if you wish. Chapter 3 (Multiplication) needs to be studied before long division in Chapter 5. Many topics from chapters 7 and 8 (Fractions and Decimals) can also be studied earlier in the school year. However, finding parts with division should be studied only after mastering division.

I wish you success in teaching maths!

Maria Miller, the author

Chapter 1: Addition, Subtraction, Patterns, and Graphs Introduction

The first chapter of *Math Mammoth Grade 4* covers addition and subtraction topics, problem solving, patterns, graphs and money.

At first, we revise the "technical aspects" of adding and subtracting: mental maths techniques and adding and subtracting in columns. We also study some patterns. The lesson on Pascal's triangle is intended to be fun and fascinating—after all, Pascal's triangle is full of patterns!

In the next lesson, we study the connection between addition and subtraction and bar models. Bar models help students write addition and subtraction sentences with unknowns, and solve them. This is teaching the students *algebraic thinking*: how to write and solve simple equations.

The lesson on the order of operations contains some revision, but we also study connecting the topic with real-life situations (such as shopping). Here, the student writes the mathematical expression (number sentence) for word problems, which again, practises algebraic thinking.

Going towards applications of maths, the chapter then contains straightforward lessons on bar graphs, line graphs, rounding, estimating and money problems.

The Lessons in Chapter 1

	page	span
Addition Revision	10	3 pages
Adding in Columns	13	l page
Subtraction Revision	14	3 pages
Subtract in Columns	17	2 pages
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Patterns in Pascal's Triangle	21	2 pages
Bar Models in Addition and Subtraction	23	4 pages
Order of Operations	27	2 pages
Making Bar Graphs	29	2 pages
Line Graphs	31	3 pages
Rounding	34	3 pages
Estimating	37	2 pages
Money and Discounts	39	3 pages
Calculate and Estimate Money Amounts	42	3 pages
Revision, Chapter 1	45	2 pages

Helpful Resources on the Internet

Use these free online resources to supplement the "bookwork" as you see fit.

Disclaimer: These links were valid at the time of writing this book, and to the best of our knowledge we believe these websites to have what is described. However, we cannot guarantee that the links have not changed. Parental supervision is needed.

Calculator Chaos

Most of the keys have fallen off the calculator but you have to make certain numbers using the keys that are left.

http://www.mathplayground.com/calculator_chaos.html

ArithmeTiles

Use the four operations and numbers on neighbouring tiles to make target numbers. **http://www.primarygames.com/math/arithmetiles/index.htm**

Choose Math Operation

Choose the mathematical operation(s) so that the number sentence is true. Practise the role of zero and one in basic operations or operations with negative numbers. Helps develop number sense and logical thinking.

http://www.homeschoolmath.net/operation-game.php

MathCar Racing

Keep ahead of the computer car by thinking logically, and practise any of the four operations at the same time.

http://www.funbrain.com/osa/index.html

Fill and Pour

Fill and pour liquid with two containers until you get the target amount. A logical thinking puzzle. http://nlvm.usu.edu/en/nav/frames_asid_273_g_2_t_4.html

Division and Order of operations and

Division and Addition - Order of Operations Two mystery picture games. http://www.dositey.com/2008/math/m/mystery2AD.htm

Order of Operations Quiz

A 10-question online quiz that includes two different operations and possibly brackets in each question. You can also modify the quiz parameters yourself. http://www.thatquiz.org/tq-1/?-j8f-la

The Order of Operations Millionaire

Answer multiple-choice questions that have to do with the order of operations, and win a million. Can be played alone or in two teams.

http://www.math-play.com/Order-of-Operations-Millionaire/order-of-operations-millionaire.html

Exploring Order of Operations (Object Interactive)

The program shows an expression, and you click on the correct operation (either +, -, \times , \div or exponent) to be done first. The program then solves that operation, and you click on the *next* operation to be performed, etc., until it is solved. Lastly, the resource includes a game where you click on the falling blocks in the order that order of operations would dictate.

http://www.learnalberta.ca/content/mejhm/html/object_interactives/order_of_operations/use_it.html

Order of Operations Practice

A simple online quiz of 10 questions. Uses brackets and the four operations. http://www.onlinemathlearning.com/order-of-operations-practice.html

Quick Calculate

Practise your arithmetic of all four operations plus the order of operations. http://themathgames.com/arithmetic-games/addition-subtraction-multiplication-division/quick-calculate-game.php

Estimate Addition Quiz

Scroll down the page to find this quiz plus some others. Fast loading. http://www.quiz-tree.com/Math_Practice_main.html

Shop 'Til You Drop

Get as many items as you can and be left with the least amount of change, and practise your addition skills. The prices are in English pounds and pennies. http://www.channel4learning.com/sites/puzzlemaths/shop.shtml

Change Maker

Determine how many of each denomination you need to make the exact change. Good and clear pictures! Playable in US, Canadian, Mexican, UK, or Australian money. http://www.funbrain.com/cashreg/index.html

Bar Chart Virtual Manipulative

Build your bar chart online using this interactive tool. http://nlvm.usu.edu/en/nav/frames_asid_190_g_1_t_1.html?from=category_g_1_t_1.html

An Interactive Bar Grapher

Graph data sets in bar graphs. The colour, thickness and scale of the graph are adjustable. You can put in your own data, or you can use or alter pre-made data sets. http://illuminations.nctm.org/ActivityDetail.aspx?ID=63

Create a Graph

A neat online tool for creating a graph from your own data. http://nces.ed.gov/nceskids/createagraph/

Math Mahjong

A Mahjong game where you need to match tiles with the same value. It uses all four operations and has three levels.

http://www.sheppardsoftware.com/mathgames/mixed_mahjong/mahjongMath_Level_1.html

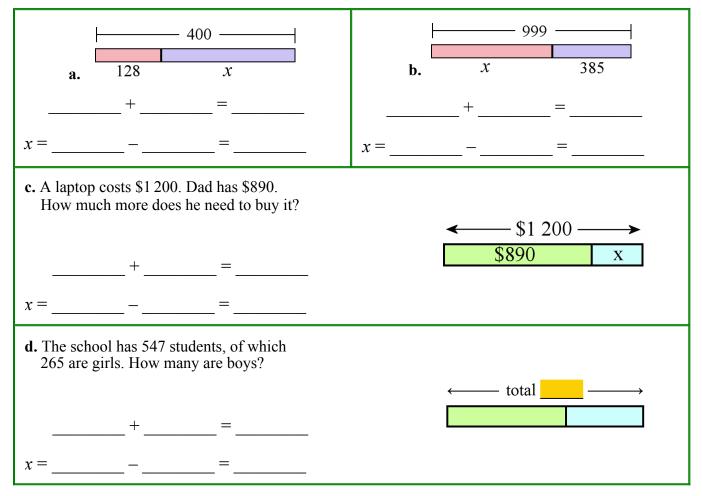
Pop the Balloons

Pop the balloons in the order of their value. You need to use all four operations. http://www.sheppardsoftware.com/mathgames/numberballoons/BalloonPopMixed.htm [This page is intentionally left blank.]

Bar Models in Addition and Subtraction

Think of this bar model as a long board, cut into two pieces. It is 56 units long in total (you can think of centimetres, for example), and the two parts are 15 and x units long.	$\leftarrow 56 \longrightarrow$
From the bar model, we can write TWO addition and TWO subtraction sentences—a <i>fact family</i> .	x + 15 = 56 15 + x = 56 56 - x = 15 56 - 15 = x
The <i>x</i> stands for a number, too. We just do not know what it is yet. It is an <i>unknown</i> .	$15 + x = 56 \qquad 56 - 15 = x$
From this bar model, we can write a missing addend problem. It means that a number to be added is "missing" or unknown.	$\begin{array}{c c} \bullet & 1510 \\ \hline 769 & x \end{array}$
We can solve it by subtracting the one part (769) from the total (1 510).	769 + x = 1510

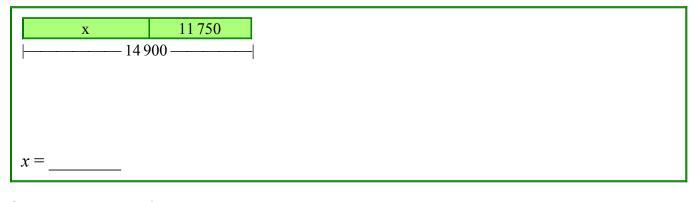
1. Write a missing addend problem that matches the bar model. Then solve it by subtracting.

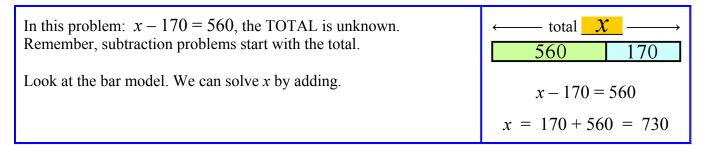


2. Write the numbers and x in the model. Remember, x is the unknown, or what the problem asks for. Write an addition using the numbers and x. Lastly solve.

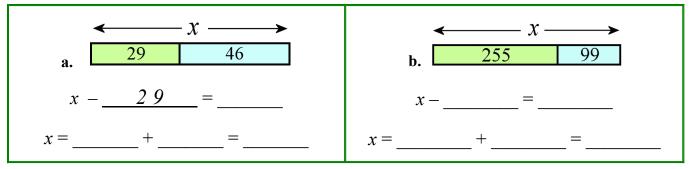
a. Of their 1 200-kilometre trip, the Ames family travelled 420 km yesterday and 370 km today. How many kilometres do they have left to travel?	b. The store is expecting a shipment of 4 000 blank CDs. Three boxes of 400 CDs arrived. How many CDs have not arrived yet?
Addition:	Addition:
Solution: <i>x</i> =	Solution: <i>x</i> =
c. A 250-cm board is divided into three parts: two 28-cm parts at the ends and a part in the middle. How long is the middle part?	d. After travelling 56 kilometres, Dad said, "Okay, 9 km more and we will be in Sidney, and from there we will have 118 km left." How many total kilometres is the trip?
Addition:	Addition:
Solution: $x = $	Solution: $x = $

3. Make a word problem that matches the model. Then solve for *x*.





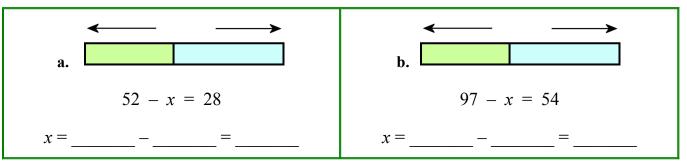
4. Write a subtraction problem that matches the bar model. Then solve it by adding.



5. The number you subtract from is missing! Solve.

a. $-4 = 20$	b. $-15 = 17$	c. $-22-7=70$
The number you subtract from is	still missing. But this time, it is den	noted by x , not by an empty line.
d. $x - 8 = 7$	e. $x - 24 = 48$	f. $x - 300 - 50 = 125$
x =	<i>x</i> =	<i>x</i> =

6. Here, the number you subtract is the unknown. Write the numbers and *x* in the bar model. Notice carefully what number is the *total*. Then write another matching subtraction that helps you solve *x*.



7. The number you subtract is still the unknown. Solve.

a. 20 = 12	b. $55 - \= 34$	c. $234 - __= 100$
d. $61 - x = 43$	e. $100 - x = 72$	f. $899 - x = 342$
<i>x</i> =	<i>x</i> =	<i>x</i> =

8. Circle the number sentence that fits the problem. Then solve for x.

a. Jane had \$15. After Dad gave Jane her allowance (<i>x</i>), Jane had \$22.	b. Matt had many drawings. He put 24 of them in the trash. Then he had 125 left.
15 + x = 22 OR $15 + 22 = x$	125 - 24 = x OR $x - 24 = 125$
x =	x =
c. Jenny had 120 marbles, but some of them got lost. Now she has 89 left.	d. Dylan gave 67 of his stickers to a friend and now he has 150 left.
·	, ,

9. Write a number sentence (addition or subtraction) with x. Solve it.

a. The teachers and students of a school filled a 450-seat auditorium. If the school had 43 teachers, how many students did it have?	+= x =
b. Mum went shopping with \$250 and came home with \$78. How much did she spend?	originally - spent = left $_____=___$ $x = ____$
c. Natalie had \$200. She bought an item for \$54 and another for \$78. How much money is left?	== x =
d. Kelly bought one item for \$23 and another for \$29, and she had \$125 left. How much did she have initially?	== x =

Puzzle Corner Find the missing numbers.		
a. $200 - 45 - \ 70 = 25$ b. $\ 5 - 55 - 120 = 40$		
c. $23 + 56 + x = 110$	d. $x + 15 + 15 + 15 + 15 = 97$	
x =	<i>x</i> =	

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Chapter 2: Large Numbers and Place Value Introduction

The second chapter of Math Mammoth Grade 4 covers large numbers (up to 1 million) and place value.

The first lessons only deal with thousands, or numbers with a maximum of four digits. These are for revision and for deepening the student's understanding of place value. It is crucial that the student understands place value with four-digit numbers before moving on to larger numbers. Then, larger numbers will be very easy to study.

Then we go on to study numbers up to one million, or numbers that have tens or hundreds of thousands. Students write them in expanded form, compare them, add and subtract them and learn more about rounding.

Lastly, we study briefly the multiples of 10, 100 and 1 000. This lesson prepares the way for some very important ideas in the next chapter (multi-digit multiplication).

The Lessons in Chapter 2

	page	span
Thousands	49	3 pages
At the Edge of Whole Thousands	52	2 pages
More Thousands	54	2 pages
Practising with Thousands	56	2 pages
Place Value with Thousands	58	2 pages
Comparing with Thousands	60	3 pages
Adding and Subtracting Big Numbers	63	4 pages
Rounding and Estimating Large Numbers	67	4 pages
Multiples of 10, 100 and 1 000	71	3 pages
Mixed Revision, Chapters 1 - 2	74	2 pages
Revision, Chapter 2	76	2 pages

Helpful Resources on the Internet

Use these free online resources to supplement the "bookwork" as you see fit. **Disclaimer:** These links were valid at the time of writing this book, and to the best of our knowledge we believe these websites to have what is described. However, we cannot guarantee that the links have not changed. Parental supervision is needed.

Place Value Payoff

Match numbers written in standard form with numbers written in expanded form in this game. http://www.quia.com/mc/279741.html

Keep My Place

Fill in the big numbers in this cross-number puzzle. http://www.counton.org/magnet/kaleidoscope2/Crossnumber/index.html

Can You Say Really Big Numbers?

Enter a really big number, try to say it out loud, and see it written. http://www.mathcats.com/explore/reallybignumbers.html

Place Value Puzzler

Place value or rounding game. Click on the asked place value in a number, or type in the rounded version of the number.

http://www.funbrain.com/tens/index.html

Rounding Sharks

You'll be asked to round numbers in the thousands to the nearest hundred. Click on the shark that has the correctly rounded number.

http://www.free-training-tutorial.com/rounding/sharks.html

Rounding Master

A Mathionare-type game where you answer rounding questions, and try to become a Rounding Master Maths Millionaire.

http://www.mrnussbaum.com/roundingmaster.htm

Estimation at AAA Math

Exercises about rounding whole numbers and decimals, front-end estimation, estimating sums and differences. Each page has an explanation, interactive practice and games. http://www.aaamath.com/B/est.htm

Maximum Capacity

Drag as many gorillas as you can into the elevator without exceeding the weight capacity. You will have to use your quick addition, estimation and number sense skills. http://www.mrnussbaum.com/maximumcapacity.htm

Home Run Derby Math

Estimate answers to maths problems. The closer you get, the further your ball will fly at-bat. In addition and subtraction, the numbers are in the thousands. In multiplication, the numbers are in the hundreds. http://www.mrnussbaum.com/derby.htm

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More Thousands

1 000 2 000 3 000 4 000 5 000	6 000 7 000 8 000	9 000 10 000 11 00	 00 12 000 13 000 1	 4 000 15 000
On this number line you see whol	e thousands from o	ne thousand to fif	teen thousand.	
The coloured digits are the "thous as the whole thousands. Read the own number. Say the word "thous separates the thousands from the l We continue with whole thousand <i>a thousand</i> thousands. That number has a new name: <i>one</i>	coloured digits as its and" for the space ast three digits. Is until we reach	1 5 3 that 8 0 2 9 9 0 9 9 9	0 0 0 Read: 0 0 0 Read:	 78 thousand 153 thousand 802 thousand 990 thousand 999 thousand sand thousand
= 1 millionThe rest of the digits tell us our hundreds, tens and ones just like you have learned.1754 <i>Read:</i> 17 thousand five hundred and forty four609230 <i>Read:</i> 609 thousand two hundred and thirty70080 <i>Read:</i> 70 thousand and eighty902005 <i>Read:</i> 902 thousand and five				

1. Draw a line in the number where there should be a space. Fill in the missing parts.

a. 164000	b. 92000	c. 309000	d. 34000	e. 780000
thousand	thousand	thousand	thousand	thousand

2. Draw a line in the number where there should be a space. Fill in. Read the numbers aloud.

a. 164453	b. 92908	c. 329033	d. 14004
<u>164</u> thousand <u>453</u>	thousand	thousand	thousand
e. 550053	f. 72001	g. 800004	h. 30036
thousand	thousand	thousand	thousand

3. Read these numbers aloud.

a. 456 098	b. 950 050	c. 23 090	d. 560 008
e. 78 304	f. 266 894	g. 1 000 000	h. 306 700

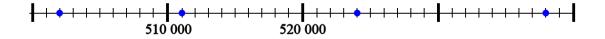
4. Think in whole thousands and add!

a. 30 000 + 5 000 = think: 30 thousand + 5 thousand	b. $200000 + 1000 =$
c. $400000 + 30000 =$	d. $710000 + 40000 =$
e. $300000 + 700000 =$	f. $700000 + 70000 =$

5. Add and subtract, thinking in whole thousands.

a. $35000 + 5000 =$	b. $711000 + 10000 =$
c. $420000 + 30000 =$	d. $700000 - 70000 =$
e. $300000 - 60000 =$	f. $1000000 - 200000 =$
g. $30000 - 5000 =$	h. $200000 - 6000 =$
i. $723000 - 400000 =$	j. $500000 - 1000 =$

6. On the number line below, 510 000 and 520 000 are marked (at the "posts"). Write the numbers that correspond to the dots.



Make a number line from 320 000 to 340 000 with tick-marks at every whole thousand, similar to the one above. Then mark the following numbers on the number line: 323 000 328 000 335 000 329 000 330 000

[This page is intentionally left blank.]

Chapter 3: Multi-Digit Multiplication Introduction

The third chapter of Math Mammoth Grade 4 covers multi-digit multiplication and some related topics.

The first lessons briefly revise the multiplication concept and the times tables. The next lesson, where students solve scales or pan balance problems, is intended to be somewhat fun and motivational. The balance problems are actually equations in disguise.

Then, the focus is on multi-digit multiplication (also called algorithm of multiplication, or multiplying in columns). We start out with multiplying by whole tens and hundreds (such as 20×4 or 500×6). After this is mastered, we study a very important concept of **multiplying in parts** (also called partial products algorithm). It means that 4×63 is done in two parts: 4×60 and 4×3 , and the results are added.

This principle underlies all other multiplication algorithms, so it is important to master. We do not want children to learn the multiplication algorithm "blindly", without understanding what is going on with it. Multiplying in parts is also tied in with an area model, which, again, is very important to understand.

Before showing the traditional form of multiplication, the lesson *Multiply in Columns—the Easy Way* shows a simplified form of the same, which is essentially just multiplying in parts. You may skip that lesson at your discretion or skim through it quickly if your child is ready to understand the standard form of the algorithm, which is taught next.

Students also study estimation, the order of operations, and multiplying with money. Many kinds of word problems abound in all of the lessons. Students are supposed to practise writing a number sentence for the word problems—essentially writing down the calculating they are doing.

The lesson "*So Many of the Same Thing*" could be entitled "Proportional Reasoning" but I wanted to avoid scaring parents and children with such a high-sounding phrase. The idea in that lesson is really simple, but it does prepare for proportions as they are taught in 7th grade and in algebra.

After that, we multiply two-digit numbers by two-digit numbers. Again, we first study partial products and tie that in with an area model. The lesson *Multiplying in Parts: Another Way* is optional. After that, the standard algorithm for multiplying a two-digit number by a two-digit number is taught, and the chapter ends.

The Lessons in Chapter 3

	page	span
Understanding Multiplication	82	3 pages
Multiplication Tables Revision	85	3 pages
Scales Problems	88	4 pages
Multiplying by Whole Tens and Hundreds	92	4 pages
Multiply in Parts, 1	96	3 pages
Multiply in Parts, 2	99	4 pages
More Practice	103	2 pages
Estimating in Multiplication	105	2 pages

	page	span
Multiply in Columns - the Easy Way	107	3 pages
Multiply in Columns - the Easy Way, Part 2	110	3 pages
Multiplying in Columns - the Standard Way	113	4 pages
Multiplying in Columns, Practice	117	3 pages
Order of Operations Again	120	3 pages
Money and Change	123	3 pages
So Many of the Same Thing	126	3 pages
Multiplying Two-Digit Numbers in Parts	129	5 pages
Multiply by Whole Tens in Columns	134	2 pages
Multiplying in Parts: Another Way	136	2 pages
The Standard Multiplication Algorithm		
with a Two-Digit Number Multiplier	138	4 pages
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Multiplication Games

A list of times tables games and activities to practise multiplication facts. http://www.homeschoolmath.net/online/multiplication.php

Math FROG MultipliACTION

Online practice of 2 by 2 digit multiplication. You enter one digit in each box. http://cemc2.math.uwaterloo.ca/mathfrog/english/kidz/mult5.shtml

Math Playground

Learn how to think algebraically with this clever weighing scales. http://www.mathplayground.com/algebraic reasoning.html

Thinking Blocks

Thinking Blocks is an engaging, interactive maths tool that helps children learn how to solve multi-step word problems. Scroll down to Multiplication and Division. http://www.mathplayground.com/thinkingblocks.html

Rectangle Multiplication

An interactive tool that illustrates multiplying in parts using the area model. Choose the "common" option for multiplying in parts.

http://nlvm.usu.edu/en/nav/frames asid 192 g 2 t 1.html

One-Digit by Two-Digits Multiplication Game

Students will multiply one-digit numbers by two-digit whole numbers, and then get to try shoot a basket. http://www.math-play.com/one-digit-by-two-digit-multiplication-game.html

Multiplication Jeopardy Game

You get to solve multi-digit multiplication questions of 1-digit by 1-digit, 1-digit by 2-digit, and 1-digit by 3-digit numbers in this game.

http://www.math-play.com/Multiplicaton-Jeopardy/Multiplication-Jeopardy.html

Interactive Pan Balance

Each of the four shapes is assigned a certain (unknown) weight. You need to figure out their weights by placing them on the two sides of the pan balance in different configurations. http://illuminations.nctm.org/Activity.aspx?id=3531

Balance Beam Activity

A virtual balance that provides balance puzzles where the student has to find the weights of various figures, practising algebraic thinking. Includes three levels. http://mste.illinois.edu/users/pavel/java/balance/

Choose Math Operation

Choose the mathematical operation(s) so that the number sentence is true. Practise the role of zero and one in basic operations or operations with negative numbers. Helps develop number sense and logical thinking.

http://www.homeschoolmath.net/operation-game.php

Order of Operations Quiz

A 10-question online quiz that includes two different operations and possibly parenthesis in each question. You can also modify the quiz parameters yourself. http://www.thatquiz.org/tq-1/?-j8f-la

The Order of Operations Millionaire

Answer multiple-choice questions that have to do with the order of operations, and win a million. Can be played alone or in two teams.

http://www.math-play.com/Order-of-Operations-Millionaire/order-of-operations-millionaire.html

Exploring Order of Operations (Object Interactive)

The program shows an expression, and you click on the correct operation (either +, --, \times , \div or exponent) to be done first. The program then solves that operation, and you click on the *next* operation to be performed, etc., until it is solved. Lastly the resource includes a game where you click on the falling blocks in the order that order of operations would dictate. http://www.learnalberta.ca/content/mejhm/html/object interactives/order of operations/use it.html

Order of Operations Practice

A simple online quiz of 10 questions. Uses brackets and the four operations. http://www.onlinemathlearning.com/order-of-operations-practice.html

Quick Calculate

Practise the arithmetic of all four operations plus order of operations. http://themathgames.com/arithmetic-games/addition-subtraction-multiplication-division/quick-calculate-game.php

Open Array Multiplication Tool

This interactive tool shows the partial products algorithm and an area model for multi-digit multiplication, allowing students to easily link the two. The model accommodates 2 digit x 2 digit problems and 1 digit x 1, 2, 3, or 4 digit problems. Requires a free registration. https://www.conceptuamath.com/app/tool/open-array-multiplication

Mental Math Tricks for Multiplication

Includes some very basic common-sense ones such as multiplying by 9 or multiplying by doubling and halving.

http://wildaboutmath.com/2007/11/11/impress-your-friends-with-mental-math-tricks

Mental Math Multiplication Guide

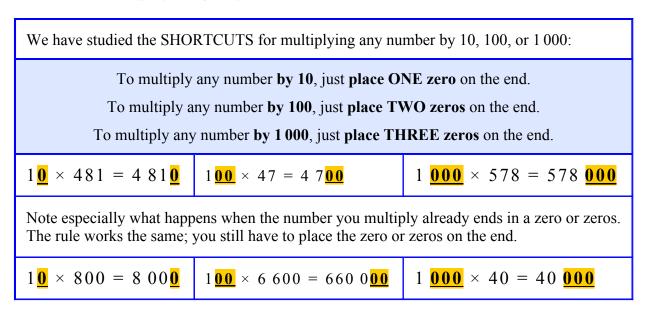
Rules of thumb and other "tricks" for mental multiplication of two-digit or bigger numbers, conveniently in one place. (This is not about single-digit multiplication; you are supposed to know those by heart of course.)

http://arscalcula.com/mental_math_multiplication_guide.shtml

Acing Math

A large collection of math games for grades K-6 that you can play with a standard deck of cards. http://www.pepnonprofit.org/uploads/2/7/7/2/2772238/acing math.pdf [This page is intentionally left blank.]

Multiplying by Whole Tens and Hundreds



1. Multiply.

a. $10 \times 315 =$	b. $100 \times 6200 =$	c. $1000 \times 250 =$
3 560 × 10 =	$10 \times 1200 =$	38 × 1 000 =
35 × 100 =	100 × 130 =	$10 \times 5000 =$

SHORTCUT for multiplying by 20 or 200 (You can probably guess this one!)		
<u>What is 20×14?</u>	<u>What is 200 × 31?</u>	
Imagine the problem without the zero. Then it becomes $2 \times 14 = 28$. Then, just place a zero on the end of the 28 you got, so it becomes 280. So, $20 \times 14 = 280$.	Imagine the problem without the zeros. Then it becomes $2 \times 31 = 62$. Then, just place <i>two</i> zeros on the end of the result you got, so you get 6 200. In other words, $200 \times 31 = 6200$.	

2. Now try it! Multiply by 20 and 200.

a.	b.	с.	d.
20 × 8 =	200 × 7 =	20 × 12 =	20 × 16 =
4 × 20 =	5 × 200 =	35 × 20 =	42 × 200 =
20 × 5 =	11 × 200 =	200 × 9 =	54 × 20 =

Why does the shortcut work? It is based on the fact that you can multiply in any order.When multiplying by 20, we can change
the 20 into
$$10 \times 2$$
. For example:
 $20 \times 14 = 10 \times 2 \times 14$ Let's try the same with 200.
For example,
 $200 \times 31 = 100 \times 2 \times 31$ In that problem, first multiply $2 \times 14 = 28$.
Then the problem becomes 10×28 , which
we know is 280.In that problem, first multiply $2 \times 31 = 62$.
The problem now becomes 100×62 , which
is 6 200: $20 \times 14 = 10 \times 2 \times 14$ $100 \times 2 \times 31$ $= 10 \times 28$ $100 \times 2 \times 31$ $= 10 \times 28$ $= 100 \times 62$ $= 280$ $= 6 200$

3. Try it yourself! Fill in.

a. 20×7	b. 20×5	c. 200 × 8	d. 200 × 25
=×2×7	=×2×5	=×2×8	=×2×25
= 10 ×	= 10 ×	= 100 ×	= 100 ×
=	=	=	=

4. Sean's house measures 20 m by 15 m. What is its area? Write a number sentence. A means area.

A =

- 5. Write a number sentence, and find the area
of Sean's garden.200 m15 m15 m
- 6. Sean was told he needed four truckloads of topsoil to cover his garden. One truckload costs 5 × \$200 plus \$50 for the delivery. How much will it cost him to cover the garden with topsoil?

SHORTCUT for multiplying by whole tens and whole hundreds

The same principle works if you multiply by whole tens (30, 40, 50, 60, 70, 80, or 90): simply multiply by 3, 4, 5, 6, 7, 8, or 9, and then place a zero on the end of the result.

Similarly, if you multiply by some whole hundred, FIRST multiply without those two zeros, and then place the two zeros on the end of the result.

$$50 \times 8 = 400$$
 $90 \times 11 = 990$ $300 \times 8 = 2400$ $12 \times 800 = 9600$

7. Multiply.

a. $40 \times 3 =$	b. $70 \times 6 =$	c. $80 \times 9 =$
8 × 20 =	50 × 11 =	30 × 15 =
d. $60 \times 11 =$	e. $200 \times 9 =$	f. $700 \times 6 =$
12 × 40 =	7 × 400 =	600 × 11 =
g. $200 \times 12 =$	h. $3 \times 1100 =$	i. 11 × 120 =
15 × 300 =	8 × 900 =	8 × 300 =

It even works this way:		
To multiply 40×70 , simply multiply 4×7 , and place two zeros on the end of the result:	To multiply 600×40 , simply multiply 6×4 , and place three zeros on the the end of the result:	To multiply 700×800 , simply multiply 7×8 , and place four zeros on the end of the result.
$40 \times 70 = 2800$	$600 \times 40 = 24000$	7 <mark>00</mark> × 8 <mark>00</mark> = 56 <mark>0 000</mark>

8. Multiply.

a. 20 × 90 =	b. $60 \times 80 =$	c. $400 \times 50 =$
70 × 300 =	30 × 900 =	200 × 200 =
d. $80 \times 800 =$	e. $100 \times 100 =$	f. $800 \times 300 =$
200 × 500 =	40 × 30 =	90 × 1 100 =

Write a number sentence for each question.

9. One hour has minutes.	
How many minutes are in 12 hours?	
How many minutes are in 24 hours?	
10. One hour has minutes, and one minute has seconds. How many seconds are there in one hour?	
11. Louis earns \$30 per hour.	
a. How much will he earn in a 8-hour workday?	
b. How much will he earn in a 40-hour workweek?	
c. How many days will he need to work in order to earn more than \$1 000?	

12. Find the missing factor. Think "backwards"! How many zeros do you need?

a. ×3=360	b. $40 \times ___= 320$	c. $\times 40 = 400$
× 50 = 450	5 ×= 600	× 2 = 180
d. \times 30 = 4800	e. $40 \times ___= 2000$	f. \times 800 = 56000
× 200 = 1 800	6 ×= 4 200	× 20 = 12 000

John wanted to prove that 40×70 is indeed 2 800 by breaking the multiplication into smaller parts. He wrote 40 as 4×10 and 70 as 7×10 , and then multiplied in a different order:

 $40 \times 70 = 4 \times 10 \times 7 \times 10$ = $10 \times 10 \times (4 \times 7) = 100 \times 28 = 2800$.

You do the same, and prove that 600×50 is indeed 30 000.

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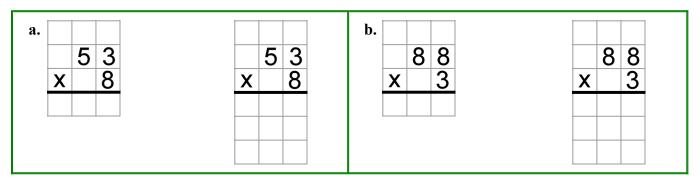
Multiplying in Columns, the Standard Way

The standard algorithm of multiplication is based on the principle that you already know: **multiplying in parts** (partial products): simply multiply ones and tens separately, and add.

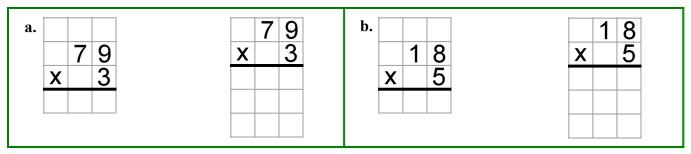
However, in the standard way, the *adding* is done at the same time as multiplying. The calculation looks more compact and takes less space than the "easy way to multiply" you have learned.

The standard way to multiply		"The easy way"
$\begin{bmatrix} 1 \\ 6 \\ 3 \\ \times \\ 4 \\ 2 \end{bmatrix}$ Multiply the ones: $4 \times 3 = 12$ Place 2 in the ones place, but write the tens digit (1) above the tens column as a little memory note. You are <i>regrouping</i> (or carrying).	$\begin{bmatrix} 1 \\ 6 \\ 3 \\ \times 4 \\ \hline 2 \\ 5 \\ 2 \end{bmatrix}$ Then multiply the tens, <i>adding</i> the 1 ten that was regrouped. $4 \times 6 + 1 = 25$ Write 25 in front of the 2. <u>Note</u> that 25 tens means 250!	$ \frac{\begin{array}{c} 6 & 3 \\ \times & 4 \\ \hline 1 & 2 \\ + & 2 & 4 & 0 \\ \hline 2 & 5 & 2 \end{array} $ In the "easy way," we multiply in parts, and the adding is done separately.
The standard v	The standard way to multiply	
$3 7 5 \times 7 5 = 35$ Multiply the ones: $7 \times 5 = 35$ Regroup the 3 tens.	$3 7 5$ $7 7$ $5 2 5$ Multiply and add the tens: $7 \times 7 + 3 = 52$	$ \begin{array}{r} 7 5 \\ \times 7 \\ \overline{)} \\ + 4 9 0 \\ \overline{)} \\ 5 2 5 \end{array} $

1. Multiply using both methods: the standard one and the easy one.



2. Multiply using both methods: the standard one and the easy one.

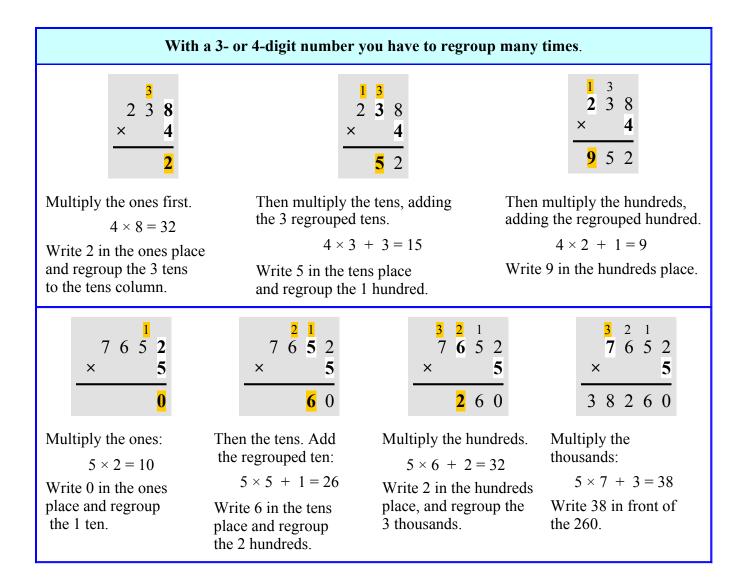


3. Multiply. Be careful with the regrouping.

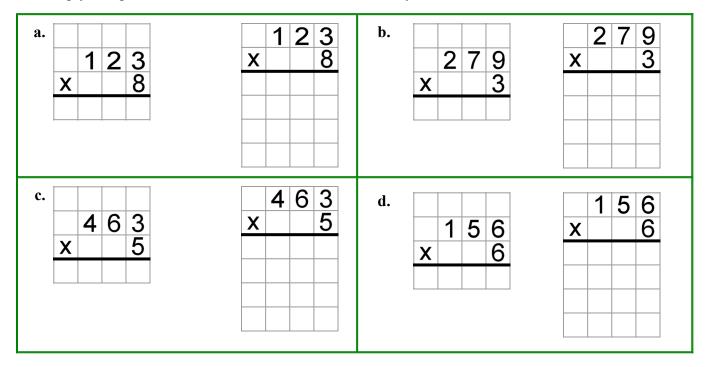
a. 51	b. 19	c. 62	d. 46
X6	X 3	X2	x7
e. 66	f. 39	g. 87	h. 67
X6	X9	X3	x 2
i. 20 x 9	j. 54 x 8	k. 34 34 X6	I. 46 x2

4. Solve. Also, write number sentences (additions, subtractions, multiplications) on the empty lines.

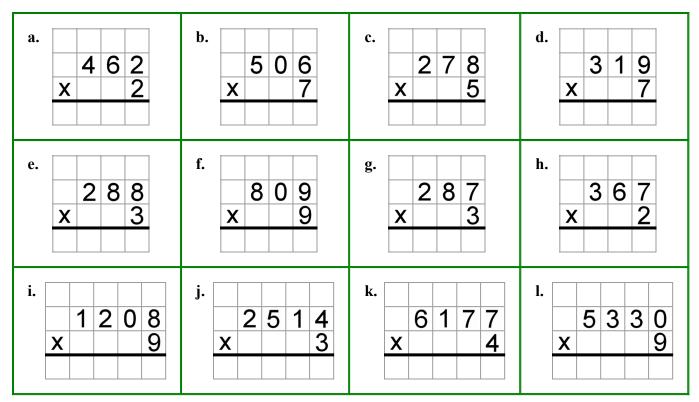
a. What is the cost of buying three dolls for \$48 each?	
And the cost for six dolls?	X
b. You earn \$77 a day. How many days do you need to work in order to have \$600 or more? Guess and check.	
	X



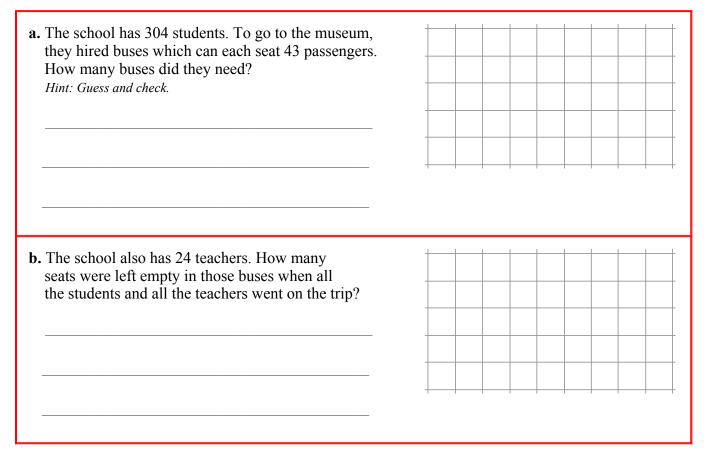
5. Multiply using both methods: the standard one and the easy one.



6. Multiply using the standard method.



7. Solve the word problems. Also, write number sentences (additions, subtractions, multiplications) on the empty lines to show what you calculate.



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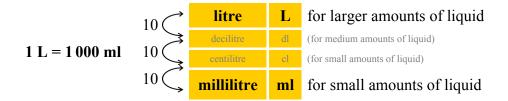
Chapter 4: Time and Measuring Introduction

The fourth chapter of *Math Mammoth Grade 4* includes lessons on time, temperature, length, weight and volume. The focus is no longer the actual act of measuring, but on conversions between the units and on word problems that involve conversions.

Students may have difficulty with the conversions, and that is why they will also be studied in 5th grade. At this point, students should be able to easily convert from a bigger unit to a smaller unit (such as converting 3 m into 300 centimetres, or 2 kg into 2 000 grams).

And while the Common Core standards of the United States do not include them for 4th grade, I have also included some problems where we convert from a smaller unit to a bigger unit (such as 4 500 ml into 4 L 500 ml or 4 000 millimetres into 4 metres), because I feel most students are capable of doing these in 4th grade.

The lessons include tables that list the units and the conversion factors. Those tables always include all the units, even when they are not in common usage. For example, for metric units of volume, the chart looks like this:



The lesson only deals with millilitres and litres. However, the chart *also* shows the two other units (decilitres and centilitres) in order to help familiarise the students with these two basic ideas of the metric system:

- 1. The units always differ by a factor of ten;
- 2. The units are *named* consistently with the same prefixes (milli-, centi-, deci-, deka-, hecto-, and kilo-). These prefixes and their meanings are not yet studied in detail in fourth grade. You may, of course, at your discretion, explain them to the student.

The Lessons in Chapter 4

	page	span
Time Units	150	3 pages
The 24-Hour Clock	153	2 pages
Elapsed Time or How Much Time Passes	155	5 pages
Measuring Temperature: Celsius	160	4 pages
Temperature Line Graphs	164	2 pages
Measuring Length	166	2 pages
More Measuring in Centimetres	168	2 pages
Metric Units for Measuring Length	170	3 pages
Metric Units of Weight	173	3 pages
Metric Units of Volume	176	3 pages
Mixed Revision, Chapters 1 - 4	179	2 pages
Revision, Chapter 4	181	2 pages

Helpful Resources on the Internet

Use these free online resources to supplement the "bookwork" as you see fit.

Disclaimer: These links were valid at the time of writing this book, and to the best of our knowledge we believe these websites to have what is described. However, we cannot guarantee that the links have not changed. Parental supervision is needed.

Measure It!

Practise measuring lines with either centimetres or inches. Multiple choice questions. http://www.funbrain.com/measure

Sal's Sub Shop

Customers order subs, and you need to cut them to the given measurements - sometimes in metric units, sometimes in inches.

http://www.mrnussbaum.com/sal.htm

Measurement Game for Kids

Measure the length and weight of various parcels using the interactive scales and ruler so you can give them a stamp with the correct postage rate. Uses grams and centimetres.

http://www.kidsmathgamesonline.com/geometry/measurement.html

Reading Scales

You can illustrate a variety of measuring devices, such as scales, measuring cup, thermometer, and speedometer, and how to read them. Generate examples using different scales on different devices at the press of a button.

http://www.teacherled.com/2008/01/28/reading-scales

Reading Scales

Weigh objects on this virtual balance scales, using weights of 10 g, 50 g, 250 g and 500 g. http://www.teacherled.com/resources/oldscales/oldscalesload.html

Measures

An online activity about metric measuring units and how to read scales, a measuring cup and a ruler. http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/maths/measures

Hours Versus Minutes Game BBC SkillsWise

An online quiz to practise minutes versus hours. For example, you have to tell whether 76 minutes or 1 hour is more.

http://www.bbc.co.uk/skillswise/game/ma25time-game-hours-vs-minutes

24-Hour Snap Game

Two times are given, one using the 24-hour clock, and another using the am/pm system. Snap or do not snap the two times together.

http://www.bbc.co.uk/skillswise/game/ma25time-game-24-hour-snap

A Dictionary of Units of Measurement

Explains the common measuring systems and has lots of background information on their history. http://www.unc.edu/~rowlett/units/

Bitesize Measures

Facts, problems and quizzes about measuring length, mass, and capacity (in metric units). http://www.bbc.co.uk/bitesize/ks2/maths/shape_space/measures/read/1/

Measurements

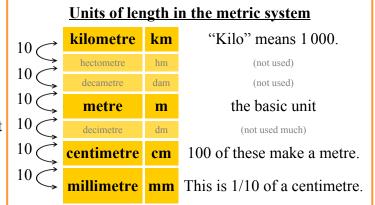
Online lessons with interactive exercises on metric prefixes, symbols, number values, metric mass, length, volume, US length and volume and temperature conversions. http://www.aaamath.com/B/mea.htm

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Metric Units for Measuring Length

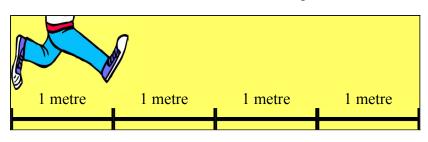
The **basic unit** for measuring length in the metric system is <u>the metre</u>. All the other units for measuring length have the word "metre" in them.

Each unit is 10 times the smaller unit. For example, 1 kilometre is 10 hectometres. But we do not commonly use hectometres, decametres, or decimetres. You only need to learn the bolded units in the chart.



1. Outside, or in a long corridor or room, draw two lines that start at the same place.

Using a measuring tape, mark on the one line 1 m, 2 m, 3 m and 4 m. Can you take "hops" 1 metre long?





2. Measure how tall you and other people are in centimetres. Write it also using whole metres and centimetres.

Name	How tall
	$___ cm = _\underline{1} m ___ cm.$

Conversions between units

Remember what millimetres look like on your ruler. 10 millimetres make 1 cm.

And 100 centimetres is 1 metre. "Centi" actually means a hundred (from the Latin word *centum*).

Lastly, 1 kilometre means one thousand metres, because "kilo" means 1 000!

1 km = 1 000 m	1 m = 100 cm	1 cm = 10 mm

3. One metre is 100 cm. Convert between metres and centimetres.

a. 5 m = cm	b. 4 m 6 cm = cm	c. 800 cm = m
8 m = cm	$9 \text{ m } 19 \text{ cm} = ___ \text{ cm}$	$239 \text{ cm} = _\m \text{ m} _\m \text{ cm}$
12 m = cm	$10 \text{ m} 80 \text{ cm} = ___ \text{ cm}$	$407 \text{ cm} = _\m \text{m} _\m \text{cm}$

4. One centimetre is 10 mm. Convert between centimetres and millimetres.

a. 5 cm = mm	b. 2 cm 8 mm = mm	c. 50 mm = cm mm
8 cm = mm	7 cm 5 mm = mm	72 mm = cm mm
14 cm = mm	$10 \text{ cm } 4 \text{ mm} = ___ \text{mm}$	145 mm = $\ cm \ mm$

5. One kilometre is 1 000 m. Convert between kilometres and metres.

a.	b.	с.
5 km = m	2 km 800 m = m	2 000 m = km
23 km = m	6 km 50 m = m	4 300 m = km m
1 km 200 m = m	13 km 579 m = m	$18700 \text{ m} = \k \text{m} \m \text{m}$

6. Calculate. Give your answer using whole kilometres and metres.

a. 5 km 200 m + 8 km 900 m

b. 3 km 600 m + 2 km 800 m

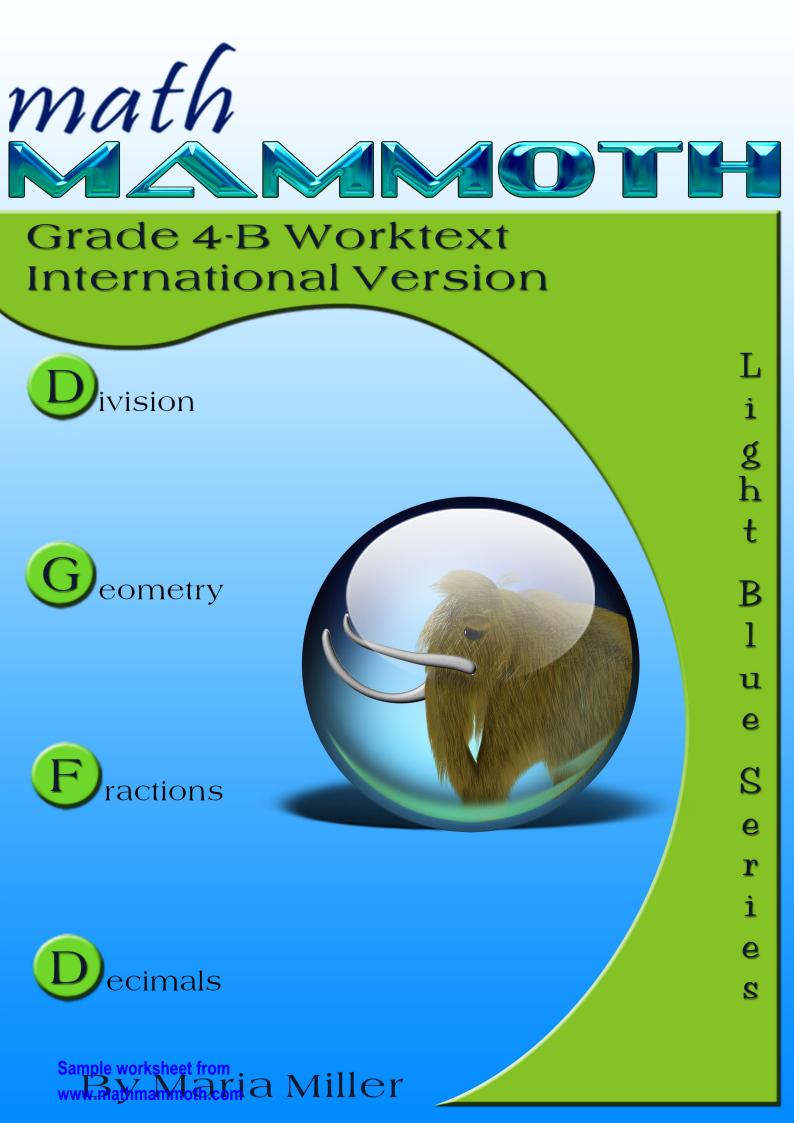
c. 1500 m + 2 km 600 m

 $\textbf{d.}~6\times700~m$

7. Solve.

a. Find the perimeter of this rectangle.		2 m
	80 cm	
b. Find the perimeter of this rectangle.		7 mm
	1 cm 5	5 mm
c. One side of a square measures 5 cm 6 mm. What is its perimeter?		
d. <i>A challenge</i> . A square has a perimeter of 6 cm. How long is its side?		
8. Solve the problems.		
a. How many millimetres are in a <i>metre</i> ?		
b John jogg around a track that is 1 km 200 m		
b. John jogs around a track that is 1 km 800 m long twice a day, five days a week.		
How long a distance does he jog in a day?		
In a week?		
c. George is 1 m 34 cm tall and Jeff is 142 cm tall.		
How much taller is Jeff?		
d. Betty's wallpaper has butterflies that are 8 cm wide. She will		
put the wallpaper in her room. How many complete butterflies	30	3 c - 3 c -
can she have on a wall that is 1 metre long?		
How about if the wall is 3 metres long?	and a	When when
Ű	6 6	**

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Foreword

Math Mammoth International Version Grade 4-A and *Grade 4-B* worktexts comprise a complete maths curriculum for the fourth grade mathematics studies.

This curriculum is essentially the same as the *Math Mammoth Grade 4* sold in the United States (US version), only customised for international use. The US version is aligned to the "Common Core" Standards, so it may not be properly aligned to the fourth grade standards in your country. However, you can probably find material for any missing topics in the neighbouring grades of Math Mammoth.

The International version of Math Mammoth differs from the US version in these aspects:

- The curriculum teaches metric measurement units, not customary (imperial) units.
- The spelling conforms to British international standards.
- The format is adapted for printing on the setting of A4.

In 4th grade, students focus on multi-digit multiplication and division, learning to use bigger numbers, solving multi-step word problems that involve several operations, and they get started in studying fractions and decimals. This is accompanied by studies in geometry and measuring.

The year starts out with a revision of addition and subtraction, patterns and graphs. We illustrate word problems with bar diagrams and study finding missing addends, which teaches algebraic thinking. Children also learn addition and subtraction terminology, the order of operations and statistical graphs.

Next come large numbers—up to millions, and the concept of place value. At first the student revises thousands and some mental maths with them. Next are presented numbers up to one million, calculations with them, the concept of place value and comparing. In the end of the chapter we find out more about millions and an introduction to multiples of 10, 100 and 1 000.

The third chapter is all about multiplication. After briefly revising the concept and the times tables, the focus is on learning multi-digit multiplication (multiplication algorithm). The children also learn why it works when they multiply in parts. We also study the order of operations again, touch on proportional reasoning and do more money and change related word problems.

In part B, we first study division. The focus is on learning long division and using division in word problems. In geometry, we first revise area and perimeter, and then concentrate on the topic of angles. Students measure and draw angles, solve simple angle problems and classify triangles according to their angles. They also study parallel and perpendicular lines.

Fractions and decimals are presented last in the school year. These two chapters practise only some of the basic operations with fractions and decimals. The focus is still on conceptual understanding and on building a good foundation towards 5th grade maths, where fractions and decimals will be in focus.

When you use these books as your only or main mathematics curriculum, they can be like a "framework", but you do have some liberty in organising the study schedule. Chapters 1, 2 and 3 should be studied in that order, but you can be flexible with chapters 4 (Time and Measuring) and 6 (Geometry), and schedule them somewhat earlier or later if you wish. Chapter 3 (Multiplication) needs to be studied before long division in Chapter 5. Many topics from chapters 7 and 8 (Fractions and Decimals) can also be studied earlier in the school year. However, finding parts with division should be studied only after mastering division.

I wish you success in teaching maths!

Maria Miller, the author

Chapter 5: Division Introduction

The fifth chapter of *Math Mammoth Grade 4* includes lessons on division, long division, remainder, part problems, average and problem solving. It is a long chapter, as division and long division are "in focus" in fourth grade.

We start out by revising basic division by single-digit numbers. Then students study division terms and dividing by whole tens and hundreds.

The lesson *Finding Fractional Parts with Division* shows an important relationship between fractions and division. For example, we can find 3/4 of a number by first finding 1/4 (divide by 4), then multiplying that result by 3.

Next we briefly study order of operations again, this time including divisions in the problems.

In the lesson *The Remainder, Part 1*, we study the concept of remainder, first using pictures and small numbers. In the second lesson on remainder, we still use small numbers, but students work the problems using the long division symbol or "corner", as I like to call it. That is of course preparing them for long division.

Next, long division is taught in several small steps over many lessons. We start with the situation where each of the thousands, hundreds, tens and ones can be divided evenly by the divisor. Then, the remainder in the ones is introduced. Next comes the situation where we have a remainder in the tens. Finally, when we have a remainder in the hundreds, and so on. We also have lots of word problems to solve.

After long division is mastered, we study the concept of average and problem solving involving a fractional part of a whole. I have included many bar diagrams and pictorial representations of these problems to help the students.

The last section deals with elementary number theory topics. We study some basic divisibility rules (though not all of them), prime numbers and find all factors of a given two-digit number.

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The Lessons in Chapter 5

	page	span
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Finding Fractional Parts with Division	17	3 pages
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Long division

MathFrog Dividerama!

Interactive long division practice. Guided help available optionally. http://cemc2.math.uwaterloo.ca/mathfrog/english/kidz/div5.shtml

Snork's Long Division Game

Interactive and guided long division practice that only accepts correct answers and truly guides the student step-by-step through long division problems. http://www.kidsnumbers.com/long-division.php

Mr. Martini's Classroom: Long Division

An interactive long division tool. http://www.thegreatmartinicompany.com/longarithmetic/longdivision.html

Double-Division.org

Another form of long division algorithm - takes the guesswork away from estimating how many times the divisor goes into what needs to be divided. Also called 1-2-4-8 division. http://www.doubledivision.org/

Short Division

A page that explains short division in detail. Short division is the same algorithm as long division, but some steps are only done in one's head, not written down.

http://www.themathpage.com/ARITH/divide-whole-numbers.htm

Factors and primes

Arrays and Factors

Drag rectangles to show the factorisations of a given number on a grid. http://www.shodor.org/interactivate/activities/FactorizeTwo/

Factor Game

Choose a number from the game board, and your opponent gets all the numbers that are its proper factors. If a player chooses a number with no proper factors remaining, that player loses a turn. You can adjust the number of rows and columns on the game board to get a more challenging (and interesting) game. This game can easily be adapted to be played offline, with paper and coloured pencils. http://illuminations.nctm.org/Activity.aspx?id=4134

Factor Feeder

"Eat" factors of the given number while avoiding numbers that are not its factors in this Pac man-style game. Use arrow keys to move.

http://www.hoodamath.com/games/factorfeeder.html

Sliding Tile Factorization Game

Slide a number over another to capture it, if it is a factor of the other. Number 1 is only supposed to be used to capture a prime number.

http://www.visualmathlearning.com/Games/sliding_factors.html

Octopus Factors

Move counters up the legs of an octopus but only when the number on the circle is a multiple of the number on the card.

http://www.counton.org/games/map-numbers/octopus/

Factors Millionaire Game

A millionaire game where the questions have to do with factors, prime numbers and the greatest common factor.

http://www.math-play.com/Factors-Millionaire/Factors-Millionaire.html

Not a Factor

Choose a number that is NOT a factor of the given number. http://www.helpingwithmath.com/resources/games/target_factors01/not_factor.html

Snake

Eat factors, multiples and prime numbers in this remake of the classic game. http://www.arcadediner.com/Snake

Product game

The players choose factors and the product of those gets coloured on the game board. The player who gets four products in a row wins. You can play against the computer or with a friend. This game can easily be adapted to be played offline, with paper and coloured pencils. http://illuminations.nctm.org/Activity.aspx?id=4213

Primes, Factors and Divisibility—Explorer at CountOn.org

Lessons explaining divisibility tests, primes and factors. http://www.counton.org/explorer/primes

If the student has not yet mastered the basic division facts, the following games can be used for practice.

A+ math games

Practise all four basic operations with maths bingo (matho), hidden picture games, or concentration games.

http://www.aplusmath.com/games/

Math Magician games

Flashcard problems in all 4 operations. Answer 20 questions in 1 minute. http://www.oswego.org/ocsd-web/games/Mathmagician/cathymath.html

Division Practice at AAAMath

Learn or practise basic division facts, and more. http://www.aaastudy.com/div39hx3.htm

Cross the Swamp

Help Little Ron move from log to log across the swamp and practise multiplication/division or addition/subtraction.

http://www.bbc.co.uk/schools/starship/maths/crosstheswamp.shtml

Math Car Racing

Keep ahead of the computer car by thinking logically, and practise any of the four operations. http://www.funbrain.com/osa/index.html

Arithmetic Game

Find numbers to fit an equation that may use all four operations. http://www.primarygames.com/math/arithmeticgame/index.htm

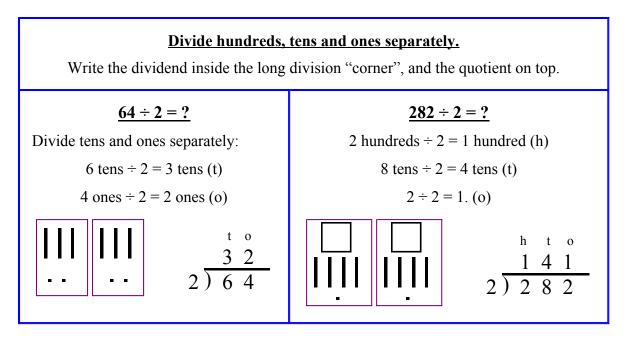
Primary Games

A collection of games. The following links open the evaluation versions of some division-related games. The game collections themselves are sold at http://www.primarygames.co.uk/

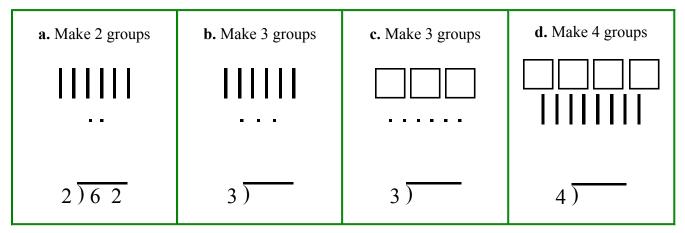
- Eggs on Legs http://www.primarygames.co.uk/PG5/Eggs/Div/eggsdiv.html
- DiviPods http://www.primarygames.co.uk/pg4/Divipods/divipods.html
- Division Divers http://www.primarygames.co.uk/pg3/ddivers/ddivers.html
- Sum Sense Division http://www.primarygames.co.uk/pg2/sumsense/sumdiv.html

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Long Division 1



1. Make groups. Divide. Write the dividend inside the "corner" if it is missing.



2. Divide thousands, hundreds, tens and ones separately.

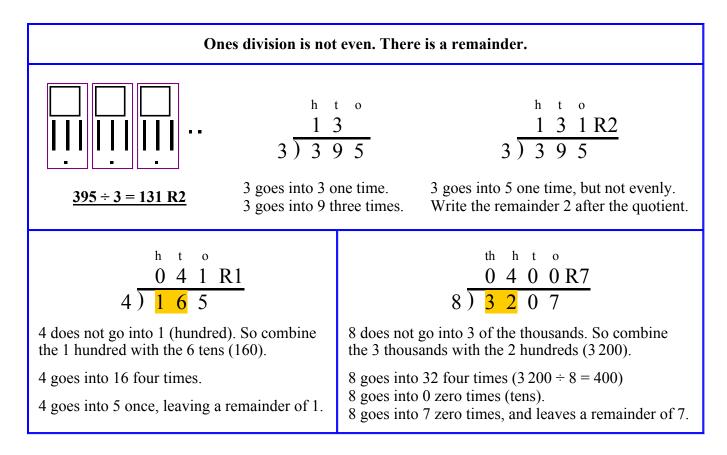
a.
$$4)\overline{84}$$
 b. $3)\overline{393}$
 c. $3)\overline{660}$
 d. $4)\overline{8040}$

 e. $3)\overline{66}$
 f. $6)\overline{6036}$
 g. $3)\overline{330}$
 h. $4)\overline{4804}$

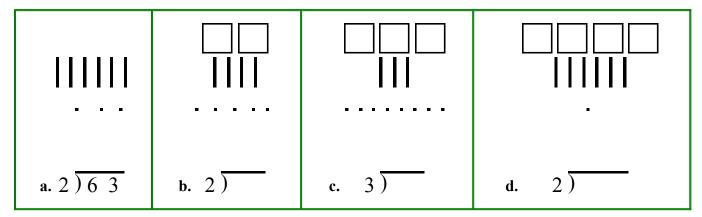
$\begin{array}{r} h \ t \ o \\ 0 \\ 4 \end{array}$	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$				
4 does not go into 2. You ca quotient in the hundreds play 4 does go into 24, six times.	ce or omit it. But	5 does not go into 3. You can put zero in the quotient. But 5 does go into 35, seven times.				
Explan	ation:	Explanation:				
The 2 of 248 is of course 20 200 by 4, the result would b why the quotient will not ha	e less than 100, so that is	$3000 \div 5$ will not give any whole thousands to the quotient because the answer is less than 1 000.				
But then you combine the 2 That makes 24 tens, and you by 4. The result 6 tens goes Check the final answer: 4 ×	CAN divide 24 tens as part of the quotient.	But 3 thousands and 5 hundreds make 35 hundreds together. You can divide $3500 \div 5 = 700$, and place 7 as part of the quotient in the hundreds place.				
		Check the final answer: $5 \times 701 = 3505$.				
If the divisor does not "go into" the first digit of the dividend, look at the <u>first two digits</u> of the dividend.						

3. Divide. Check your answer by multiplying the quotient and the divisor.

a.	3)123	b.	4)284
c.	6)360	d.	8)248
e.	2)184	f.	7)427
g.	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	h.	4) <mark>24</mark> 04
i.	7)4970	j.	5)4505



4. Divide into groups. Find the remainder.



5. Divide. Indicate the remainder if any.

a.
$$4\overline{)847}$$
 b. $2\overline{)69}$
 c. $3\overline{)367}$
 d. $4\overline{)89}$

 e. $2\overline{)121}$
 f. $6\overline{)1805}$
 g. $7\overline{)215}$
 h. $8\overline{)2482}$

In the problems before, you just wrote down the remainder of the ones. Usually, we write down the subtraction that actually finds the remainder. Look carefully:

$$\begin{array}{c} h & t & 0 \\ \hline 0 & 6 & 1 \\ \hline 4 & 2 & 4 & 7 \\ \hline -2 & -2 \\ \hline \end{array}$$

When dividing the ones, 4 goes into 7 one time.

Multiply $1 \times 4 = 4$, write that four under the 7,

and subtract. This finds us the remainder of 3.

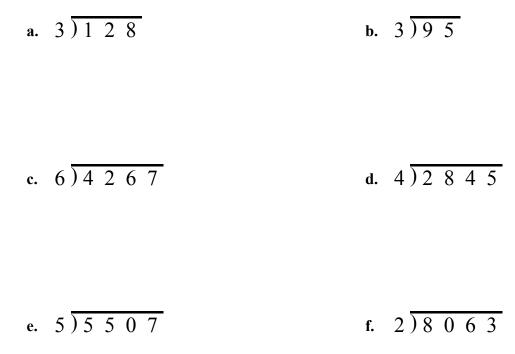
Check: $4 \times 61 + 3 = 247$

$$\begin{array}{r} \text{th h t o} \\ 0 \ 4 \ 0 \ 2 \\ 4 \ 1 \ 6 \ 0 \ 9 \\ \hline - 8 \\ 1 \end{array}$$

When dividing the ones, 4 goes into 9 two times. Multiply $2 \times 4 = 8$, write that eight under the 9, and subtract. This finds us the remainder of 1.

Check: $4 \times 402 + 1 = 1609$

6. Practise some more. Subtract to find the remainder in the ones. Check your answer by multiplying the divisor times the quotient, and then adding the remainder. You should get the dividend.



7. Divide these numbers mentally. Remember, you can always check divisions by multiplying!

a. $440 \div 4 =$	b. $3600 \div 400 =$	c. $824 \div 2 =$
820 ÷ 2 =	369 ÷ 3 =	560 ÷ 90 =

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Divisibility

A number <i>a</i> is <i>divisible</i> by another number <i>b</i> if the division $a \div b$ is exact (no remainder).					
For example, $18 \div 3 = 6$. So, <u>18 is divisible by 3</u> . Also, <u>18 is divisible by 6</u> , because we can write the other division $18 \div 6 = 3$. So, 18 is divisible by both 6 and 3.					
We say 6 and 3 are <i>divisors</i> or <i>factors</i> of 18.					
You can use long division to check if a number is divisible by another. $67 \div 4 = 16$, R3. There is a remainder, so 67 is <u>not</u> divisible by 4. Also, from this we learn that neither 4 nor 16 is a factor (divisor) of 67.	$ \begin{array}{r} 1 & 6 \\ 4 & 6 & 7 \\ - & 4 \\ 2 & 7 \\ - & 2 & 4 \\ \hline 3 \\ \end{array} $				

1. Divide and determine if the numbers are divisible by the given number.

a. $21 \div 3 =$	b. 40 ÷ 6 =	c. 17 ÷ 5	d. 84 ÷ 7 =
Is 21 divisible by 3?	Is 40 divisible by 6?	= Is 5 a divisor of	Is 7 a factor of 84?
		17?	

2. Answer the questions. You may need long division.

a. Is 9	a. Is 98 divisible by 4?								b. Is 603 divisible by 7?				c. Is	s 3 a	fact	tor c	of 11	256	?		
_						,															
_																					
													ļ								

In any multiplication, the numbers that are multiplied are called <i>factors</i> and the result is called a <i>product</i> .	factor		factor		product 42	
So, since $6 \times 7 = 42$, 6 and 7 are <i>factors</i> of 42.	,		U		.2	
From this multiplication fact we can write two divisions: 42 So, this also means that 42 is divisible by both 6 and 7.	÷6=	7	and	42	$\div 7 = 6.$	
Yet one more new word that ties in with all of this: <i>multiple</i> .						
We say 42 is a multiple of 6, because 42 is some number tim	es 6, n	am	ely 7	× 6		
And of course 42 is also a multiple of 7, because it is some number times 7!						

3. Fill in.

Here is a multiplication fact: $8 \times 9 = 72$. So, 8 is a	of 72, and so is 9.
---	---------------------

Also, 72 is a ______ of 8, and also 72 is a ______ of 9.

And, 72 is _____ by 8 and also by 9.

4. Fill in.

a. Is 5 a factor of 55?	b. Is 8 a divisor of 45?
Yes, because $_\ \times _\ = _\$.	No, because ÷ =
c. Is 36 a multiple of 6?	d. Is 34 a multiple of 7?
, because × =	, because÷=
e. Is 7 a factor of 46?	f. Is 63 a multiple of 9?
, because	, because

<u>Multiples of 6</u> are all the numbers we get when we multiply 6 by other numbers. For example, if we multiply 0×6 , 7×6 , 11×6 , 109×6 , and so on, the resulting numbers are all multiples of six.

In fact, the skip-counting pattern of 6 gives us a list of multiples of 6:

0, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84 and so on.

5. **a.** Make a list of multiples of 11, starting at 0 and at least until 154.

b. Make a list of multiples of 111, starting at 0. Make it as long as you can in this space!

Divisibility by 2

Numbers that are divisible by 2 are called *even* numbers. Numbers that are NOT divisible by 2 are called *odd* numbers.

Even numbers end in 0, 2, 4, 6, or 8. Every second number is even.

Divisibility by 5

Numbers that end in 0 and 5 are divisible by 5.

For example, 10, 35, 720 and 3 675 are such numbers.

6. Mark with "x" if the numbers are divisible by 2 or 5.

number	divisible		number	divisible		number	divisible		numbor	divisible	
number	by 2	by 5									
750			755			760			765		
751			756			761			766		
752			757			762			767		
753			758			763			768		
754			759			764			769		

Divisibility by 10

Numbers that end in 0 are divisible by 10.

For example, 10, 60, 340 and 2 570 are such numbers.

7. Mark an "x" if the numbers are divisible by 2, 5, or 10.

number	divisible			number	divisible			number	divisible		
number	by 2	by 5	by 10	number	by 2	by 5	by 10	number	by 2	by 5	by 10
860				865				870			
861				866				871			
862				867				872			
863				868				873			
864				869				874			

If a number is divisible by 10, it ends in zero, so it is ALSO divisible by _____ and _

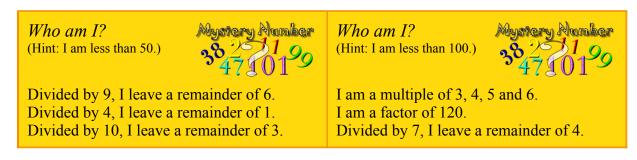
8. a	. Write a	list of numbers	divisible by 2,	from 0 to 60.
------	-----------	-----------------	-----------------	---------------

	This is also a list of		of 2.
--	------------------------	--	-------

- **b.** In the list above, *underline* those numbers that are divisible by 4. What do you notice?
- **c.** In the list above, *colour* those numbers that are divisible by 6. What do you notice?
- d. Which numbers are divisible by both 4 and 6?
- 9. **a.** Write a list of numbers divisible by 3, from 0 to 60.

This is also a list of ______ of 3.

- **b.** In the list above, *underline* those numbers that are divisible by 6. What do you notice?
- **c.** In the list above, *colour* those numbers that are divisible by 9. What do you notice?
- 10. Use the lists you made in (7) and (8). Find numbers that are divisible by *both* 2 and 9.
- 11. What number is a factor of every number?
- 12. Twenty is a multiple of 4. It is also a multiple of 5. It is also a multiple of four other numbers. Which ones?



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Chapter 6: Geometry Introduction

We start fourth grade geometry by revising the concepts of area and the perimeter of rectangles (from third grade). Students get to apply these concepts in problem solving, including problems where they write simple equations and explore possible perimeters for a given fixed area.

The focus of this chapter is angles. Children learn about lines, rays and angles, and about acute, right, obtuse and straight angles. Next they learn how to measure and draw angles with a protractor. We also study angle problems where students write simple equations, and estimate some common angles.

The lesson *Parallel and Perpendicular Lines* ties in with the topic of angles, because perpendicular means to be at a right angle. Next we study parallelograms and other quadrilaterals in more detail, paying attention to the angles and side lengths in them.

We study triangles, and classify them according to the angles. Classifying triangles according to their sides (equilateral versus isosceles triangles) is left for the 5th grade. The last topic for this chapter (an easy one) is line symmetry.

The study of geometry is full of strange-sounding words to learn. I encourage you to let the student(s) keep a *geometry notebook*, where they will write every new concept or term, and draw a picture or pictures and text to explain the term. The students could also do the drawing exercises from this chapter in this notebook. It will then become their very own geometry book, and while working with it, it helps them to learn and remember the terms and concepts better.

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The Lessons in Chapter 6

page	span
79	5 pages
84	4 pages
88	5 pages
93	5 pages
98	2 pages
100	5 pages
105	5 pages
110	5 pages
115	3 pages
118	4 pages
122	3 pages
125	2 pages
127	4 pages
	79 84 88 93 98 100 105 110 115 118 122 125

Helpful Resources on the Internet

Use these free online resources to supplement the "bookwork" as you see fit. **Disclaimer:** These links were valid at the time of writing this book, and to the best of our knowledge we believe these websites to have what is described. However, we cannot guarantee that the links have not changed. Parental supervision is needed.

Area and perimeter

Free Worksheets for Area and Perimeter

Create worksheets for the area and the perimeter of rectangles/squares with images, word problems, or problems where the student writes an expression for the area using the distributive property. Options also include area and perimeter problems for irregular rectangular areas and more.

http://www.homeschoolmath.net/worksheets/area_perimeter_rectangles.php

Shape Explorer

Find the perimeter and area of odd shapes on a rectangular grid. http://www.shodor.org/interactivate/activities/ShapeExplorer/

Math Playground: Measuring the Area and Perimeter of Rectangles

Amy and her brother, Ben, explain how to find the area and perimeter of rectangles and show you how changing the perimeter of a rectangle affects its area. After the lesson, you will use an interactive ruler to measure the length and width of 10 rectangles, and to calculate the perimeter and area of each. http://www.mathplayground.com/area perimeter.html

Math Playground: Party Designer

You need to design areas for the party, such as crafts table, food table, seesaw and so on, so that they have the given perimeters and areas. http://www.mathplayground.com/PartyDesigner/PartyDesigner.html

nttp://www.matnpiayground.com/r artyDesigner/r artyDesigner

Geometry Area/Perimeter Quiz from ThatQuiz.org

An online quiz, about the area and perimeter of rectangles, triangles and trapeziums. You can modify the quiz parameters to your liking, for example to omit a certain shape, or instead of solving for perimeter/area, you solve for an unknown side when perimeter/area is given. http://www.thatquiz.org/tq-4/?-j1200b-lc-p0

Area vs. Perimeter Quiz

Do you sometimes mix up area and perimeter? Take this 10-question online quiz and practice NOT mixing up the two concepts. Includes some challenge questions. http://www.mrmaisonet.com/index.php?/Area-Quizzes/Area-vs-Perimeter.html

Area and Perimeter of Rectangles

A 10-question quiz with varying questions concerning area and perimeter of rectangles. https://www.ck12.org/assessment/ui/views/test.view.new.html?practice/Area-and-Perimeter-of-Rectangles-Practice?type=practice

FunBrain: Shape Surveyor Geometry Game

A simple and easy game that practises finding either the perimeter or area of rectangles. http://www.funbrain.com/poly/index.html

Angles

Turtle Pond

Guide a turtle to a pond using commands, which include turning him in certain angles, or moving him a specific distance.

http://illuminations.nctm.org/Activity.aspx?id=3534

Banana hunt at Primary Games

Help the monkey find bananas and learn to estimate angles. http://www.primarygames.co.uk/pg2/bhunt/bhunt.html

Ladybug Leaf

Guide the ladybug by giving her commands to turn 90° or 45°, right or left, or to move forward/backward. http://nlvm.usu.edu/en/nav/frames asid 287 g 2 t 3.html

LadyBug Mazes

Similar to the Ladybug Leaf, but this time you guide the ladybug through the maze. http://nlvm.usu.edu/en/nav/frames_asid_141_g_2_t_3.html

Shapes/Polygons

Interactive Quadrilaterals

See all the different kinds of quadrilaterals "in action". You can drag the corners, see how the angles change and observe what properties do not change. http://www.mathsisfun.com/geometry/quadrilaterals-interactive.html

Polygon Matching Game

Learn all the common polygons by playing this fun, timed matching game. http://www.mathplayground.com/matching_shapes.html

Polygon Vocabulary

A matching game. http://www.quia.com/cc/2758.html

Shapes Identification Quiz from ThatQuiz.org

An online quiz in a multiple-choice format, asking to identify common two-dimensional shapes. You can modify the quiz parameters to your liking. http://www.thatquiz.org/tq-f/?-jlofy-l1-p0

General

Interactivate! Tessellate

An online, interactive tool for creating your own tessellations. Choose a shape, then edit its corners or edges. The program automatically changes the shape so that it will tessellate (tile) the plane. Then push the tessellate button to see your creation!

http://www.shodor.org/interactivate/activities/Tessellate

Patch Tool

An online activity where the student designs a pattern using geometric shapes. http://illuminations.nctm.org/Activity.aspx?id=3577

Polygon Playground

Drag various colourful polygons to the work area to make your own creations! http://www.mathcats.com/explore/polygons.html

Interactive Tangram Puzzle

Place the tangram pieces so they form the given shape. http://nlvm.usu.edu/en/nav/frames_asid_112_g_2_t_1.html

Logic Tangram game

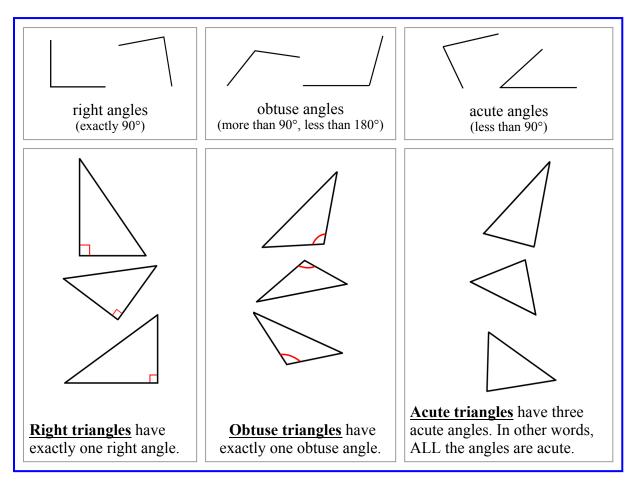
Note: this uses four pieces only. Use logic and spatial reasoning skills to assemble the four pieces into the given shape.

http://www.mathplayground.com/tangrams.html

Geometry Worksheets and Quizzes

Worksheets about complementary and supplementary angles, parallel, perpendicular and intersecting lines, types of angles, basic shapes, area and perimeter of rectangles and parts of a circle. http://www.dadsworksheets.com/worksheets/basic-geometry.html [This page is intentionally left blank.]

Triangles



- 1. **a.** Draw a right *angle*. Then make it into a right *triangle* by drawing in the third side.
 - **b.** Draw another, different right triangle.
 - **c.** A right triangle has one right angle. Are the other two angles in a right triangle acute, right, or obtuse?



A right triangle has one right angle. The other two angles are _

2. **a.** Draw an obtuse angle. Then make it into an obtuse triangle by drawing in the third side.

- **b.** Draw another, different obtuse triangle.
- c. An obtuse triangle has one obtuse angle. Are the other two angles in an obtuse triangle acute, right, or obtuse?

An obtuse triangle has one obtuse angle. The other two angles are _____

- 3. **a.** Draw an acute triangle. The side lengths can be any.
 - **b.** Measure its angles.

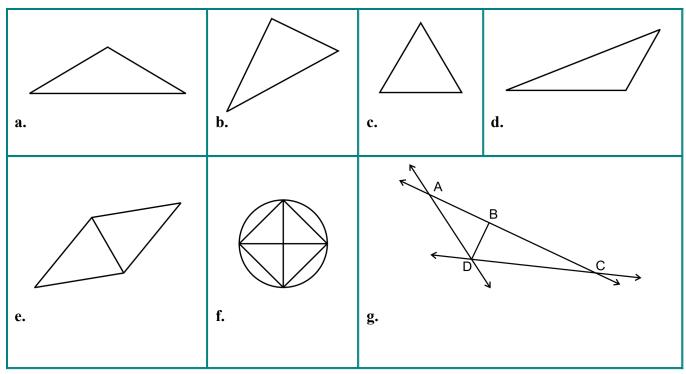
They measure _____°,

_____° and _____°.

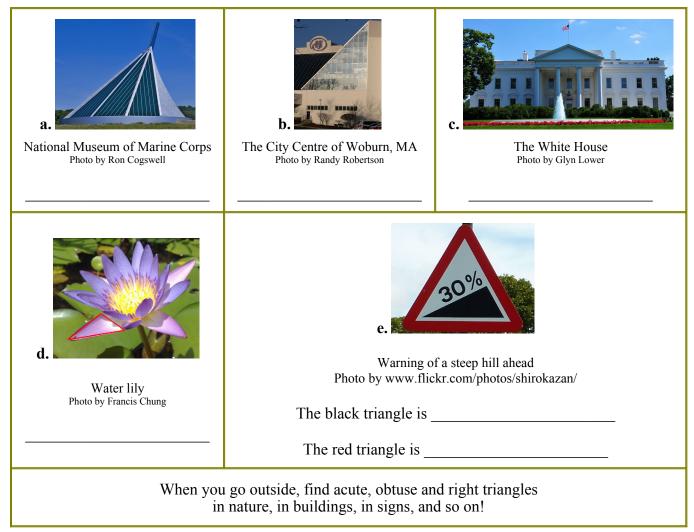
4. Observe all you have done thus far in this lesson, and fill in.

Right triangles have exactly 1	,
and the other two angles are	
Obtuse triangles have exactly 1	,
and the other two angles are	
Acute triangles have angles.	

5. Label the triangles in the pictures as right, acute, or obtuse.



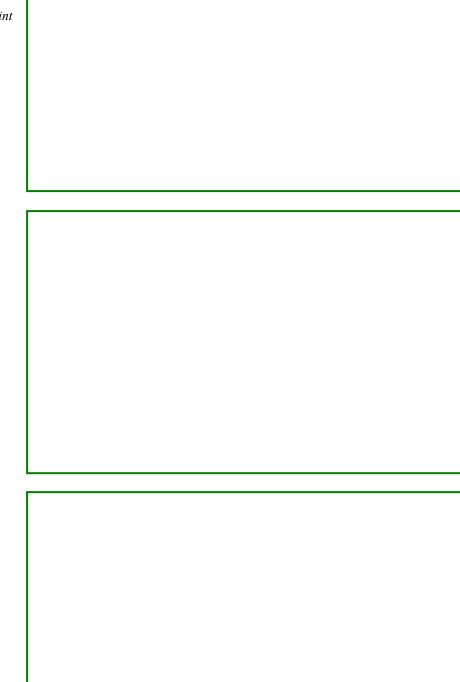
6. Label the triangles in the pictures as right, acute, or obtuse.



7. **a.** Draw a triangle with 85° and 40° angles.

Hint: First draw a 85° angle. Then, mark a point anywhere on one side of that angle to be the second vertex of the triangle. Use that point as a vertex for the 40° angle, and draw the 40° angle.

- **b.** Measure the third angle. It is _____ degrees.
- **c.** What kind of triangle is it? (acute, right, obtuse)
- **d.** What is the angle sum?
- 8. **a.** Draw a triangle with 125° and 40° angles.
 - **b.** Measure the third angle. It is _____ degrees.
 - **c.** What kind of triangle is it? (acute, right, obtuse)
 - **d.** What is the angle sum?
- 9. **a.** Draw a triangle with 55° and 35° angles.
 - **b.** Measure the third angle. It is <u>degrees</u>.
 - **c.** What kind of triangle is it? (acute, right, obtuse)
 - **d.** What is the angle sum?



New Terms

an acute triangle
a right triangle

Chapter 7: Fractions Introduction

In the third grade, students studied the concept of a fraction, equivalent fractions, and compared some easy fractions. In fourth grade, it is time to expand the fraction topics. We study

- mixed numbers
- adding and subtracting like fractions and mixed numbers with like fractional parts (the denominators are the same)
- equivalent fractions
- comparing fractions
- multiplying a fraction by a whole number

Then in fifth grade, students study *all* of the four operations with fractions. Our studies here are still laying groundwork for that, emphasising conceptual understanding and using visual models a lot.

These lessons are also important because they are the basis for understanding decimal numbers, the topic of the next chapter. Decimals are just another way of writing fractions with denominators 10, 100, 1000 etc.

The topics in this chapter are studied with the help of visual models in order to emphasise the concepts. We must avoid presenting fraction maths as a list of computational rules. Students easily confuse the various fraction rules, because there are so many, such as:

- a rule for converting a mixed number to a fraction, and vice versa
- a rule for adding like fractions
- a rule for finding a common denominator
- a rule for changing fractions to like fractions
- a rule for adding unlike fractions
- a rule for simplifying fractions
- a rule for finding equivalent fractions
- a rule for multiplying fractions
- a rule for dividing fractions
- a few rules for doing the four operations with mixed numbers

There is a place for the rules, as *shortcuts* for ideas that are already understood, but we do not start with them. The goal is to let the big ideas sink in conceptually first, followed by some shortcuts.

The Lessons in Chapter 7

page	span
135	3 pages
138	4 pages
142	4 pages
146	3 pages
149	5 pages
154	3 pages
157	4 pages
161	3 pages
164	2 pages
166	2 pages
168	2 pages
	135 135 138 142 146 149 154 157 161 164 166

Helpful Resources and Games on the Internet

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naga

anon

General

Visual Fractions

Great site for studying all aspects of fractions: identifying, renaming, comparing, addition, subtraction, multiplication, division. Each topic is illustrated by either a number line or a circle with a Java applet. Also a couple of games, for example: make cookies for Grampy. http://www.visualfractions.com/

Conceptua Math Fraction Tools

Free and interactive fraction tools for identifying fractions, adding and subtracting, estimating, comparing, equivalent fractions, finding common denominators and more. Each activity uses several fraction models such as fraction circles, horizontal and vertical bars, number lines, etc. that allow students to develop conceptual understanding of fractions. Free registration required. https://www.conceptuamath.com/app/tool-library

Fraction Games at Sheppard Software

Games for addition and subtraction of fractions, simplifying fractions, equivalent fractions and a fraction of a set.

http://www.sheppardsoftware.com/math.htm#fractions

Who Wants pizza?

This site explains the concept of fractions, addition, and multiplication with a pizza example, then has some interactive exercises.

http://math.rice.edu/~lanius/fractions/index.html

Fractioncity

Make "fraction streets" and help students with comparing fractions, equivalent fractions, addition of fractions of like and unlike denominators while they drive toy cars on the streets. This is not an online activity but has instructions of how to do it at home or at school. http://www.teachnet.com/lesson/math/fractioncity.html

Fraction Worksheets: Equivalent Fractions, Simplifying, Convert to Mixed Numbers

Create custom-made worksheets for some other fraction operations. http://www.homeschoolmath.net/worksheets/fraction-b.php

Fractions and mixed numbers

Identifying Fractions at Conceptua Fractions

A tool that shows fractions or mixed numbers using a pie, a bar, dots and a number line. A free registration required.

https://www.conceptuamath.com/app/tool/identifying-fractions

Visualizing Fractions

The computer shows a fraction, and you divide the pie and colour in the pieces. http://nlvm.usu.edu/en/nav/frames_asid_103_g_2_t_1.html

Pattern Blocks—Parts as Wholes

Click on the "Activities" in the top menu, and click on arrows until you find Parts as Wholes activity. http://nlvm.usu.edu/en/nav/frames_asid_170_g_2_t_3.html

Fraction Models

Adjust the numerator and the denominator, and the applet shows the fraction as a pie/rectangle/set model, as a decimal and as a percent. http://illuminations.nctm.org/Activity.aspx?id=3519

Clara Fraction's Ice Cream Shop

Convert improper fractions to mixed numbers and scoop the right amount of ice cream flavours onto the cone.

http://mrnussbaum.com/icecream/

Addition and subtraction

MathSplat

Click on the right answer for addition problems or the bug splats on your windshield! http://fen.com/studentactivities/MathSplat/mathsplat.htm

Action Fraction

A racing game with several levels where you answer questions about adding and subtracting fractions. The levels advance from using like fractions to using unlike fractions and eventually subtraction. http://www.solvemymath.com/math_games/arithmetic_games/action_fraction/

Fraction Worksheets: Addition and Subtraction

Create custom-made worksheets for the four operations with fractions and mixed numbers. Choose "Like Fractions" for this level.

http://www.homeschoolmath.net/worksheets/fraction.php

Fruit Shoot Fractions Addition

Click on the fruit with the correct answer to a fraction problem you are given. Options include adding fractions with like or unlike denominators and simplifying. To match the topics students learn in this section, choose adding 2 or 3 fractions with like denominators. You can also choose your mode (untimed or timed) and speed (slow vs. fast fruit).

http://www.sheppardsoftware.com/mathgames/fractions/FruitShootFractionsAddition.htm

Comparing Fractions

Comparison Shoot Out

Choose level 2 or 3 to compare fractions and shoot the soccer ball to the goal. http://www.fuelthebrain.com/games/comparison-shootout/

Comparing Fractions—XP Math

Simple timed practice with comparing two fractions. http://xpmath.com/forums/arcade.php?do=play&gameid=8

Ordering Fractions at Conceptua Fractions

An interactive tool where students place numbers, visual models and decimals on a number line. https://www.conceptuamath.com/app/tool/comparing-fractions

Equivalent fractions

Equivalent Fractions from National Library of Virtual Manipulatives (NLVM)

See the equivalency of two fractions as the applet divides the whole into more pieces. http://nlvm.usu.edu/en/nav/frames_asid_105_g_2_t_1.html

Equivalent Fractions

Draw two equivalent fractions for the given fraction. Choose either a square or a circle for the shape. http://illuminations.nctm.org/Activity.aspx?id=3510

Fraction Frenzy

Click on pairs of equivalent fractions, as fast as you can. See how many levels you can get! http://www.learningplanet.com/sam/ff/index.asp

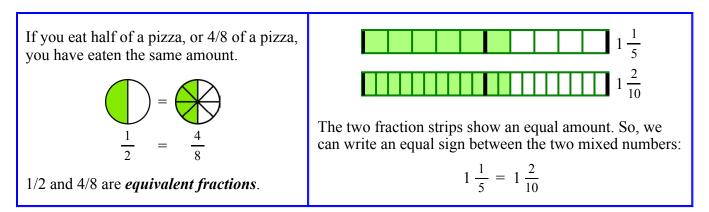
Fresh Baked Fractions

Practise equivalent fractions by clicking on a fraction that is not equal to others. http://www.funbrain.com/fract/index.html

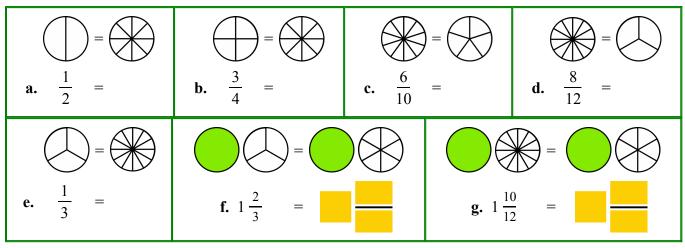
Free Equivalent Fractions Worksheets

Create custom-made worksheets for equivalent fractions that can either include pie images or not. http://www.homeschoolmath.net/worksheets/equivalent_fractions.php

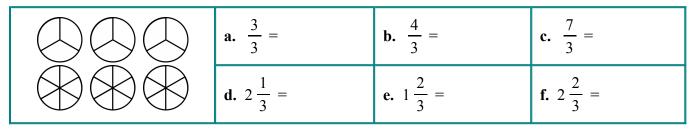
Equivalent Fractions



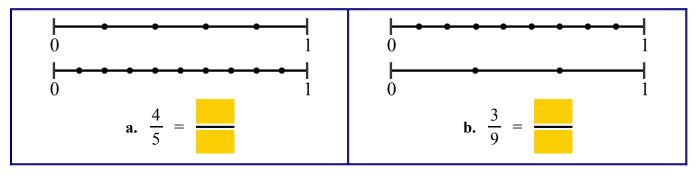
1. Colour in the first fraction. Shade the same *amount of pie* in the second picture. Write the second fraction.



2. Write the fractions that have thirds using sixths instead. You can shade parts in the pictures.

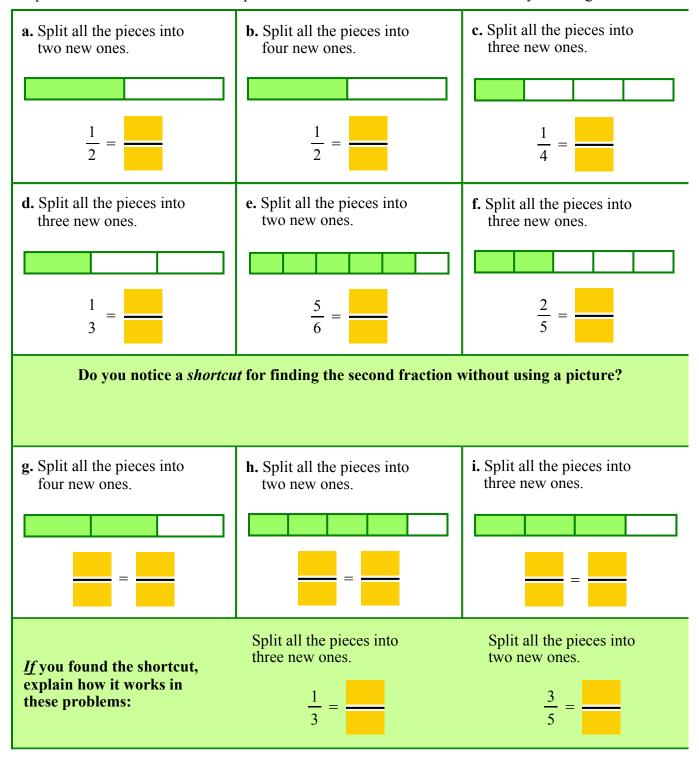


3. Mark the equivalent fractions on the number lines.



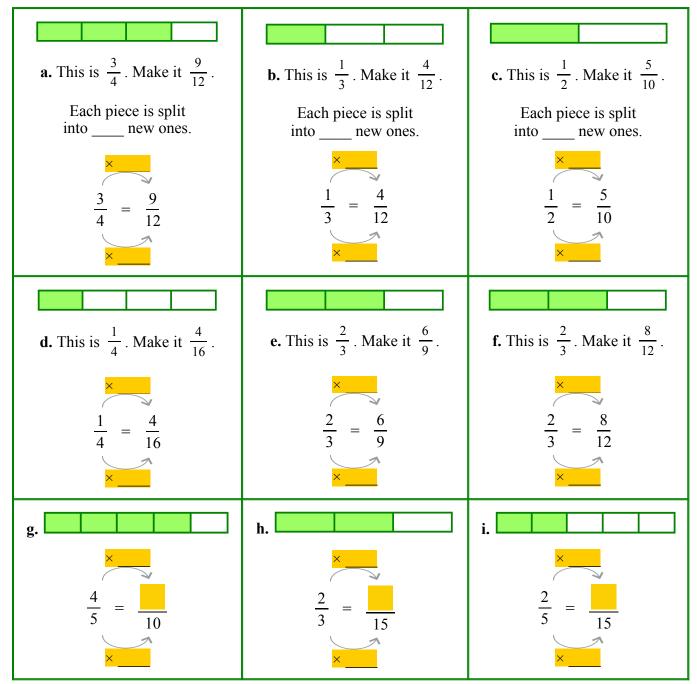
The fraction strip illustrates $\frac{2}{5}$. If you split each piece (both the coloured and white pieces) into *two* new pieces, what fraction do you get? You get $\frac{4}{10}$ – four coloured pieces, and ten pieces total. You have *two* times as many coloured pieces, and *two* times as many total pieces as before.

4. Split both the coloured and white pieces as instructed. Write the fraction after you change it.

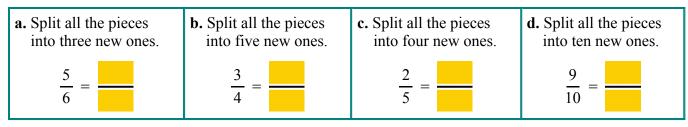


The fraction strip illustrates $\frac{1}{2}$. If we split each piece (both the coloured and the white piece) into <i>three</i> new pieces, we get $\frac{3}{6}$.
We now have <i>three</i> times as many coloured pieces, and <i>three</i> times as many total pieces as before. We can show this in writing this way: We multiply both the top and bottom number in a fraction by 3. We get an equivalent fraction—it is the same amount , $\frac{1}{2} = \frac{3}{6}$
just cut into more pieces. <i>This does not mean we multiply the whole</i> <i>fraction by 3.</i>

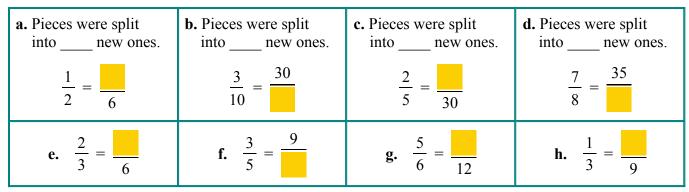
5. Split the pieces. Fill in the missing parts.



6. Write the equivalent fraction. Use multiplication.



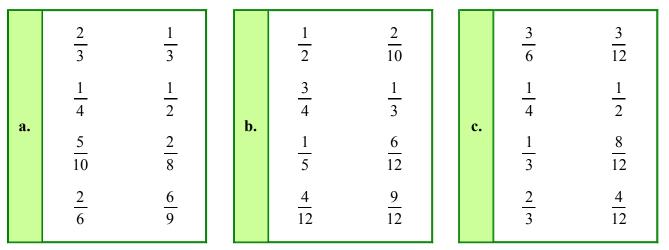
7. Figure out how many new pieces the existing pieces were split into. Fill in the missing parts.



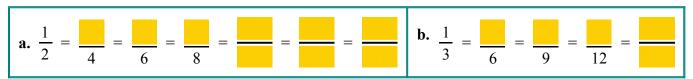
8. Write the fractions that have tenths with hundredths instead.

a. $\frac{1}{10} = \frac{1}{100}$	b. $\frac{3}{10} =$	c. $\frac{6}{10} =$	d. $\frac{4}{10} =$	e. $\frac{13}{10} =$
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9. Connect the equivalent fractions with a line.

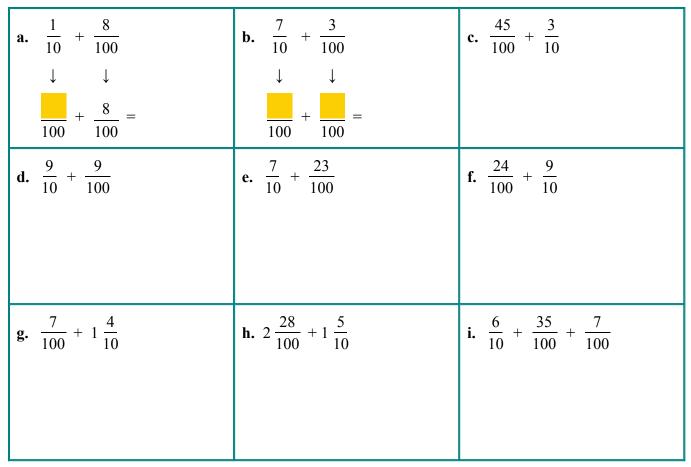


10. Write chains of equivalent fractions!



We can use equivalent fractions to add fractions that have different denominators. Example. Add $\frac{2}{10} + \frac{17}{100}$. First, write 2/10 as 20/100 (an equivalent fraction). Then you can add, because the fractions now have the same denominator: $\frac{20}{100} + \frac{17}{100} = \frac{37}{100}$.

11. Add.



12. Draw a picture showing that 1/3 and 4/12 are equivalent fractions.

Puzzle Corner	Add. This is challenging. <i>Hint: You cannot simply add the top numbers and the bottom numbers. Use equivalent fractions.</i>		
a. $\frac{3}{4} + \frac{1}{2}$	b. $\frac{1}{5} + \frac{3}{10}$	c. $\frac{2}{3} + \frac{2}{9}$	

Chapter 8: Decimals Introduction

In fourth grade, we study decimal numbers with one or two decimal digits, and add and subtract them. It is important that the student grasps these simple topics well, because we are laying a groundwork towards fifth and sixth grade, where decimal operations and using decimals take a "center stage."

For now, the focus is first of all understanding the fact that decimals are simply fractions with the denominator 10 or 100. Then with that in mind (decimals are fractions), we study comparing, adding and subtracting them.

Notice:

- In the addition problem 0.5 + 0.9, we get 14 tenths, which is 1.4. A common student misconception is to add 0.5 + 0.9 = 0.14.
- In a problem such as 0.5 + 0.11, a common student misconception is to get 0.16. Such students are thinking of the decimal parts as if they were "whole numbers." To solve 0.5 + 0.11 correctly, students can rewrite 0.5 as 0.50, and then the problem becomes 0.50 + 0.11 = 0.61.

In the lesson Using Decimal Numbers, students use decimals with some metric measuring units, including converting between units. This topic will also be studied further in 5th grade.

The Lessons in Chapter 8

	page	span
Decimal Numbers—Tenths	172	2 pages
Adding and Subtracting with Tenths	174	2 pages
Two Decimal Digits—Hundredths	176	4 pages
Adding and Subtracting Hundredths	180	4 pages
Adding and Subtracting Decimals in Columns	184	3 pages
Using Decimals with Measuring Units	187	2 pages
Mixed Revision, Chapters 1 - 8	189	2 pages
Revision, Chapter 8	191	2 pages

Helpful Resources on the Internet

Use these free online resources to supplement the "bookwork" as you see fit. <u>Disclaimer</u>: These links were valid at the time of writing this book, and to the best of our knowledge we believe these websites to have what is described. However, we cannot guarantee that the links have not changed. Parental supervision is needed.

Mathematical Interactivities

http://mathematics.hellam.net/

Find several games related to fractions and decimals in the Number Puzzles section, including:

- Decimal Challenge Guess the decimal number between 0 and 10. Each time feedback tells whether your guess was too high or too low. http://www.interactivestuff.org/sums4fun/decchall.html
- Switch Put the sequence of decimal numbers into ascending order by switching them around. Refresh the page from your browser to get another problem to solve. http://www.interactivestuff.org/sums4fun/switch.html
- Scales Move the pointer to match the decimal number given to you. Refresh the page from your browser to get another problem to solve. http://www.interactivestuff.org/sums4fun/scales.html

A Decimal Puzzle

Make every circle add up to 3. http://nlvm.usu.edu/en/nav/frames_asid_187_g_2_t_1.html?open=instructions&from=category_g_2_t_1.html

Fraction/Decimal Worksheets

Change fractions to decimal numbers or decimal numbers to fractions. http://www.homeschoolmath.net/worksheets/fraction-decimal.php

Modelling Decimals (Area and Grid Models)

An interactive "gizmo" for modelling decimals in a grid or on a number. By subscription, but you can try the gizmo for free for 5 minutes.

http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=1007

Adding Decimals (Base 10 Blocks)

An interactive "gizmo" for modelling decimal addition with regrouping. By subscription, but you can try the gizmo for free for 5 minutes.

http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=1023

Subtracting Decimals (Base 10 Blocks)

An interactive "gizmo" for modelling decimal subtraction with regrouping. By subscription, but you can try the gizmo for free for 5 minutes.

http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=1030

Beat the Clock

Type in the decimal to show how much of the square is shaded in this timed game. http://www.decimalsquares.com/dsGames/games/beatclock.html

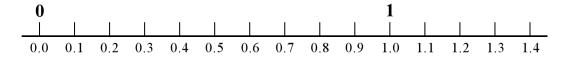
Decimal Darts

Try to pop the balloons with darts by estimating at which height the balloons are. http://www.decimalsquares.com/dsGames/games/darts.html

Adding and Subtracting with Tenths

You <i>already</i> know how to add or subtract decimals with tenths. They are just fractions with a denominator of 10. Compare these additions that are written with decimals or fractions.	$0.1 + 0.5 = 0.6$ $\frac{1}{10} + \frac{5}{10} = \frac{6}{10}$	$8.4 - 2.3 = 6.1$ $8\frac{4}{10} - 2\frac{3}{10} = 6\frac{1}{10}$
There is one tricky part though: $0.6 + 0.7$ is <u>NOT</u> 0.13 !!	0.6 + 0.7 = 1.3	1.5 + 0.9 = 2.4
To see why, add the fractions. Notice that six tenths and seven tenths make more than one whole!	$\frac{6}{10} + \frac{7}{10} = \frac{13}{10} = 1\frac{3}{10}$	$1\frac{5}{10} + \frac{9}{10} = 2\frac{4}{10}$

1. Write an addition or subtraction sentence for each "number line jump."



a. You are at 0.7 and you jump *five tenths* to the right.

b. You are at 0.6 and you jump *eight tenths* to the right.

c. You are at 1.1 and you jump *eight tenths* to the left.

d. You are at 1.3 and you jump *four tenths* to the left.

e. You are at 0.2 and you jump *eleven tenths* to the right.

2. Solve the fraction additions, and then write them using decimals.

a. $\frac{2}{10} + \frac{7}{10} =$	b. $\frac{5}{10}$ + $\frac{6}{10}$ =	c. $\frac{9}{10}$ + $\frac{8}{10}$ =
0.2 +		

3. Add and subtract.

a.	b.	с.	d.
0.9 + 0.2 =	0.5 + 0.7 =	0.8 + 0.7 =	1.8 - 0.9 =
1.9 + 0.2 =	3.5 + 0.7 =	0.8 + 2.7 =	5.8 - 0.9 =

4. Fill in the missing parts.

a.	b.	с.	d.
2.3 + 0.9 =	1.5 + 0.7 =	6.6 - 0.5 =	4.7 - 1.7 =

5. Write the numbers.

a. 3 tenths, 5 ones	d. Write the numbers in order.
b. 7 tens, 8 ones, 4 tenths	9 8.9 9.1 9.0 9.9 1.9
c. 4 tenths, 3 ones, 6 tens	

6. Continue the patterns by adding or subtracting the same number repeatedly.

a. 0.1	b. 1.1	c. 2.5	d. 3.6
+ 0.2 =	+ 0.5 =	+ 0.3 =	- 0.4 =
+ 0.2 =	+ 0.5 =	+ 0.3 =	- 0.4 =
+ 0.2 =	+ 0.5 =	+ 0.3 =	- 0.4 =
+ 0.2 =	+ 0.5 =	+ 0.3 =	- 0.4 =
+ 0.2 =	+ 0.5 =	+ 0.3 =	- 0.4 =
+ 0.2 =	+ 0.5 =	+ 0.3 =	- 0.4 =

7. Remember? 1 millimetre is one-tenth of a centimetre. Or, 1 mm = 0.1 cm.		
a. Draw a line that is 4.7 cm long.	b. Measure the line in centimetres. Use a decimal.	
1 2 3 4 5	1 2 3 4 5	

8. Convert. In (c), add and give your answer in centimetres.

a. $0.5 \text{ cm} = _ mm$ **b.** $7 \text{ mm} = _ cm$ **c.** $5 \text{ mm} + 0.9 \text{ cm} = _ cm$ $1.2 \text{ cm} = _ mm$ $35 \text{ mm} = _ cm$ $4 \text{ cm} + 3.4 \text{ cm} = _ cm$

9. The two sides of a rectangle measure 6.5 cm and 3.6 cm. Draw the rectangle on blank paper. What is its perimeter?