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Foreword

Math Mammoth Grade 6, Canadian Version, comprises a complete math curriculum for the sixth grade mathematics studies. This curriculum is essentially the same as the U.S. version of Math Mammoth Grade 6, only customised for Canadian audiences in a few aspects (listed below). The curriculum meets the Common Core Standards in the United States, but it may not perfectly align to the sixth grade standards in your province.

The Canadian version of Math Mammoth differs from the US version in these aspects:

- The curriculum uses metric measurement units, not both metric and customary (imperial) units.
- The spelling conforms to British international standards.
- The pages are formatted for Letter paper size.
- Large numbers are formatted with a space as the thousands separator (such as 12 394).
(The decimals are formatted with a decimal point, as in the US version.)

The worktext 6-A covered the first half of the topics for 6th year: a revision of the four basic operations, an introduction to algebra, and decimals, ratios and percent.

This part B covers the remainder of the topics for 6th year: prime factorisation, the greatest common factor, least common multiple; fractions; integers and graphing in the coordinate plane; geometry and statistics.

Chapter 6 first revises prime factorisation and then applies those principles to using the greatest common factor to simplify fractions and the least common multiple to find common denominators. Chapter 7 provides a thorough revision of the fraction operations from fifth year and includes ample practice in solving problems with fractions.

Chapter 8 introduces students to integers (signed numbers). Students plot points in all four quadrants of the coordinate plane, reflect and translate simple figures, and learn to add and subtract with negative numbers. (The multiplication and division of integers will be studied in 7th year.)

The next chapter, Geometry, focuses on calculating the area of polygons. The final chapter is about statistics. Beginning with the concept of a statistical distribution, students learn about measures of centre and measures of variability. They also learn how to make dot plots, histograms, boxplots, and stem-and-leaf plots as ways to summarise and analyse distributions.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching math!

Maria Miller, the author

User Guide

Note: You can also find the information that follows online, at **https://www.mathmammoth.com/userguides/** .

Basic principles in using Math Mammoth Complete Curriculum

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. The two books (parts A and B) are like a “framework”, but you still have a lot of liberty in planning your child’s studies. You can even use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously. In sixth grade, chapters 1 and 2 should be studied before the other chapters, but you can be flexible with all the other chapters and schedule them earlier or later.

Math Mammoth is not a scripted curriculum. In other words, it is not spelling out in exact detail what the teacher is to do or say. Instead, Math Mammoth gives you, the teacher, various tools for teaching:

- **The two student worktexts** (parts A and B) contain all the lesson material and exercises. They include the explanations of the concepts (the teaching part) in blue boxes. The worktexts also contain some advice for the teacher in the “Introduction” of each chapter.

  The teacher can read the teaching part of each lesson before the lesson, or read and study it together with the student in the lesson, or let the student read and study on his own. If you are a classroom teacher, you can copy the examples from the “blue teaching boxes” to the board and go through them on the board.
- There are hundreds of **videos** matched to the curriculum available at https://www.mathmammoth.com/videos/ . There isn’t a video for every lesson, but there are dozens of videos for each grade level. You can simply have the author teach your child or student!
- Don’t automatically assign all the exercises. Use your judgement, trying to assign just enough for your student’s needs. You can use the skipped exercises later for revision. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.
- For each chapter, there is a **link list to various free online games** and activities. These games can be used to supplement the math lessons, for learning math facts, or just for some fun. Each chapter introduction (in the student worktext) contains a link to the list corresponding to that chapter.
- The student books contain some **mixed revision lessons**, and the curriculum also provides you with additional **cumulative revision lessons**.
- There is a **chapter test** for each chapter of the curriculum, and a comprehensive end-of-year test.
- The **worksheet maker** allows you to make additional worksheets for most calculation-type topics in the curriculum. This is a single html file. You will need Internet access to be able to use it.
- You can use the free online exercises at https://www.mathmammoth.com/practice/
  This is an expanding section of the site, so check often to see what new topics we are adding to it!
- Some grade levels have **cut-outs** to make fraction manipulatives or geometric solids.
- And of course there are answer keys to everything.

How to get started

Have ready the first lesson from the student worktext. Go over the first teaching part (within the blue boxes) together with your child. Go through a few of the first exercises together, and then assign some problems for your child to do on their own.

Repeat this if the lesson has other blue teaching boxes. Naturally, you can also use the videos at https://www.mathmammoth.com/videos/

Many children can eventually study the lessons completely on their own — the curriculum becomes self-teaching. However, children definitely vary in how much they need someone to be there to actually teach them.

Pacing the curriculum

The lessons in Math Mammoth complete curriculum are NOT intended to be done in a single teaching session or class. Sometimes you might be able to go through a whole lesson in one day, but more often, the lesson itself might span 3-5 pages and take 2-3 days or classes to complete.

Therefore, it is not possible to say exactly how many pages a student needs to do in one day. This will vary. However, it is helpful to calculate a general guideline as to how many pages per week you should cover in the student worktext in order to go through the curriculum in one school year (or whatever span of time you want to allot to it).

The table below lists how many pages there are for the student to finish in this particular grade level, and gives you a guideline for how many pages per day to finish, assuming a 180-day school year.

Example:

Grade level	Lesson pages	Number of school days	Days for tests and revisions	Days for the student book	Pages to study per day	Pages to study per week
6-A	166	92	10	82	2.0	10
6-B	157	88	10	78	2.0	10
Grade 6 total	323	180	20	160	2.0	10

The table below is for you to fill in. First fill in how many days of school you intend to have. Also allow several days for tests and additional revision before the test — at least twice the number of chapters in the curriculum. For example, if the particular grade has 8 chapters, allow at least 16 days for tests & additional revision. Then, to get a count of "pages/day", divide the number of pages by the number of available days. Then, multiply this number by 5 to get the approximate page count to cover in a week.

Grade level	Lesson pages	Number of school days	Days for tests and revisions	Days for the student book	Pages to study per day	Pages to study per week
6-A	166					
6-B	157					
Grade 6 total	323					

Now, let's assume you determine that you need to study about 2 pages a day, 10 pages a week in order to get through the curriculum. As you study each lesson, keep in mind that sometimes most of the page might be filled with blue teaching boxes and very few exercises. You might be able to cover 3 pages on such a day. Then some other day you might only assign one page of word problems. Also, you might be able to go through the pages quicker in some chapters, for example when studying graphs, because the large pictures fill the page so that one page does not have many problems.

When you have a page or two filled with lots of similar practice problems ("drill") or large sets of problems, feel free to **only assign 1/2 or 2/3 of those problems**. If your child gets it with less amount of exercises, then that is perfect! If not, you can always assign him/her the rest of the problems some other day. In fact, you could even use these unassigned problems the next week or next month for some additional revision.

In general, 1st-2nd graders might spend 25-40 minutes a day on math. Third-fourth graders might spend 30-60 minutes a day. Fifth-sixth graders might spend 45-75 minutes a day. If your child finds math enjoyable, he/she can of course spend more time with it! However, it is not good to drag out the lessons on a regular basis, because that can then affect the child’s attitude towards math.

Working space, the usage of additional paper and mental math

The curriculum generally includes working space directly on the page for students to work out the problems. However, feel free to let your students to use extra paper when necessary. They can use it, not only for the “long” algorithms (where you line up numbers to add, subtract, multiply, and divide), but also to draw diagrams and pictures to help organise their thoughts. Some students won’t need the additional space (and may resist the thought of extra paper), while some will benefit from it. Use your discretion.

Some exercises don’t have any working space, but just an empty line for the answer (e.g. 200 + _____ = 1 000). Typically, I have intended that such exercises to be done using MENTAL MATH.

However, there are some students who struggle with mental math (often this is because of not having studied and used it in the past). As always, the teacher has the final say (not me!) as to how to approach the exercises and how to use the curriculum. We do want to prevent extreme frustration (to the point of tears). The goal is always to provide SOME challenge, but not too much, and to let students experience success enough so that they can continue enjoying learning math.

Students struggling with mental math will probably benefit from studying the basic principles of mental calculations from the earlier levels of Math Mammoth curriculum. To do so, look for lessons that list mental math strategies. They are taught in the chapters about addition, subtraction, place value, multiplication, and division. My article at https://www.mathmammoth.com/lessons/practical_tips_mental_math also gives you a summary of some of those principles.

Using tests

For each chapter, there is a **chapter test**, which can be administered right after studying the chapter. **The tests are optional.** Some families might prefer not to give tests at all. The main reason for the tests is for diagnostic purposes, and for record keeping. These tests are not aligned or matched to any standards.

In the digital version of the curriculum, the tests are provided both as PDF files and as html files. Normally, you would use the PDF files. The html files are included so you can edit them (in a word processor such as Word or LibreOffice), in case you want your student to take the test a second time. Remember to save the edited file under a different file name, or you will lose the original.

The end-of-year test is best administered as a diagnostic or assessment test, which will tell you how well the student remembers and has mastered the mathematics content of the entire grade level.

Using cumulative revisions and the worksheet maker

The student books contain mixed revision lessons which revise concepts from earlier chapters. The curriculum also comes with additional cumulative revision lessons, which are just like the mixed revision lessons in the student books, with a mix of problems covering various topics. These are found in their own folder in the digital version, and in the Tests & Cumulative Revisions book in the print version.

The cumulative revisions are optional; use them as needed. They are named indicating which chapters of the main curriculum the problems in the revision come from. For example, “Cumulative Revision, Chapter 4” includes problems that cover topics from chapters 1-4.

Both the mixed and cumulative revisions allow you to spot areas that the student has not grasped well or has forgotten. When you find such a topic or concept, you have several options:

1. Check if the worksheet maker lets you make worksheets for that topic.
2. Check for any online games and resources in the Introduction part of the particular chapter in which this topic or concept was taught.
3. If you have the digital version, you could simply reprint the lesson from the student worktext, and have the student restudy that.
4. Perhaps you only assigned 1/2 or 2/3 of the exercise sets in the student book at first, and can now use the remaining exercises.
5. Check if our online practice area at https://www.mathmammoth.com/practice/ has something for that topic.
6. Khan Academy has free online exercises, articles, and videos for most any math topic imaginable.

Concerning challenging word problems and puzzles

While this is not absolutely necessary, I heartily recommend supplementing Math Mammoth with challenging word problems and puzzles. You could do that once a month, for example, or more often if the student enjoys it.

The goal of challenging story problems and puzzles is to **develop the student's logical and abstract thinking and mental discipline**. I recommend starting these in fourth grade, at the latest. Then, students are able to read the problems on their own and have developed mathematical knowledge in many different areas. Of course I am not discouraging students from doing such in earlier grades, either.

Math Mammoth curriculum contains lots of word problems, and they are usually multi-step problems. Several of the lessons utilise a bar model for solving problems. Even so, the problems I have created are usually tied to a specific concept or concepts. I feel students can benefit from solving problems and puzzles that require them to think "out of the box" or are just different from the ones I have written.

I recommend you use the free Math Stars problem-solving newsletters as one of the main resources for puzzles and challenging problems:

Math Stars Problem Solving Newsletter (grades 1-8)
https://www.homeschoolmath.net/teaching/math-stars.php

I have also compiled a list of other resources for problem solving practice, which you can access at this link:

https://l.mathmammoth.com/challengingproblems

Another idea: you can find puzzles online by searching for "brain puzzles for kids," "logic puzzles for kids" or "brain teasers for kids."

Frequently asked questions and contacting us

If you have more questions, please first check the FAQ at https://www.mathmammoth.com/faq-lightblue

If the FAQ does not cover your question, you can then contact us using the contact form at the Math Mammoth.com website.

I wish you success in teaching math!

Maria Miller, the author

Chapter 6: Prime Factorisation, GCF and LCM
Introduction

The topics of this chapter belong to a branch of mathematics known as *number theory*. Number theory has to do with the study of whole numbers and their special properties. In this chapter, we revise prime factorisation and study the greatest common factor (GCF) and the least common multiple (LCM).

The main application of factoring and the greatest common factor in arithmetic is in simplifying fractions, so that is why I have included a lesson on that topic. However, it is not absolutely necessary to use the GCF when simplifying fractions, and the lesson emphasises that fact.

The concepts of factoring and the GCF are important to understand because they will be carried over into algebra, where students will factor polynomials. In this chapter, we lay the groundwork for that by using the GCF to factor simple sums, such as 27 + 45. For example, a sum like 27 + 45 factors into 9(3 + 5).

Similarly, the main use for the least common multiple in arithmetic is in finding the smallest common denominator for adding fractions, and we study that topic in this chapter in connection with the LCM.

Primes are fascinating "creatures," and you can let students read more about them by accessing the Internet resources mentioned below. The really important, but far more advanced, application of prime numbers is in cryptography. Some students might be interested in reading additional material on that subject—please see the list for Internet resources.

Keep in mind that the specific lessons in the chapter can take several days to finish. They are not "daily lessons." Instead, use the general guideline that sixth graders should finish about 2 pages daily or 9-10 pages a week in order to finish the curriculum in about 40 weeks. Also, I recommend not assigning all the exercises by default, but that you use your judgement, and strive to vary the number of assigned exercises according to the student's needs. Please see the user guide at **https://www.mathmammoth.com/userguides/** for more guidance on using and pacing the curriculum.

You can find some free videos for the topics of this chapter at **https://www.mathmammoth.com/videos/** (choose 6th grade).

The Lessons in Chapter 6

The Sieve of Eratosthenes and Prime Factorisation

Remember? A number is a **prime** if it has no other factors besides 1 and itself.

For example, 13 is a prime, since the only way to write it as a multiplication is $1 \cdot 13$. In other words, 1 and 13 are its only factors.

And, 15 is not a prime, since we can write it as $3 \cdot 5$. In other words, 15 has other factors besides 1 and 15, namely 3 and 5.

To find all the prime numbers less than 100 we can use the *sieve of Eratosthenes.*

Here is an online interactive version: https://www.mathmammoth.com/practice/sieve-of-eratosthenes

1. Cross out 1, as it is not considered a prime.
2. Cross out all the even numbers except 2.
3. Cross out all the multiples of 3 except 3.
4. You do not have to check multiples of 4. Why?
5. Cross out all the multiples of 5 except 5.
6. You do not have to check multiples of 6. Why?
7. Cross out all the multiples of 7 except 7.
8. You do not have to check multiples of 8 or 9 or 10.
9. The numbers left are primes.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

List the **primes between 0 and 100** below:

2, 3, 5, 7, ____

Why do you not have to check numbers that are bigger than 10? Let's think about multiples of 11. The following multiples of 11 have already been crossed out: $2 \cdot 11$, $3 \cdot 11$, $4 \cdot 11$, $5 \cdot 11$, $6 \cdot 11$, $7 \cdot 11$, $8 \cdot 11$ and $9 \cdot 11$. The multiples of 11 that have not been crossed out are $10 \cdot 11$ and onward... but they are not on our chart! Similarly, the multiples of 13 that are less than 100 are $2 \cdot 13$, $3 \cdot 13$, ..., $7 \cdot 13$, and all of those have already been crossed out when you crossed out multiples of 2, 3, 5 and 7.

1. You learned this in 4th and 5th grades... find all the factors of the given numbers. Use the checklist to help you keep track of which factors you have tested.

a. 54

Check 1 2 3 4 5 6 7 8 9 10

factors: ______________________________

b. 60

Check 1 2 3 4 5 6 7 8 9 10

factors: ______________________________

c. 84

Check 1 2 3 4 5 6 7 8 9 10

factors: ______________________________

d. 97

Check 1 2 3 4 5 6 7 8 9 10

factors: ______________________________

For your reference, here are some of the common divisibility tests for whole numbers.

A number is...

divisible by 2 if it ends in 0, 2, 4, 6, or 8.

divisible by 5 if it ends in 0 or 5.

divisible by 10 if it ends in 0.

divisible by 100 if it ends in "00".

A number is...

divisible by 3 if the sum of its digits is divisible by 3.

divisible by 4 if the number formed from its last two digits is divisible by 4.

divisible by 6 if it is divisible by both 2 and 3.

divisible by 9 if the sum of its digits is divisible by 9.

Use the various divisibility tests when building a factor tree for a composite number.

```
 135           2 7          135
 / \       5 ) 1 3 5        / \
5 · ?        - 1 0         5 · 27
               3 5             / \
             - 3 5            3 · 9
                 0               / \
                                3 · 3
```

We start out by noticing that 135 is **divisible by 5.** From long division, we get 135 = 5 · 27. The final factorisation is 135 = 3 · 3 · 3 · 5 or $3^3 \cdot 5$.

```
 441           4 9          441
 / \       9 ) 4 4 1        / \
9 · ?        - 3 6         9 · 49
               8 1        / \  / \
             - 8 1       3 · 3 · 7 · 7
                 0
```

Adding the digits of 441, we get 9, so it is **divisible by 9.** We divide to get 441 = 9 · 49. The end result is 441 = 3 · 3 · 7 · 7 or $3^2 \cdot 7^2$.

2. Find the prime factorisation of these composite numbers. Use a notebook for long divisions. Give each factorisation below the factor tree.

a. 124

```
  /   \
2 ·  ___
     /  \
    2
```

124 =

b. 260

```
   /    \
 10  ·  ___
 / \    / \
```

260 =

c. 96

```
  /   \
3 ·  ___
     /  \
```

96 =

d. 90

90 =

e. 165

165 =

f. 95

95 =

3. Mark an “x” if the number is divisible by 2, 3, 4, 5, 6, or 9.

Divisible by	2	3	4	5	6	9
128						
765						

Divisible by	2	3	4	5	6	9
209						
6 042						

4. Find the prime factorisation of the numbers. Use a notebook for long divisions. Give each factorisation below the factor tree.

Note: in (a), the last two digits of 912 are “12” so it is **divisible by 4**.

a. 912
/ \
4 · ______

912 =

b. 528

528 =

c. 76

76 =

d. 126

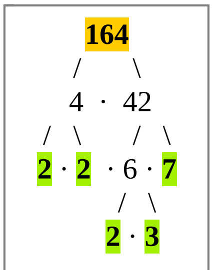
126 =

e. 272

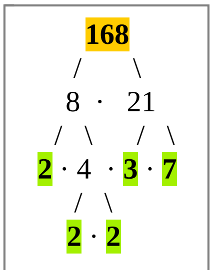
272 =

5. Mia and Alex found the prime factorisation of 164 and 168, and were completely surprised that they got the same factorization for both!

Investigate the situation. Is there something fishy going on somewhere?



$164 = 2^3 \cdot 3 \cdot 7$



$168 = 2^3 \cdot 3 \cdot 7$

6. Find all the primes between 100 and 110. How? You need to check, for each number, whether it is divisible by 2, 3, 4, 5, 6, 7, 8, 9, or 10.

7. Find the prime factorisation of these composite numbers.

a. 196	**b.** 380	**c.** 336
196 =	**380 =**	**336 =**
d. 306	**e.** 116	**f.** 720
306 =	**116 =**	**720 =**
g. 675	**h.** 990	**i.** 945
675 =	**990 =**	**945 =**

Puzzle Corner Find all the primes between 0 and 200. Use the sieve of Eratosthenes again (you need to make a grid in your notebook).

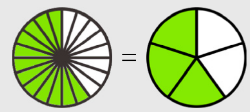
This time, you need to cross out 1, and then every even number except 2, every multiple of 3 except 3, every multiple of 5 except 5, every multiple of 7 except 7, every multiple of 11 except 11 and every multiple of 13 except 13.

Using Factoring When Simplifying Fractions

You have seen the process of **simplifying fractions** before.

In simplifying fractions, we divide both the numerator and the denominator by the same number. The fraction becomes *simpler*, which means that the numerator and the denominator are now *smaller* numbers than they were before.

However, this does NOT change the actual value of the fraction. It is the "same amount of pie" as it was before. It is just cut differently.



Every four slices have been joined together.

$$\frac{12}{20} \overset{\div 4}{=} \frac{3}{5} \quad (\div 4)$$

Why does this work?

It is based on finding common factors and on how fraction multiplication works. In our example above, the fraction $\frac{12}{20}$ can be written as $\frac{4 \cdot 3}{4 \cdot 5}$. Then we can **cancel out** those fours: $\frac{\cancel{4} \cdot 3}{\cancel{4} \cdot 5} = \frac{3}{5}$.

The reason this works is because $\frac{4 \cdot 3}{4 \cdot 5}$ is equal to the fraction multiplication $\frac{4}{4} \cdot \frac{3}{5}$. And in that, 4/4 is equal to 1, which means we are only left with 3/5.

Example 1. Often, the simplification is simply written or indicated this way →

Notice that here, the 4's that were cancelled out do ***not*** get indicated in any way! You only think it: "I divide 12 by 4, and get 3. I divide 20 by 4, and get 5."

$$\frac{\overset{3}{\cancel{12}}}{\underset{5}{\cancel{20}}} = \frac{3}{5}$$

Example 2. Here, 35 and 55 are both divisible by 5. This means we can cancel out those 5's, but notice this is not shown in any way. We simply cross out 35 and 55, think of dividing them by 5, and write the division result above and below.

$$\frac{\overset{7}{\cancel{35}}}{\underset{11}{\cancel{55}}} = \frac{7}{11}$$

1. Simplify the fractions, if possible.

a. $\frac{12}{36}$	**b.** $\frac{45}{55}$	**c.** $\frac{15}{23}$	**d.** $\frac{13}{6}$
e. $\frac{15}{21}$	**f.** $\frac{19}{15}$	**g.** $\frac{17}{24}$	**h.** $\frac{24}{30}$

2. Leah simplified various fractions like you see below. She did not get them right though.
   Explain to her what she is doing wrong.

$$\frac{24}{84} = \frac{20}{80} = \frac{1}{4} \qquad \frac{27}{60} = \frac{7}{40} \qquad \frac{14}{16} = \frac{10}{12} = \frac{6}{8} = \frac{3}{4}$$

Chapter 7: Fractions
Introduction

This chapter begins with a revision of fraction arithmetic from fifth grade—specifically, addition, subtraction, simplification, and multiplication of fractions. Then it focuses on division of fractions.

The introductory lesson on the division of fractions presents the concept of reciprocal numbers and ties the reciprocity relationship to the idea that division is the appropriate operation to solve questions of the form, "How many times does this number fit into that number?" For example, we can write a division from the question, "How many times does 1/3 fit into 1?" The answer is, obviously, 3 times. So we can write the division $1 \div (1/3) = 3$ and the multiplication $3 \cdot (1/3) = 1$. These two numbers, 3/1 and 1/3, are reciprocal numbers because their product is 1.

Students learn to solve questions like that through using visual models and writing division sentences that match them. Thinking of fitting the divisor into the dividend (measurement division) also gives us a tool to check whether the answer to a division problem is reasonable.

Naturally, the lessons also present the shortcut for fraction division—that each division can be changed into a multiplication by taking the reciprocal of the divisor, which is often called the "invert (flip)-and-multiply" rule. However, that "rule" is just a shortcut. It is necessary to memorise it, but memorising a shortcut doesn't help students make sense conceptually out of the division of fractions—they also need to study the concept of division and use visual models to better understand the process involved.

In two lessons that follow, students apply what they have learned to solve problems involving fractions or fractional parts. A lot of the problems in these lessons are revision in the sense that they involve previously learned concepts and are similar to problems students have solved earlier, but many involve the division of fractions, thus incorporating the new concept presented in this chapter.

Consider mixing the lessons from this chapter (or from some other chapter) with the lessons from the geometry chapter (which is a fairly long chapter). For example, the student could study these topics and geometry on alternate days, or study a little from both each day. Such, somewhat spiral, usage of the curriculum can help prevent boredom, and also to help students retain the concepts better.

Also, don't forget to use the resources for challenging problems:
https://l.mathmammoth.com/challengingproblems
I recommend that you at least use the first resource listed, Math Stars Newsletters.

The Lessons in Chapter 7

Revision: Add and Subtract Fractions and Mixed Numbers

Example 1. Add $\frac{5}{6} + 2\frac{5}{8}$.

We need to convert unlike fractions into equivalent fractions that have a common denominator before we can add them. The common denominator must be a **multiple of both 6 and 8** (a *common* multiple).

Naturally, $6 \cdot 8 = 48$ is one common multiple of 6 and 8. We could use 48. However, it is better to use 24, which is the *least* common multiple (LCM) of 6 and 8, because it leads to easier calculations.

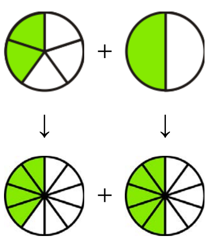
The common denominator is 24:

$$\frac{5}{6} + 2\frac{5}{8}$$

$$\downarrow \qquad \downarrow$$

$$\frac{20}{24} + 2\frac{15}{24} = 2\frac{35}{24} = 3\frac{11}{24}$$

1. Write the addition sentences.

a. 

$\frac{\square}{\square} + \frac{\square}{\square}$

$\downarrow \qquad \downarrow$

$\frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

b. $\frac{3}{4} + \frac{1}{9}$

$\downarrow \qquad \downarrow$

$\frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

c. $\frac{7}{10} + \frac{1}{4}$

$\downarrow \qquad \downarrow$

$\frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

2. Add and subtract. Use the common denominator you found in the previous exercise. Remember, the *best* possible choice for the common denominator (but not the only one) is the LCM of the denominators.

a. $\frac{5}{16} + \frac{1}{6}$

b. $3\frac{1}{12} + 1\frac{4}{9}$

c. $\frac{5}{6} - \frac{3}{8}$

d. $2\frac{5}{12} + \frac{4}{5}$

e. $5\frac{11}{15} - 2\frac{3}{20}$

f. $\frac{45}{100} + \frac{9}{20}$

Regroup in subtraction, if necessary.

Here we regroup **one** as 13/13. This leaves 9 wholes. There is already 1/13 in the column of the fractional parts, so in total we get 14/13.

$$\begin{array}{r} \overset{9}{\cancel{10}}\ \overset{14/13}{\cancel{\tfrac{1}{13}}} \\ -\ 5\ \tfrac{5}{13} \\ \hline 4\ \tfrac{9}{13} \end{array}$$

We can use the same idea (regrouping) when the fractions are written horizontally.

Take one of the 7 wholes, think of it as 9/9, and regroup that with the fractional parts (with 2/9). Instead of 7 wholes, we are left with 6, and instead of 2/9, we get 11/9.

$$7\tfrac{2}{9} - 3\tfrac{8}{9}$$
$$\downarrow \quad \downarrow$$
$$6\tfrac{11}{9} - 3\tfrac{8}{9} = 3\tfrac{3}{9}$$

3. Subtract.

a. $\begin{array}{r} 7\ \tfrac{3}{9} \\ -\ 2\ \tfrac{7}{9} \\ \hline \end{array}$	**b.** $\begin{array}{r} 18\ \tfrac{1}{10} \\ -\ 5\ \tfrac{9}{10} \\ \hline \end{array}$	**c.** $\begin{array}{r} 10\ \tfrac{1}{15} \\ -\ 3\ \tfrac{8}{15} \\ \hline \end{array}$	**d.** $16\tfrac{3}{9} - 9\tfrac{8}{9}$ **e.** $7\tfrac{3}{14} - 2\tfrac{10}{14}$

4. Subtract. First write equivalent fractions with the same denominator.

a. $\begin{array}{r} 3\ \tfrac{3}{4} \\ -\ 1\ \tfrac{1}{6} \\ \hline \end{array} \begin{array}{c} \rightarrow \\ \rightarrow \end{array} \begin{array}{r} 3\ \tfrac{\square}{\square} \\ -\ 1\ \tfrac{\square}{\square} \\ \hline \end{array}$	**b.** $\begin{array}{r} 3\ \tfrac{3}{8} \\ -\ 1\ \tfrac{5}{12} \\ \hline \end{array} \begin{array}{c} \rightarrow \\ \rightarrow \end{array} \begin{array}{r} \\ - \qquad \\ \hline \end{array}$	**c.** $\begin{array}{r} 8\ \tfrac{9}{11} \\ -\ 5\ \tfrac{1}{2} \\ \hline \end{array} \begin{array}{c} \rightarrow \\ \rightarrow \end{array} \begin{array}{r} \\ - \qquad \\ \hline \end{array}$

5. Figure out and explain how these subtractions were done!

<u>**Emma's way:**</u> $9\tfrac{2}{17} - 3\tfrac{8}{17}$

$$= (9 - 3) + \left(\tfrac{2}{17} - \tfrac{8}{17}\right) = 6 - \tfrac{6}{17} = 5\tfrac{11}{17}$$

<u>**Joe's method:**</u> $5\tfrac{3}{14} - 2\tfrac{9}{14}$

$$\downarrow$$

$$5\tfrac{3}{14} - 2\tfrac{3}{14} - \tfrac{6}{14}$$

$$= 3 - \tfrac{6}{14} = 2\tfrac{8}{14}$$

When adding or subtracting three ore more fractions, find a common denominator for all of them. You can always use the product of the denominators as your common denominator. However, it *may be* more efficient to use the LCM of the denominators if it is smaller.

Example 2. Here, we *could* use 6 · 7 · 2 = 84 as a common denominator.

However, in this case, the LCM of 6, 7 and 2 is 42, so it is better (leads to easier calculations) than using 84.

Another option would be to add the first two fractions (5/6 and 5/7) to get 65/42, and then to subtract the third fraction, 1/2, from that result.

$$\frac{5}{6} + \frac{5}{7} - \frac{1}{2}$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$\frac{35}{42} + \frac{30}{42} - \frac{21}{42} = \frac{44}{42} = 1\,\frac{1}{21}$$

6. Add or subtract the fractions.

a. $\frac{5}{12} + \frac{1}{6} + \frac{1}{3}$	**b.** $\frac{2}{7} + \frac{1}{2} - \frac{1}{4}$
c. $\frac{1}{10} + \frac{2}{5} + \frac{1}{3}$	**d.** $\frac{19}{20} - \frac{1}{3} - \frac{1}{4}$
e. $\frac{7}{8} - \frac{1}{5} + \frac{2}{3}$	**f.** $\frac{7}{6} - \frac{3}{5} + \frac{3}{4}$

7. Joe started working at an automobile company 23 ½ years ago. However, during that time, he has taken ¼ of a year off for paternity leave, and spent another 1 ⅓ years laid off due to a recession. So, how long has he actually been working for the company?

While you can often compare two fractions using mental math strategies, sometimes the fractions are so close to each other that you need to rewrite both using a common denominator, then compare.

Example 3. Which is more, $\frac{7}{8}$ or $\frac{11}{13}$?

Let's write both using the common denominator 104 (on the right):

We see that 7/8 is more.

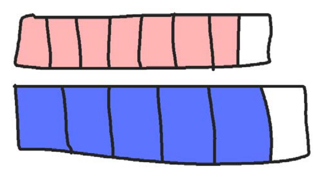
$$\frac{7}{8} \quad \frac{11}{13}$$
$$\downarrow \quad \downarrow$$
$$\frac{91}{104} > \frac{88}{104}$$

8. Compare the fractions, writing < or > between them. Use a common denominator only if you need to.

a. $\frac{1}{2}$ ☐ $\frac{5}{9}$	**b.** $\frac{15}{65}$ ☐ $\frac{15}{34}$	**c.** $\frac{6}{15}$ ☐ $\frac{1}{2}$	**d.** $\frac{1}{120}$ ☐ $\frac{1}{75}$
e. $\frac{3}{5}$ ☐ $\frac{8}{13}$ ↓ ↓ $\frac{\_\_}{\_\_}$ ☐ $\frac{\_\_}{\_\_}$	**f.** $\frac{2}{3}$ ☐ $\frac{8}{11}$ ↓ ↓ $\frac{\_\_}{\_\_}$ ☐ $\frac{\_\_}{\_\_}$	**g.** $\frac{11}{15}$ ☐ $\frac{3}{4}$ ↓ ↓ $\frac{\_\_}{\_\_}$ ☐ $\frac{\_\_}{\_\_}$	**h.** $\frac{10}{2\,000}$ ☐ $\frac{2}{1\,000}$ ↓ ↓ $\frac{\_\_}{\_\_}$ ☐ $\frac{\_\_}{\_\_}$

9. Julie is convinced that 5/6 is more than 7/8 — she even sketched a picture where it looks like it is so.

How would you convince (prove to) her otherwise?



10. Order the fractions from the smallest to the biggest.

a. $\frac{1}{4}, \frac{1}{2}, \frac{3}{8}, \frac{3}{7}$ ____ < ____ < ____ < ____ < ____	**b.** $\frac{2}{3}, \frac{7}{5}, \frac{5}{4}, \frac{3}{2}$ ____ < ____ < ____ < ____ < ____
c. $\frac{2}{3}, \frac{8}{5}, \frac{3}{5}, \frac{3}{4}, \frac{5}{4}$ ____ < ____ < ____ < ____ < ____	**d.** $\frac{5}{6}, \frac{7}{12}, \frac{5}{8}, \frac{2}{9}, \frac{6}{5}$ ____ < ____ < ____ < ____ < ____

Puzzle Corner

Solve the equations. *Hint:* if the fractions confuse you, *first* think how the equation would be solved if it had whole numbers. Then solve the original equation the same way.

a. $8\frac{4}{7} + x = 10\frac{2}{5}$	**b.** $5\frac{1}{9} - x = 2\frac{1}{3}$

Add and Subtract Fractions: More Practice

These exercises simply give you more practice on adding and subtracting fractions and mixed numbers. Use them as directed by your teacher.

1. Add or subtract. Give your answer in lowest terms, and as a mixed number, if applicable.

a. $\frac{17}{18} + \frac{2}{9}$	**b.** $\frac{11}{30} + \frac{7}{12}$	**c.** $\frac{13}{22} + \frac{3}{4}$
d. $6\frac{7}{10} - 1\frac{3}{20}$	**e.** $4\frac{7}{8} - 1\frac{1}{3}$	**f.** $15\frac{9}{10} - 3\frac{31}{100}$

2. Subtract. First write equivalent fractions with the same denominator.

a. $5\frac{1}{2} - 1\frac{7}{12}$ → $5\frac{\square}{\square} - 1\frac{\square}{\square}$	**b.** $12\frac{1}{9} - 5\frac{2}{3}$	**c.** $33\frac{1}{3} - 17\frac{6}{7}$
d. $8\frac{1}{9} - 2\frac{7}{12}$	**e.** $86\frac{6}{7} - 45\frac{1}{8}$	**f.** $53\frac{1}{6} - 40\frac{6}{7}$

Chapter 8: Integers
Introduction

In chapter 8, students are introduced to integers, the coordinate plane in all four quadrants and integer addition and subtraction. The multiplication and division of integers will be studied next year.

Integers are introduced using the number line to relate them to the concepts of temperature, elevation and money. We also study briefly the ideas of absolute value (an integer's distance from zero) and the opposite of a number.

Next, students learn to locate points in all four quadrants and how the coordinates of a figure change when it is reflected across the x or y-axis. Students also move points according to given instructions and find distances between points with the same first coordinate or the same second coordinate.

Adding and subtracting integers is presented through two main models: (1) movements along the number line and (2) positive and negative counters. With the help of these models, students should not only learn the shortcuts, or "rules", for adding and subtracting integers, but also understand *why* these shortcuts work.

A lesson about subtracting integers explains the shortcut for subtracting a negative integer from three different viewpoints (as a manipulation of counters, as movements on a number-line and as a distance or difference). There is also a roundup lesson for addition and subtraction of integers.

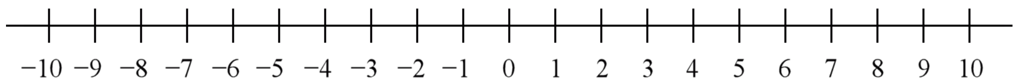
The last topic in this chapter is graphing. Students will plot points on the coordinate grid according to a given equation in two variables (such as $y = x + 2$), this time using also negative numbers. They will notice the patterns in the coordinates of the points and the pattern in the points drawn in the grid and also work through some real-life problems.

You will find free videos covering many topics of this chapter at **https://www.mathmammoth.com/videos/** (choose 6th grade).

The Lessons in Chapter 8

Integers

When we continue the number-line towards the left from zero, we come to the **negative numbers**.



The **negative whole numbers** are −1, −2, −3, −4 and so on.
The **positive whole numbers** are 1, 2, 3, 4 and so on. You can also write them as +1, +2, +3, *etc*.
Zero is neither positive nor negative.
All of the negative and positive whole numbers and zero are called **integers**.

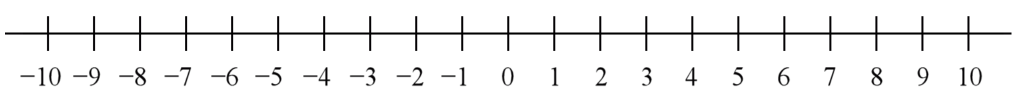
Read −1 as "negative one" and −5 as "negative five". Some people read −5 as "minus five".
That is very common, and it is not wrong, but be sure that you do not confuse it with subtraction.

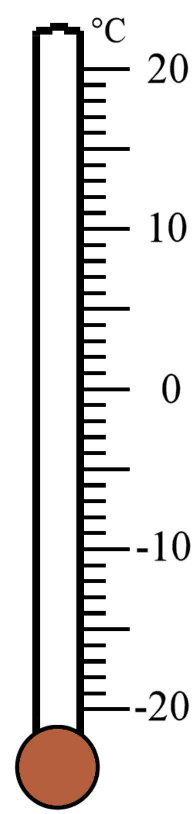
Put a "−" sign in front of negative numbers. This sign can also be elevated: ⁻5 is the same as −5.

Often, we need to put brackets around negative numbers in order to avoid confusion with other symbols. Therefore, ⁻5, −5 and (−5) all mean "negative five".

Negative numbers are commonly used with temperature. They are also used to express debt. If you owe $5, you write that as −$5. Another use is with elevation below sea level. For example, just as 200 m can mean an elevation of 200 metres above sea level, −100 m would mean 100 metres *below* sea level.

1. Plot the integers on the number-line.   **a.** −7   **b.** +6   **c.** −4   **d.** −2





2. Write an integer appropriate to each situation.

   a. Daniel owes $23.

   b. Mary earned $250.

   c. The aeroplane flew at the altitude of 8 800 metres.

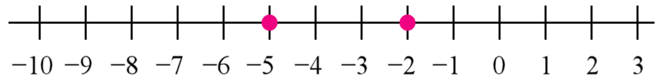
   d. The temperature in the freezer is 18 degrees Celsius below zero.

   e. A dolphin dove 9 m below sea level.

3. The temperature changed from what it was before. Find the new temperature.
   You can draw the mercury on the thermometer to help you.

before	1°C	2°C	−2°C	−4°C	−12°C	−8°C
change	drops 3°C	drops 7°C	drops 1°C	rises 5°C	rises 4°C	rises 3°C
now						

Which is more, −5 or −2?

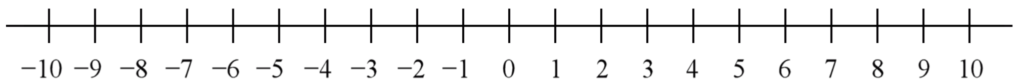


Which is *warmer*, −5°C or −2°C? Clearly −2°C is. Temperatures just get colder and colder the more you move towards the negative numbers. We can write a comparison: −2°C > −5°C.

Which is the *better* money situation, to have −$5 (owe $5) or to have −$2 (owe $2)? Clearly, it is better to owe only $2 because you can pay that off easier. We can write: −$5 < −$2.

Which is the *higher* elevation, −5 m or −2 m? Of course, 2 m below sea level, or −2m, is higher.

On the number line, the number that is ***farther to the right*** is the **greater** number. So, −5 < −2.



4. Compare. Write **<** or **>** between the numbers. You can plot the integers on the number line to help you.

a. −2 □ −3	**b.** 8 □ −8	**c.** −3 □ 0	**d.** 4 □ −3	**e.** −5 □ −9
f. −10 □ −30	**g.** −4 □ 1	**h.** 0 □ −13	**i.** −2 □ −7	**j.** −11 □ −14

5. You can use the number line to help you. Which integer is ...

a. 2 more than −4 **b.** 5 more than −3 **c.** 3 less than 1 **d.** 6 more than −11

6. Find the number that is 5 less than ... **a.** 0 **b.** −3 **c.** 3

7. Express the situations using integers. Then write > or < to compare them.

a. Shelly owes $10 and Mary owes $8.

b. One fish was swimming 3 m below the surface of the water, and another fish was swimming 4 m below the surface of the water.

c. The temperature this morning was 10°C below zero. Now it is 6°C below zero.

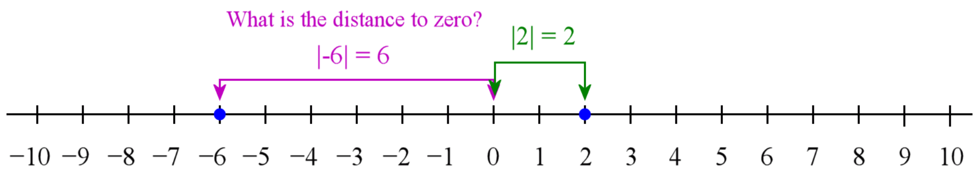
d. Henry has $5. Emma owes $5.

e. The temperature during the day was 10°C but at night it was 2°C below zero.

8. Write the numbers in order from the least to the greatest.

a. −2 0 −4 4	**b.** −3 −6 5 3
c. −20 −10 −14 −9	**d.** −3 0 −6 −8

The **absolute value** of a number is its distance from zero.



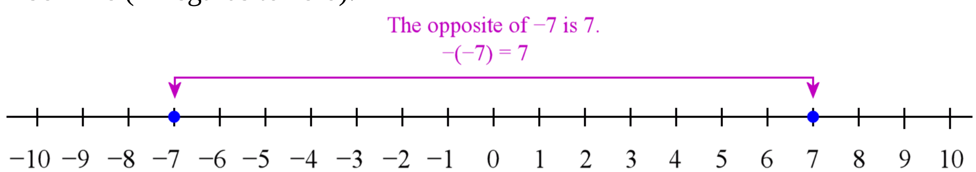
We denote the absolute value of a number using vertical bars around the number.

So, | −4 | means "the absolute value of 4", which is 4. Similarly, | 87 | = 87.

9. Find the absolute values of these numbers.

a. | −5 | **b.** | −12 | **c.** | 7 | **d.** | 0 | **e.** | 68 |

The **opposite** of a number is the number that is at the same distance from zero as it is, but on the *opposite* side of the number-line (in regards to zero).



We denote the opposite of a number using the minus sign. For example, −4 means the opposite of 4, which is −4. Or, −(−2) means the opposite of negative two, which is 2.

The opposite of zero is zero itself. In symbols, −0 = 0.

"But wait," you might ask, "doesn't −4 mean negative four, not the 'opposite of four'?"

It can mean either. Sometimes the context will help you to differentiate between the two (to tell which is which). Other times it's unnecessary to differentiate because, after all, the opposite of four is negative four: −4 = −4. ☺

So there are three different meanings for the minus sign:

1. To indicate subtraction: 7 − 2 = 5.
2. To indicate negative numbers: "negative 7" is written −7.
3. To indicate the opposite of a number: −(−14) is the opposite of negative 14.

10. Think of the minus sign as signifying "the opposite of". Simplify.

a. −5 **b.** −(−9) **c.** −10 **d.** −0 **e.** −(−100)

11. Write using mathematical symbols, and simplify (solve) if possible.

a. the opposite of 6

b. the opposite of the absolute value of 6

c. the absolute value of negative 6

d. the absolute value of the opposite of 6

e. the opposite of −6.

f. the absolute value of the opposite of −6

Coordinate Grid

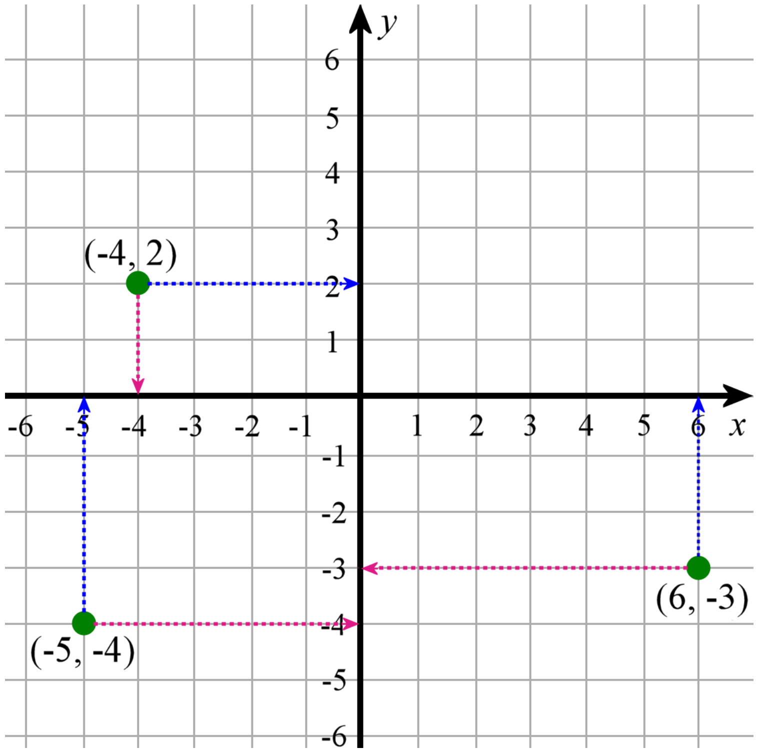
This is the *coordinate grid* or *coordinate plane*. We have extended the *x*-axis and the *y*-axis to include negative numbers now. The axes cross each other at the *origin*, or the point (0, 0).

The axes divide the coordinate plane into four parts, called *quadrants*. Previously you have worked in only the so-called first quadrant, but now we will use all four quadrants.

The coordinates of a point are found in the same manner as before. Draw a vertical line (either up or down) from the point towards the *x*-axis. Where this line crosses the *x*-axis tells you the point's *x*-coordinate.

Similarly, draw a horizontal line (either right or left) from the point towards the *y*-axis. Where this line crosses the y-axis tells you the point's *y*-coordinate.

We list first the point's *x*-coordinate and then the *y*-coordinate. Look at the examples in the picture.



1. Write the *x*- and *y*-coordinates of the points.

A (____ , ____)

B (____ , ____)

C (____ , ____)

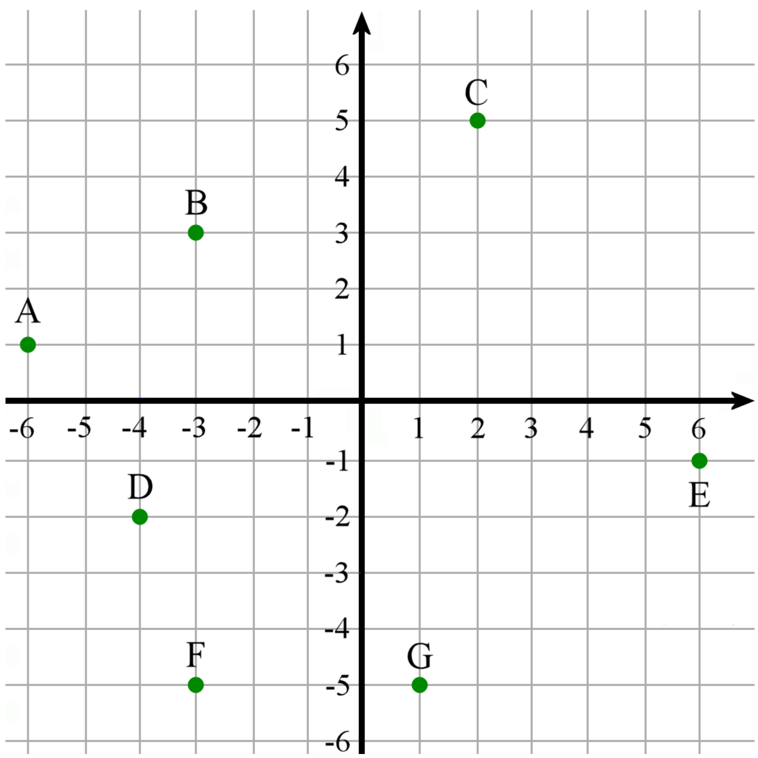
D (____ , ____)

E (____ , ____)

F (____ , ____)

G (____ , ____)

Self-check: Add the *x*-coordinates of all points. You should get −7.



Chapter 9: Geometry
Introduction

The main topics in this chapter include:

- area of triangles
- area of polygons
- nets and surface area of prisms and pyramids
- volume of rectangular prisms with sides of fractional length

However, the chapter starts out with some revision of topics from earlier grades, as we revise the different types of quadrilaterals and triangles and students do some basic drawing exercises. In these drawing problems, students will need a ruler to measure lengths and a protractor to measure angles.

One focus of the chapter is the area of polygons. To reach this goal, we follow a step-by-step development. First, we study how to find the area of a right triangle, which is very easy, as a right triangle is always half of a rectangle. Next, we build on the idea that the area of a parallelogram is the same as the area of the related rectangle, and from that we develop the usual formula for the area of a parallelogram as the product of its base times its height. This formula then gives us a way to generalise finding the area of any triangle as *half* of the area of the corresponding parallelogram.

Finally, the area of a polygon can be determined by dividing it into triangles and rectangles, finding the areas of those and summing them. Students also practise their new skills in the context of a coordinate grid. They draw polygons in the coordinate plane and find the lengths of their sides, perimeters and areas.

Nets and surface area is another major topic. Students draw nets and determine the surface area of prisms and pyramids using nets. They also learn how to convert between different area units, not using conversion factors or formulas, but using logical reasoning where they learn to determine those conversion factors themselves.

Lastly, we study the volume of rectangular prisms, this time with edges of fractional length. (Students have already studied this topic in fifth grade with edges that are a whole number long.) The basic idea is to prove that the volume of a rectangular prism *can* be calculated by multiplying its edge lengths even when the edges have fractional lengths. To that end, students need to think how many little cubes with edges ½ or ⅓ unit go into a larger prism. Once we have established the formula for volume, students solve some problems concerning the volume of rectangular prisms.

There are quite a few videos available to match the lessons in this chapter at
https://www.mathmammoth.com/videos/ (choose 6th grade).

Also, don't forget to use the resources for challenging problems:
https://l.mathmammoth.com/challengingproblems

I recommend that you at least use the first resource listed, Math Stars Newsletters.

The Lessons in Chapter 9

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better, and even just for some fun. Enjoy!

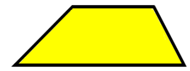
https://l.mathmammoth.com/gr6ch9

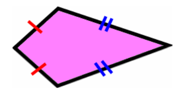
SCAN ME

Quadrilaterals Revision

Revision the definitions of various quadrilaterals below.

- A **rectangle** has four right angles.
- A **square** is a rectangle with four congruent sides.
- A **trapezium** has *at least* one pair of parallel sides. It may have two!
- A **parallelogram** has two pairs of parallel sides.
- A **rhombus** is a parallelogram that has four congruent sides (a diamond).
- A **kite** has two pairs of congruent sides that touch each other. *The single tick marks show the one pair of congruent sides, and the double tick marks show the other pair.*
- In a **scalene** quadrilateral, all sides are of different lengths (no two sides are congruent).



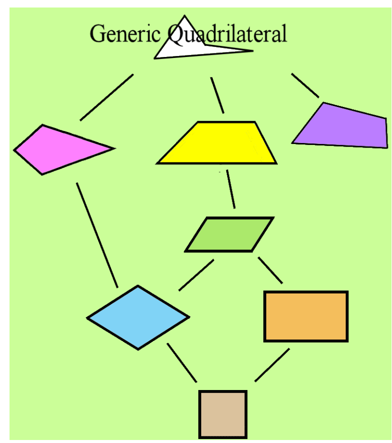


The chart shows the seven different types of quadrilaterals as a "family," descending from the generic quadrilateral at the top.

If a quadrilateral is listed under another, it means the two have like a "parent-child" relationship: the quadrilateral listed lower (the child) has its parent's characteristics.

Number in the chart the following types of quadrilaterals:

1. rhombus	5. scalene quadrilateral
2. kite	6. trapezium
3. rectangle	7. parallelogram
4. square	



1. Match each description to a name of a quadrilateral.

- All of the sides measure 6 cm; angles measure 50°, 130°, 50° and 130°.
- Two of the sides measure 12 cm and two measure 8 cm; angles measure 60°, 120°, 60° and 120°; opposite sides are parallel.
- None of the sides are congruent.
- The sides, listed in order, measure 70 cm, 70 cm, 120 cm and 120 cm.
- The sides measure 20 cm, 35 cm, 20 cm, and 55 cm. The two longest sides are parallel.
- All of the sides measure 16 cm; the angles all measure 90°.

square

scalene quadrilateral

trapezium

rhombus

kite

parallelogram

2. Find the correct type of quadrilateral for each definition.

a. A quadrilateral with four congruent sides (but we know nothing about its angles).

b. A quadrilateral where the opposite sides are parallel (that is all we know about it!).

c. A quadrilateral with one pair of parallel sides.

d. A quadrilateral with two pairs of congruent sides, where the two congruent sides are touching (adjacent), and also the other two congruent sides are touching.

Triangles Revision

Remember, we can classify a triangle both according to its angles and according to its sides.	
According to angles: • A **right triangle** has one right angle. • An **obtuse triangle** has one obtuse angle. • An **acute triangle** has three acute angles.	According to sides: • An **equilateral triangle** has three congruent sides. • An **isosceles triangle** has two congruent sides. • In a **scalene triangle**, none of the sides are congruent.

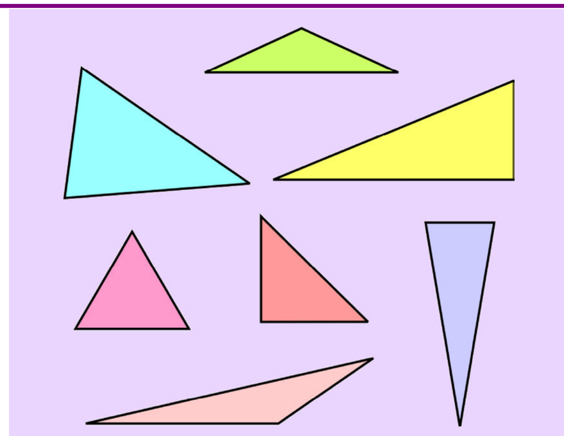
1. Match the classifications and the triangles.

obtuse scalene triangle

right scalene triangle

acute scalene triangle

obtuse isosceles triangle



right isosceles triangle

acute isosceles triangle

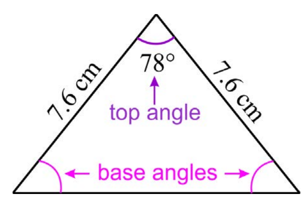
equilateral triangle

2. Which are impossible?

isosceles scalene triangle *isosceles acute triangle* *isosceles obtuse triangle*

scalene right triangle *equilateral obtuse triangle*

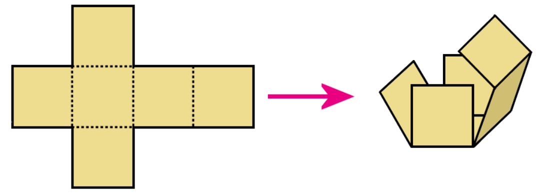
3. **a.** Draw an isosceles triangle with a 78° top angle and two 7.6-cm sides.
Start out by drawing the 78° angle, then measure the two 7.6-cm sides.



b. Measure the base angles of your triangle. They should be congruent; however, it is very hard to draw so accurately that you would get the same angle measurements for the base angles, so don't worry if they differ a little bit.

Nets and Surface Area 1

This picture shows a flat figure, called a **net**, that can be folded up to form a solid, in this case a cube.

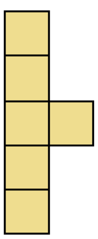


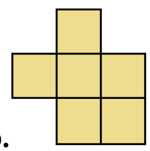
Each face of a cube is a square. If we find the total area of its faces, we will have found the **surface area** of the cube.

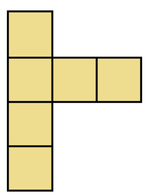
Let’s say that each edge of this cube measures 2 cm. Then one face would have an area of 2 cm · 2 cm = 4 cm^2, and the total surface area of the six faces of the cube would be 6 · 4 cm^2 = 24 cm^2.

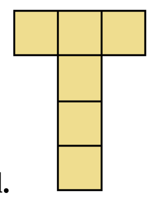
What is its volume? Remember, **volume** has to do with how much space a figure takes up, and not with “flat” area. Volume is measured in *cubic* units, whereas area is measured in *square* units. The volume of this cube is 2 cm · 2 cm · 2 cm = $(2\text{ cm})^3$ = 8 cm^3.

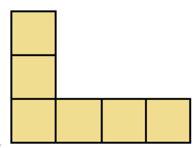
1. Which of these patterns are nets of a cube? In other words, which ones can be folded into a cube? You can copy the patterns on paper, cut them out and fold them.

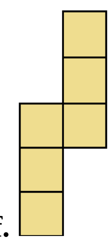
a. 

b. 

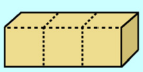
c. 

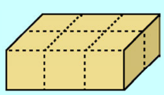
d. 

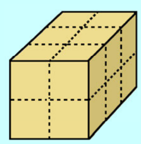
e. 

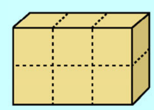
f. 

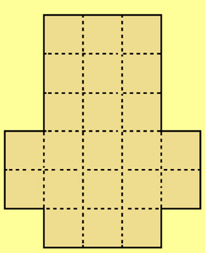
2. Match each rectangular prism (a), (b), (c) and (d) with the correct net (1), (2), (3) and (4). Again, if you would like, you can copy the nets onto paper, cut them out, and fold them.

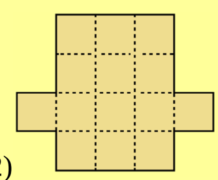
a. 

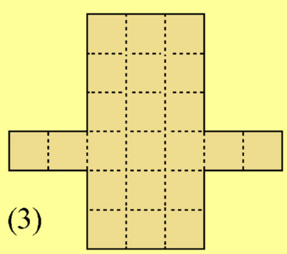
b. 

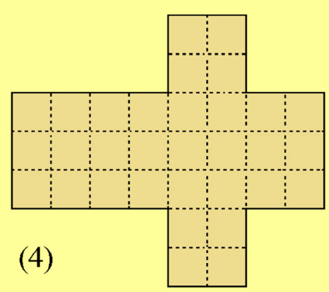
c. 

d. 

(1) 

(2) 

(3) 

(4) 

3. Find the surface area (A) and volume (V) of each rectangular prism in problem #2 if the edges of the little cubes are 1 cm long.

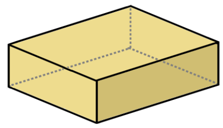
a. A = ________ cm^2 **b.** A =________ cm^2 **c.** A =________ cm^2 **d.** A =________ cm^2

V = ________ cm^3 V = ________ cm^3 V = ________ cm^3 V = ________ cm^3

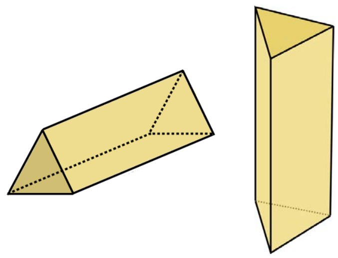
A **prism** has two identical polygons as its top and bottom faces. These polygons are called the *bases* of the prism. The bases are connected with faces that are parallelograms (and often rectangles).

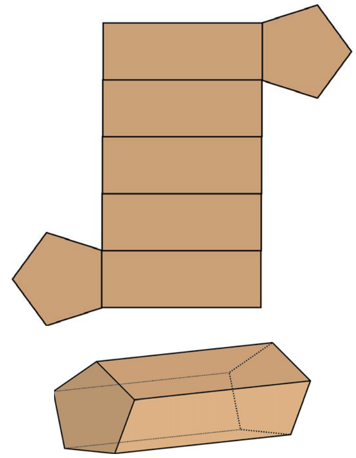
Prisms are named after the polygon used as the bases.

A rectangular prism.
The bases are rectangles.



Two **triangular prisms**.
One is lying down, where the base is facing you.
The other is “standing up”.

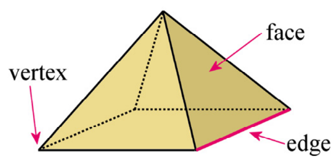




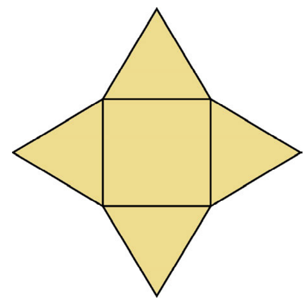
A pentagonal prism and its net. The bases are pentagons. Again, the base is not “on the bottom” but facing you.

A **pyramid** has a polygon as its bottom face (the base), and triangles as other faces.

Pyramids are named after the polygon at the base: a triangular pyramid, square pyramid, rectangular pyramid, pentagonal pyramid, and so on.



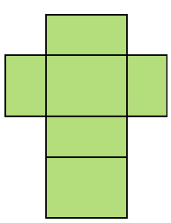
A **square pyramid** and its net.



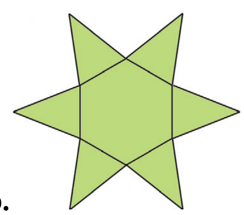
See interactive solids and their nets at the link below:
https://www.mathsisfun.com/geometry/polyhedron-models.html

4. Name the solid that can be built from each net.

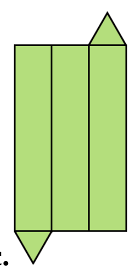
a.



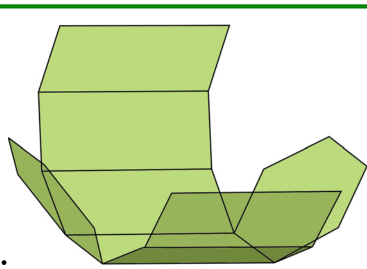
b.



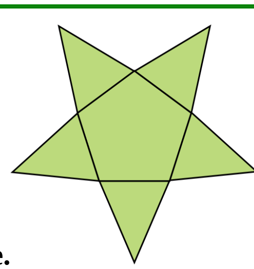
c.



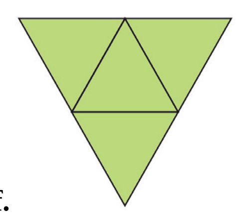
d.



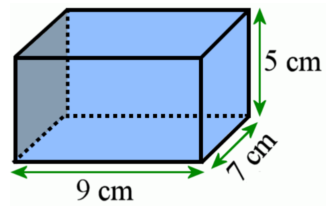
e.



f.



5. Which expression, (1), (2), or (3), can be used to calculate the surface area of this prism correctly? (You do *not* have to actually calculate the surface area.)

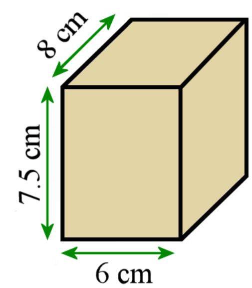


1. $2 \cdot 35\text{ cm}^2 + 2 \cdot 63\text{ cm}^2 + 2 \cdot 45\text{ cm}^2$

2. 5 cm · 9 cm · 7 cm

3. 5 cm · 7 cm + 9 cm · 7 cm + 9 cm · 5 cm

6. Ryan organised the calculation of the surface area of this prism into three parts. Write down the intermediate calculations, and solve. This way, your teacher (or others) can follow your work. Remember also to include the units (cm or cm^2)!



Top and bottom:

Back and front:

The two sides:

Total:

7. The surface area of a cube is 96 square centimetres.

a. What is the area of one face of the cube?

b. How long is each edge of the cube?

c. Find the volume of the cube.

Puzzle Corner

Consider the rectangular prisms in problem #2. If the edges of the little cubes were double as long, how would that affect the surface area? Volume?

You can use the table below to investigate the situation.

Prism a.	Prism b.	Prism c.	Prism d.
A = ________ cm^2	A =________ cm^2	A =________ cm^2	A =________ cm^2
V = ________ cm^3	V = ________ cm^3	V = ________ cm^3	V = ________ cm^3

Chapter 10: Statistics
Introduction

The fundamental theme in our study of statistics is the concept of *distribution*. In the first lesson, students learn what a distribution is—basically, it is *how* the data is distributed. The distribution can be described by its centre, spread and overall shape. The shape is read from a graph, such as a dot plot or a bar graph.

Two major concepts when summarising and analysing distributions are its centre and its variability. First we study the centre, in the lessons about mean, median and mode. Students not only learn to calculate these values, but also relate the choice of measures of centre to the shape of the data distribution and the type of data.

In the lesson *Measures of Variation* we study range, interquartile range and mean absolute deviation. The last one takes many calculations, and the lesson gives instructions on how to calculate it using a spreadsheet program, such as Excel.

Then in the next lessons, students learn to make several different kinds of graphs: histograms, boxplots and stem-and-leaf plots. In those lessons, students continue summarising distributions by giving their shape, a measure of centre and a measure of variability.

There are some videos available for these topics at **https://www.mathmammoth.com/videos/** (choose 6th grade).

The Lessons in Chapter 10

Understanding Distributions

A **statistical question** is a question where we expect a range of *variability* in the answers to the question.

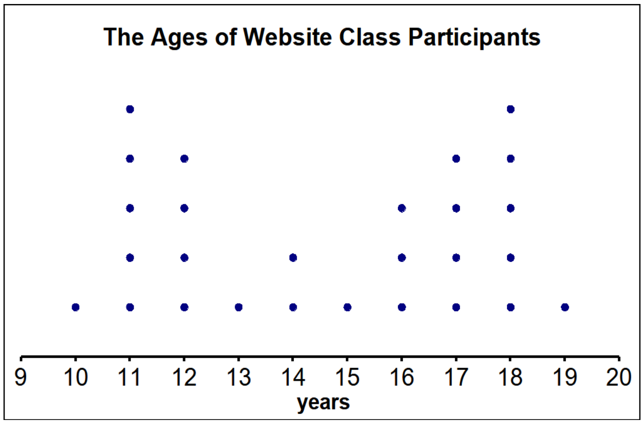
For example, "How old am I?" is *not* a statistical question (there is only one answer), but "How old are the students in my school?" *is* a statistical question because we expect the students' ages not to be all the same.

"How much does this TV cost?" is *not* a statistical question because we expect there to be just one answer.

"How much does this TV cost in various stores around town?" *is* a statistical question, because we expect a number of different answers: the prices in different stores will vary.

To answer a statistical question we collect a set of **data** (many answers). The data can be displayed in some kind of a graph, such as a bar graph, a histogram, or a dot plot.

This is a **dot plot** showing the ages of the participants in a website-building class. Each dot in the plot signifies one observation. For example, we can see there was one 13-year old and two 14-year olds in the class.



The dot plot shows us the **distribution** of the data: it shows how many times (the frequency) each particular value (age in this case) occurs in the data.

This distribution is actually **bimodal,** or "double-peaked". This means it has two "centres": one around 11-12 years, and another around 17-18 years.

1. Are these statistical questions? If not, change the question so that it becomes a statistical question.

a. What colour are my teacher's eyes?

b. How much money do the students in this university spend for lunch?

c. How much money do working adults in Romania earn?

d. How many children in the United States use a cell phone regularly?

e. What is the minimum wage in Ohio?

f. How many sunny days were there in August, 2020, in London?

g. How many pets does my friend have?

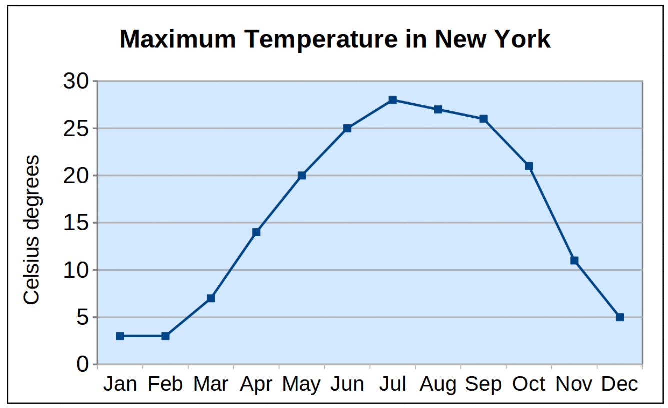
2. Is this graph based on a statistical question?

Why or why not?

3. The line graph shows the maximum temperatures in New York for each month of a certain year.

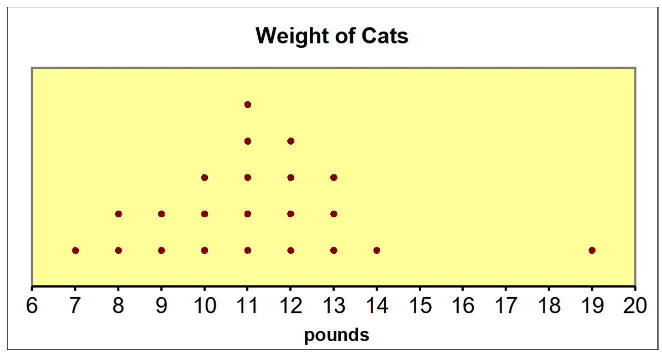
Is this graph based on a statistical question?

Why or why not?



4. The title of this dot plot is not the best. But, could the plot be based on a statistical question?

If yes, give it a better, more specific, title. Imagine what situation and what question might have produced the data.



5. Change each question from a non-statistical question to a statistical question, and vice versa.

a. What shampoo do you use?

b. How cold was it yesterday where you live?

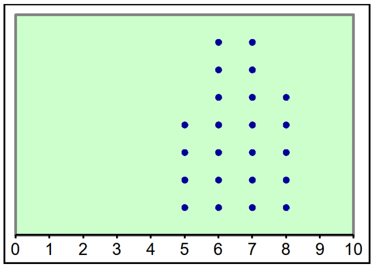
c. How old are people in Germany when they marry (the first time)?

d. How long does it take for our company’s packages to reach the customers?

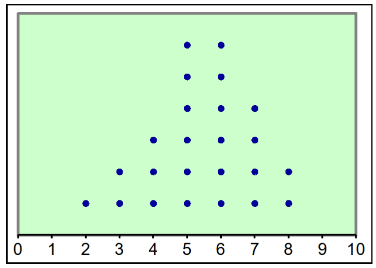
We are often interested in the **centre**, **spread** and **overall shape** of the distribution. Those three things can summarize for us what is important about the distribution.

The **centre** of a distribution has to do with where its peak is. We can use mean, median and mode to characterise the central tendency of a distribution. We will study those in detail in the next lesson.

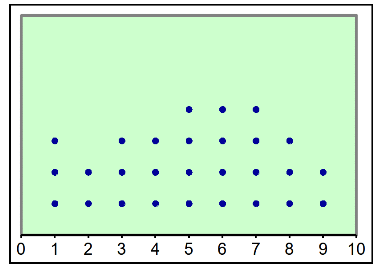
These three dot plots show how the **spread** of a distribution can vary. This means how the data items themselves are spread — whether they are "spread" all over, or tightly concentrated near some value, or somewhat concentrated around some value. We will study more about spread in another lesson.



little spread

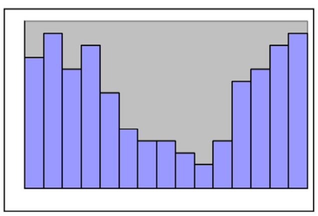


medium spread

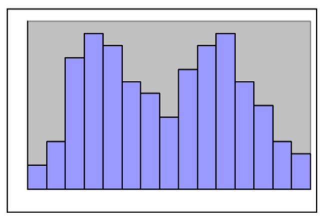


large spread

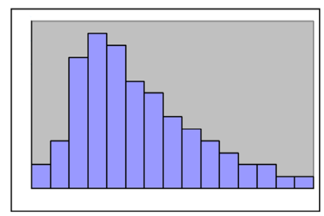
The distribution can have many varying overall **shapes.** For example:



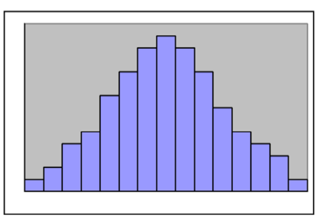
U-shaped



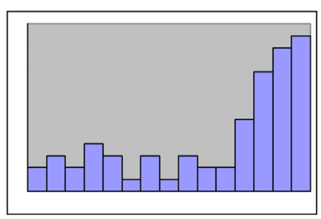
double-peaked (bimodal)



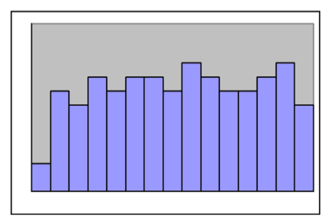
asymmetrical, right-tailed
(a.k.a. right-skewed)



bell-shaped



J-shaped
(can also be mirrored where
most of the values are at the left)

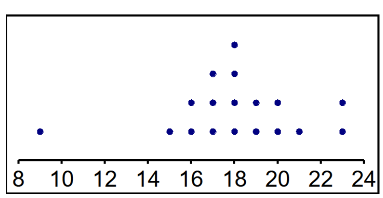


rectangular

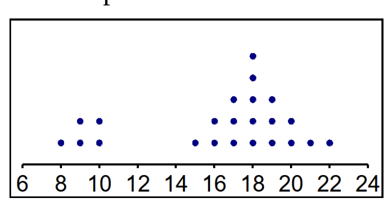
In addition to its overall shape, a distribution may have a gap, an outlier, or a cluster:

This distribution has a **gap** from 19 to 22:

In this distribution, 9 is an **outlier —** a data item whose value is considerably less or more than all the others.

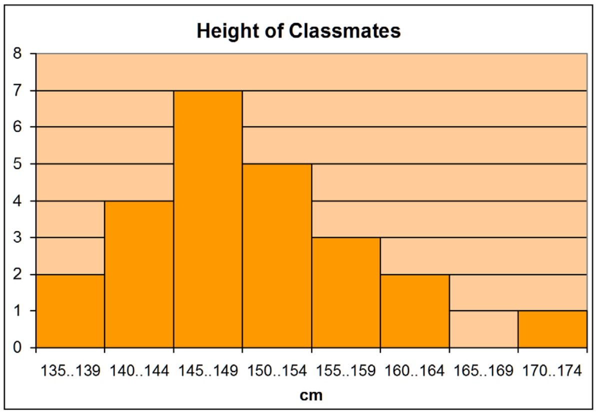


This distribution has a bell shape overall (with a peak at 18), but also a **cluster** or a smaller peak at 8-10.



6. Anne asked her classmates the question, “How tall are you?” The histogram shows the distribution of her data.

a. Describe the overall shape of the distribution, and also include if there are any striking deviations from the overall pattern (gaps, outliers, or clusters).



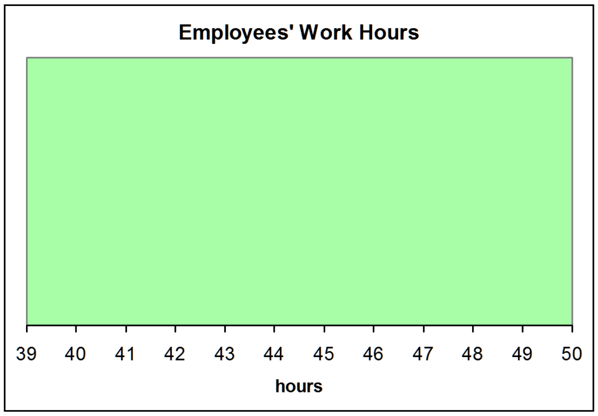
b. Where is the peak of the distribution?

c. How many observations are there?

7. Make a dot plot from this data (weekly work hours of a restaurant’s employees). You need to place a dot for each observation.

48	45	46	41	42	42	43	43	42	42	41
41	45	49	40	41	41	42	46	47	42	40

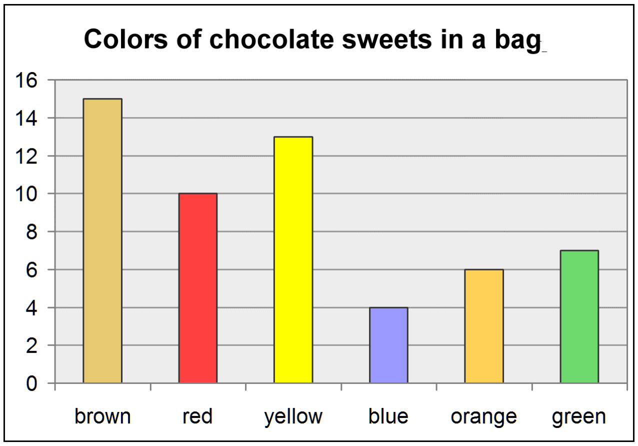
a. Describe the overall shape of the distribution. Also include if there are any striking deviations from the overall pattern (gaps, outliers, or clusters).



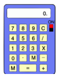
b. Where is the peak of the distribution?

c. How many observations are there?

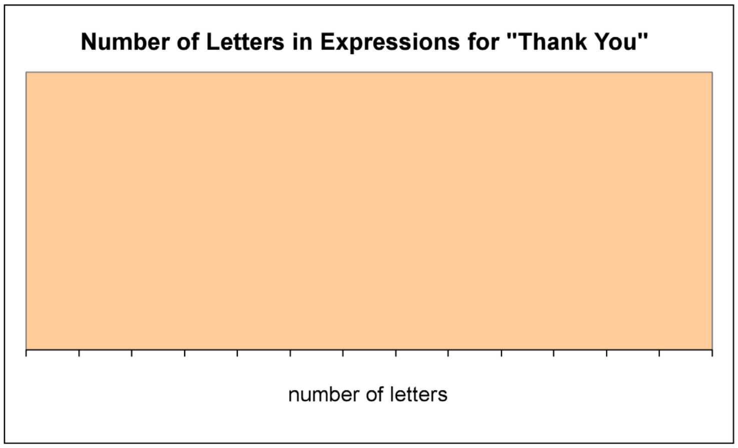
8. **a.** Does this graph show a statistical distribution? Why or why not?



b. Calculate what percentage of the candies are red and what percentage are green.



9. First, count the number of letters in these expressions for "Thank You" from various languages and fill in the empty column in the table. Next, label the number line below the dot plot so that all of the data will fit. Finally, plot the data.



Language	Spelling	Number of letters
Africaans	dankee	
Arabic	shukran	
Chinese, Cantanese	do jeh	
Chinese, Mandarin	xie xie	
Czech	dêkuji	
Danish	tak	
English	thank you	
Finnish	kiitos	
French	merci	
German	danke	
Greek	efharisto	
Hawaiian	mahalo	
Hebrew	toda	
Hindi	sukria	
Italian	grazie	
Japanese	arigato	
Korean	kamsa hamnida	
Norwegian	takk	
Philippines (Tagalog)	salamat po	
Polish	dziekuje	
Portuguese	obrigado	
Russian	spasibo	
Spanish	gracias	
Sri Lanka (Sinhak)	istutiy	
Swahili	asante	
Swedish	tack	
Thai	khop khun krab	
Turkish	tesekkür ederim	
Vietnamese	ca'm on	

a. Describe the overall shape of the distribution. Also include if there are any striking deviations from the overall pattern (gaps, outliers, or clusters).

b. Where is the peak of the distribution?

c. How many observations are there?

Mean, Median and Mode

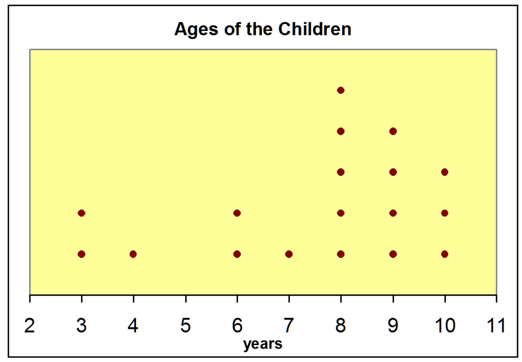
Mean, median and **mode** are all measures for the *centre* of a data set. In other words, each of them gives us a *single number* that indicates a "middle point" of the distribution.

The **mode** is the most commonly occurring data item within the data set.

- If no item occurs more often than others, there is no mode.
  Example 1. The data set {*bear, parrot, cat, dog, lizard*} has no mode.
- If two (or three, four, *etc.*) items occur equally often, there are that many modes.
  Example 2. The data set {3, 3, 6, 6, 7, 8, 8, 10} has three modes: 3, 6 and 8.

The **median** is the *middle* item after the data is organised from the least to the greatest. Exactly half of the data is before the median, and the other half is after.

- If there is an even number of data items, the median is the average of the two items in the middle.



Example 3. Find the median of the ages of a group of children:
3, 3, 4, 6, 6, 7, 8, 8, **8**, **8**, 8, 9, 9, 9, 9, 10, 10, 10

There are 18 data items and they are already in order.
The median is the "middle item", in this case the average of the 9th and 10th ages, which are both 8. So the median is 8.
It matches well with the peak in the plot of the distribution.

What is the mode in this example?

The **mean**, or the **average**, is calculated by adding all the data items, then dividing by the number of items.

Example 4. Mia's scores on her spelling tests were 80%, 72%, 88%, 92% and 79%.
What was her average score?

We calculate the mean by adding the five scores and dividing by 5: $\frac{80 + 72 + 88 + 92 + 79}{5} = 82.2\%$

1. Find the median and mode of these data sets.

a. 20, 25, 21, 30, 29, 24, 18, 32, 25, 26, 25 (ages of participants in a parenting class)

median ________ mode ________

b. 1, 1, 0, 2, 2, 2, 3, 1, 2, 2, 1 (the number of automobiles per household, for 11 households on Meadow Street)

median ________ mode ________

c. 80, 85, 80, 90, 70, 75, 90, 85, 100, 80 (Alice's quiz scores in algebra class)

median ________ mode ________

d. sandals, crocs, tennis shoes, crocs, dress shoes, sandals (types of shoes Emma keeps on her shoe rack)

mode ________

2. Joe practises swimming. These are the times, in seconds, it took him to swim 50 m freestyle, on six different days last week: 29.76 28.45 28.12 30.73 30.48 29.57. Find his average time.

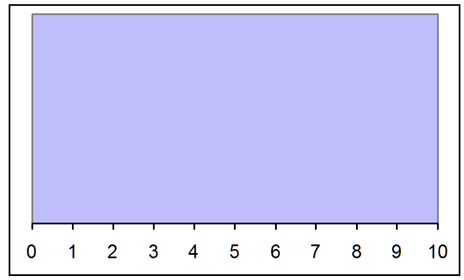
3. Find the mean, median and mode of the data sets. Draw a dot plot.

a. Ages of children in an art club:
4, 8, 2, 5, 5, 9, 3, 6, 5, 4, 4, 5, 1

mean ___________ median ___________

mode ___________

Notice: All three measures are close to each other.
This is not surprising, because this particular distribution is bell-shaped and has a very clear central peak.



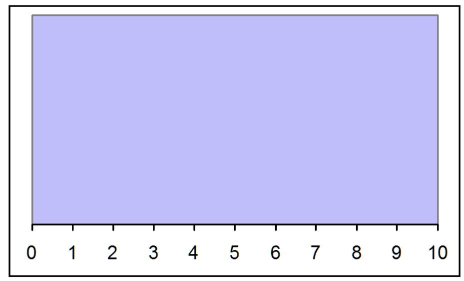
b. The number of sick days that a bakery's employees had last year:
1, 1, 1, 2, 2, 2, 2, 3, 3, 4, 6, 7, 7, 8, 8, 8, 9, 9, 9, 9, 9, 10, 10

mean ___________ median ___________

mode ___________

Shape of the distribution: ______________________________

Notice: Because of the odd shape of the distribution, median and mean do not describe the peaks at all.



4. These are the marks a group of students got in a course about electricity.

a. Make a bar graph from the data.

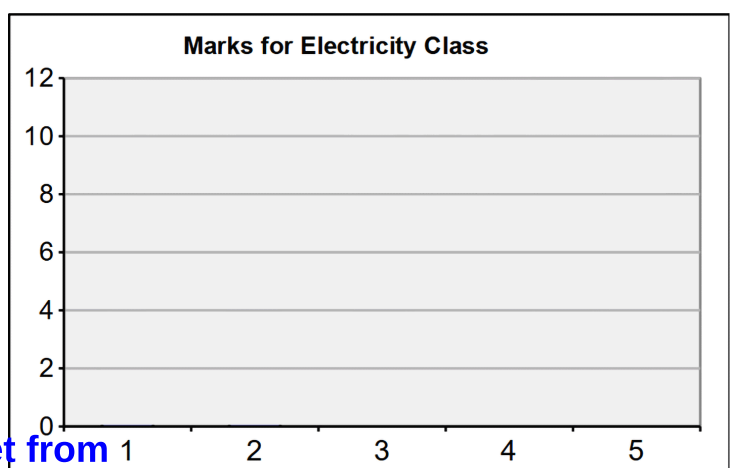
b. Before you go on, look at your graph and make a guess as to what the mean and median will be (approximately).

mean _______ median _______

c. Now find the mean, median and mode.

mean _______ median _______ mode ________

Marks	Students
1	2
2	3
3	5
4	7
5	10



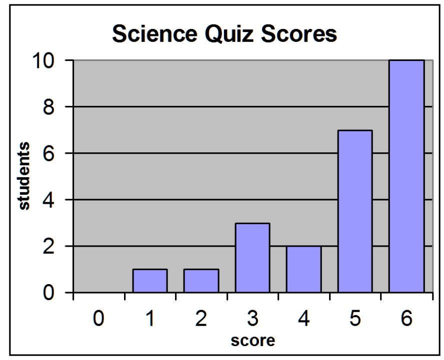
Using Mean, Median, and Mode

- The *mode* can be used with any type of data.
- The *median* can only be used if the data can be put in order.
- The *mean* can be used only if the data is numerical.

Whether you use mean, median, or mode depends both on

- the **type of data** and
- the **shape of the distribution.**

Example. This distribution of science quiz scores is heavily skewed (asymmetrical), and its “peak” is at 6. Clearly, most students did very well on the quiz.



Which of the three measures of centre — mean, median, or mode — would best describe this distribution?

Mode: We can see from the graph that the mode is 6.

Median: There are 24 students. The students’ actual scores can be read from the graph. They are 1, 2, 3, 3, 3, 4, 4, 5, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6.

The median is the average of the 12th and 13th scores, which is 5.

The mean is $\dfrac{1 + 2 + 3 \cdot 3 + 2 \cdot 4 + 7 \cdot 5 + 10 \cdot 6}{24} = 4.79167 \approx 4.79.$

Notice that the mean is less than 5, but the two highest bars on the graph are at 5 and 6. In this case, the mean does *not* describe the peak of the distribution very well because it actually falls outside the peak!

The median describes the peak reasonably well, but the mode is actually the best in this situation.

1. Fill in.

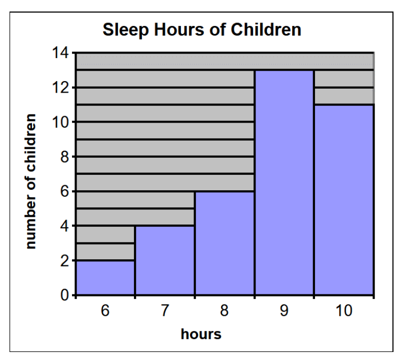
a. Is the original data numerical? ________

Calculate those centres of measure that are possible.

The mode: ____________ The median: ______________

The mean: ____________

Which measure(s) of centre describe the peak of the distribution well?



b. Is the original data numerical? ________

Calculate those centres of measure that are possible.

The mode: ____________ The median: ______________

The mean: ____________

Which measure(s) of centre describe the peak of the distribution well?