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Foreword

Math Mammoth Grade 4 (International version; Canada) comprises a complete math curriculum for the fourth grade mathematics studies. This curriculum is essentially the same as the *Math Mammoth Grade 4* sold in the United States, only customized for international use in a few aspects (listed below). The curriculum meets the Common Core Standards in the United States, but it may not properly align to the fourth grade standards in your province. However, you can probably find material for any missing topics in the neighbouring grades of Math Mammoth.

This International version of Math Mammoth for Canada differs from the US version in these aspects:

- The curriculum uses metric measurement units, not customary (imperial) units.
- The currency used in the money lessons is the Canadian dollar.
- The spelling conforms to British international standards.
- Large numbers are formatted with a space as the thousands separator (such as 12 394). (The decimals are formatted with a decimal point, as in the US version.)

The main areas of study in Math Mammoth Grade 4 are:

1. Students develop understanding and fluency with multi-digit multiplication, and use efficient multiplication procedures to solve problems.
2. They develop understanding of division to find quotients involving multi-digit dividends (long division), and they solve word problems involving division, including division with a remainder.
3. Students develop an understanding of fraction equivalence and some operations with fractions. They learn to add and subtract fractions with same denominators, and to multiply a fraction by a whole number.
4. Students learn the concept of angle. They draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Additional topics we study are place value, time, measuring, graphs, and decimals.

This book, 4-B, covers division (chapter 5), geometry (chapter 6), fractions (chapter 7), and decimals (chapter 8). The rest of the topics are covered in the 4-A worktext.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching math!

Maria Miller, the author

User Guide

Note: You can also find the information that follows online, at **https://www.mathmammoth.com/userguides/** .

Basic principles in using Math Mammoth Complete Curriculum

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. The two books (parts A and B) are like a "framework", but you still have a lot of liberty in planning your child's studies. You can even use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously. In fourth grade, chapters 3, 4, and 5 should be studied in order, but you can be flexible with all the other chapters and schedule them earlier or later.

Math Mammoth is not a scripted curriculum. In other words, it is not spelling out in exact detail what the teacher is to do or say. Instead, Math Mammoth gives you, the teacher, various tools for teaching:

- **The two student worktexts** (parts A and B) contain all the lesson material and exercises. They include the explanations of the concepts (the teaching part) in blue boxes. The worktexts also contain some advice for the teacher in the "Introduction" of each chapter.

  The teacher can read the teaching part of each lesson before the lesson, or read and study it together with the student in the lesson, or let the student read and study on his own. If you are a classroom teacher, you can copy the examples from the "blue teaching boxes" to the board and go through them on the board.

- There are hundreds of **videos** matched to the curriculum available at https://www.mathmammoth.com/videos/ . There isn't a video for every lesson, but there are dozens of videos for each grade level. You can simply have the author teach your child or student!

- Don't automatically assign all the exercises. Use your judgement, trying to assign just enough for your student's needs. You can use the skipped exercises later for revision. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.

- For each chapter, there is a **link list to various free online games** and activities. These games can be used to supplement the math lessons, for learning math facts, or just for some fun. Each chapter introduction (in the student worktext) contains a link to the list corresponding to that chapter.

- The student books contain some **mixed revision lessons**, and the curriculum also provides you with additional **cumulative revision lessons**.

- There is a **chapter test** for each chapter of the curriculum, and a comprehensive end-of-year test.

- The **worksheet maker** allows you to make additional worksheets for most calculation-type topics in the curriculum. This is a single html file. You will need Internet access to be able to use it.

- You can use the free online exercises at https://www.mathmammoth.com/practice/
  This is an expanding section of the site, so check often to see what new topics we are adding to it!

- Some grade levels have **cut-outs** to make fraction manipulatives or geometric solids.

- And of course there are answer keys to everything.

How to get started

Have ready the first lesson from the student worktext. Go over the first teaching part (within the blue boxes) together with your child. Go through a few of the first exercises together, and then assign some problems for your child to do on their own.

Repeat this if the lesson has other blue teaching boxes. Naturally, you can also use the videos at https://www.mathmammoth.com/videos/

Many children can eventually study the lessons completely on their own — the curriculum becomes self-teaching. However, children definitely vary in how much they need someone to be there to actually teach them.

Pacing the curriculum

The lessons in Math Mammoth complete curriculum are NOT intended to be done in a single teaching session or class. Sometimes you might be able to go through a whole lesson in one day, but more often, the lesson itself might span 3-5 pages and take 2-3 days or classes to complete.

Therefore, it is not possible to say exactly how many pages a student needs to do in one day. This will vary. However, it is helpful to calculate a general guideline as to how many pages per week you should cover in the student worktext in order to go through the curriculum in one school year (or whatever span of time you want to allot to it).

The table below lists how many pages there are for the student to finish in this particular grade level, and gives you a guideline for how many pages per day to finish, assuming a 180-day school year.

Example:

Grade level	Lesson pages	Number of school days	Days for tests and revisions	Days for the student book	Pages to study per day	Pages to study per week
4-A	168	88	8	80	2.1	10.5
4-B	177	92	8	84	2.1	10.5
Grade 4 total	345	180	16	164	2.1	10.5

The table below is for you to fill in. First fill in how many days of school you intend to have. Also allow several days for tests and additional revision before the test — at least twice the number of chapters in the curriculum. For example, if the particular grade has 8 chapters, allow at least 16 days for tests & additional revision. Then, to get a count of "pages/day", divide the number of pages by the number of available days. Then, multiply this number by 5 to get the approximate page count to cover in a week.

Grade level	Lesson pages	Number of school days	Days for tests and revisions	Days for the student book	Pages to study per day	Pages to study per week
4-A	168					
4-B	177					
Grade 4 total	345					

Now, let's assume you determine that you need to study about 2 pages a day, 10 pages a week in order to get through the curriculum. As you study each lesson, keep in mind that sometimes most of the page might be filled with blue teaching boxes and very few exercises. You might be able to cover 3 pages on such a day. Then some other day you might only assign one page of word problems. Also, you might be able to go through the pages quicker in some chapters, for example when studying graphs, because the large pictures fill the page so that one page does not have many problems.

When you have a page or two filled with lots of similar practice problems ("drill") or large sets of problems, feel free to **only assign 1/2 or 2/3 of those problems**. If your child gets it with less amount of exercises, then that is perfect! If not, you can always assign him/her the rest of the problems some other day. In fact, you could even use these unassigned problems the next week or next month for some additional revision.

In general, 1st-2nd graders might spend 25-40 minutes a day on math. Third-fourth graders might spend 30-60 minutes a day. Fifth-sixth graders might spend 45-75 minutes a day. If your child finds math enjoyable, he/she can of course spend more time with it! However, it is not good to drag out the lessons on a regular basis, because that can then affect the child's attitude towards math.

Working space, the usage of additional paper and mental math

The curriculum generally includes working space directly on the page for students to work out the problems. However, feel free to let your students to use extra paper when necessary. They can use it, not only for the "long" algorithms (where you line up numbers to add, subtract, multiply, and divide), but also to draw diagrams and pictures to help organize their thoughts. Some students won't need the additional space (and may resist the thought of extra paper), while some will benefit from it. Use your discretion.

Some exercises don't have any working space, but just an empty line for the answer (e.g. 200 + _____ = 1 000). Typically, I have intended that such exercises to be done using MENTAL MATH.

However, there are some students who struggle with mental math (often this is because of not having studied and used it in the past). As always, the teacher has the final say (not me!) as to how to approach the exercises and how to use the curriculum. We do want to prevent extreme frustration (to the point of tears). The goal is always to provide SOME challenge, but not too much, and to let students experience success enough so that they can continue enjoying learning math.

Students struggling with mental math will probably benefit from studying the basic principles of mental calculations from the earlier levels of Math Mammoth curriculum. To do so, look for lessons that list mental math strategies. They are taught in the chapters about addition, subtraction, place value, multiplication, and division. My article at https://www.mathmammoth.com/lessons/practical_tips_mental_math also gives you a summary of some of those principles.

Using tests

For each chapter, there is a **chapter test**, which can be administered right after studying the chapter. **The tests are optional.** Some families might prefer not to give tests at all. The main reason for the tests is for diagnostic purposes, and for record keeping. These tests are not aligned or matched to any standards.

In the digital version of the curriculum, the tests are provided both as PDF files and as html files. Normally, you would use the PDF files. The html files are included so you can edit them (in a word processor such as Word or LibreOffice), in case you want your student to take the test a second time. Remember to save the edited file under a different file name, or you will lose the original.

The end-of-year test is best administered as a diagnostic or assessment test, which will tell you how well the student remembers and has mastered the mathematics content of the entire grade level.

Using cumulative revisions and the worksheet maker

The student books contain mixed revision lessons which revise concepts from earlier chapters. The curriculum also comes with additional cumulative revision lessons, which are just like the mixed revision lessons in the student books, with a mix of problems covering various topics. These are found in their own folder in the digital version, and in the Tests & Cumulative Revisions book in the print version.

The cumulative revisions are optional; use them as needed. They are named indicating which chapters of the main curriculum the problems in the revision come from. For example, "Cumulative Revision, Chapters 1 - 4" includes problems that cover topics from chapters 1-4.

Both the mixed and cumulative revisions allow you to spot areas that the student has not grasped well or has forgotten. When you find such a topic or concept, you have several options:

1. Check if the worksheet maker lets you make worksheets for that topic.
2. Check for any online games and resources in the Introduction part of the particular chapter in which this topic or concept was taught.
3. If you have the digital version, you could simply reprint the lesson from the student worktext, and have the student restudy that.
4. Perhaps you only assigned 1/2 or 2/3 of the exercise sets in the student book at first, and can now use the remaining exercises.
5. Check if our online practice area at https://www.mathmammoth.com/practice/ has something for that topic.
6. Khan Academy has free online exercises, articles, and videos for most any math topic imaginable.

Concerning challenging word problems and puzzles

While this is not absolutely necessary, I heartily recommend supplementing Math Mammoth with challenging word problems and puzzles. You could do that once a month, for example, or more often if the student enjoys it.

The goal of challenging story problems and puzzles is to **develop the student's logical and abstract thinking and mental discipline**. I recommend starting these in fourth grade, at the latest. Then, students are able to read the problems on their own and have developed mathematical knowledge in many different areas. Of course I am not discouraging students from doing such in earlier grades, either.

Math Mammoth curriculum contains lots of word problems, and they are usually multi-step problems. Several of the lessons utilize a bar model for solving problems. Even so, the problems I have created are usually tied to a specific concept or concepts. I feel students can benefit from solving problems and puzzles that require them to think "out of the box" or are just different from the ones I have written.

I recommend you use the free Math Stars problem-solving newsletters as one of the main resources for puzzles and challenging problems:

Math Stars Problem Solving Newsletter (grades 1-8)
https://www.homeschoolmath.net/teaching/math-stars.php

I have also compiled a list of other resources for problem solving practice, which you can access at this link:

https://l.mathmammoth.com/challengingproblems

Another idea: you can find puzzles online by searching for "brain puzzles for kids," "logic puzzles for kids" or "brain teasers for kids."

Frequently asked questions and contacting us

If you have more questions, please first check the FAQ at https://www.mathmammoth.com/faq-lightblue

If the FAQ does not cover your question, you can then contact us using the contact form at the Math Mammoth.com website.

I wish you success in teaching math!

Maria Miller, the author

Chapter 5: Division
Introduction

The fifth chapter of *Math Mammoth Grade 4* includes lessons on division, long division, remainder, average, divisibility, and problem solving. It is a long chapter, because division and long division are "in focus" in fourth grade. Therefore, feel free to mix the lessons from this chapter with lessons from some other chapter, essentially using the curriculum in a somewhat spiral manner. This is especially advisable if your student has difficulties retaining the material or starts feeling bored with these topics.

For further help in teaching these topics, check out the free videos matched to the curriculum at https://www.mathmammoth.com/videos/. Remember not to automatically assign all the exercises. Instead, adjust the amount of exercises according to the student's needs. The rest can be used later for revision.

We start out by revising basic division facts by single-digit numbers (such as 24 ÷ 4 or 56 ÷ 7). After that, we study terminology of division and dividing numbers by whole tens and hundreds (such as 400 ÷ 20). Next students practise the order of operations again—this time with division as one of the operations.

Then we study the concept of remainder, preparing students for the upcoming lessons on long division. At first, the concept of remainder is presented visually. Soon, students solve simple division problems with a remainder, written with the long division symbol (or long division "corner", as I like to call it).

Next comes a set of lessons intended to teach long division in several small steps. We start with divisions where each of the digits in the dividend (thousands, hundreds, tens, and ones) can be divided evenly by the divisor (for example, 3 096 ÷ 3). As the next step, there is a remainder in the ones. Then, the divisions have a remainder in the tens. Finally, there is a remainder in the hundreds and in the thousands, and this completes the step-by-step learning process for long division. The lessons also include lots of word problems to solve.

After long division, we study the concept of average, which is a nice application of division, and problems that involve finding a fractional part of a quantity using division. For example, we can find 3/4 of a number by first finding 1/4 (dividing by 4) and then multiplying the result by 3. Students get help from visual bar models to solve the problems.

The last section deals with elementary number theory. We study basic divisibility rules (though not all of them), prime numbers, and finding all factors of a given two-digit number.

The Lessons in Chapter 5

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better, and even just for some fun. Enjoy!

https://links.mathmammoth.com/gr4ch5

SCAN ME

Dividing with Whole Tens and Hundreds

Solving division problems always involves thinking the opposite of multiplication, or "how many times."	
4 800 ÷ 60 = ? Think "backwards" of multiplication: Because 60 × 80 = 4 800, then 4 800 ÷ 60 = 80.	**4 000 ÷ 400 = ?** Or ask, "How many times does 400 go into 4 000?" Ten times. So, 4 000 ÷ 400 = 10.

1. Solve the multiplication and then write *two* matching divisions.

a.	**b.**	**c.**
300 × 7 = _____	50 × 800 = _____	60 × 40 = _____
_____ ÷ 7 = _____	_____ ÷ _____ = _____	_____ ÷ _____ = _____
_____ ÷ 300 = _____	_____ ÷ _____ = _____	_____ ÷ _____ = _____

2. Think of multiplication in order to divide. Also, compare the problems.

a. 400 ÷ 8 = _____	**b.** 5 000 ÷ 5 = _____	**c.** 4 200 ÷ 700 = _____
400 ÷ 80 = _____	5 000 ÷ 50 = _____	4 200 ÷ 70 = _____
4 000 ÷ 800 = _____	5 000 ÷ 500 = _____	420 ÷ 70 = _____
4 000 ÷ 80 = _____	500 ÷ 50 = _____	420 ÷ 7 = _____

3. Solve the unknown factor problems and the division problems. Notice something special here!

a. _____ × 6 = 540	**b.** 3 × _____ = 2 700	**c.** _____ × 40 = 2 800
540 ÷ 6 = _____	2 700 ÷ 3 = _____	2 800 ÷ 40 = _____

4. Practise some more.

a. 320 ÷ 8 = _____	**b.** 540 ÷ 60 = _____	**c.** 360 ÷ 6 = _____
320 ÷ 80 = _____	540 ÷ 6 = _____	3 600 ÷ 60 = _____
3 200 ÷ 8 = _____	5 400 ÷ 60 = _____	36 000 ÷ 6 = _____

You can divide larger numbers digit-by-digit—if the division is even in each digit.

Example 1. To find 640 ÷ 2, you can divide 6, 4, and 0 by 2 (separately). Each of those is an even division. We get 640 ÷ 2 = 320.

Example 2. We can divide digit-by-digit to solve 309 ÷ 3, because each of the digits (3, 0, and 9) can be divided by 3 evenly. We get 103.

5. Solve.

a. 426 ÷ 2 = _____ **b.** 8 044 ÷ 2 = _____ **c.** 9 303 ÷ 3 = _____

d. 550 ÷ 5 = _____ **e.** 4 008 ÷ 4 = _____ **f.** 2 820 ÷ 2 = _____

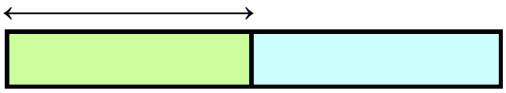
Finding half...	**...is the same as dividing by 2!**
$\frac{1}{2}$ of 280 is _____	280 ÷ 2 = _____

6. Find half of these numbers.

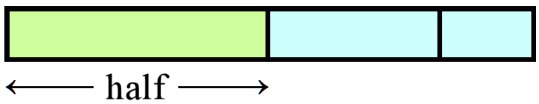
a.	**b.**	**c.**	**d.**
$\frac{1}{2}$ of 80 = _____	$\frac{1}{2}$ of 24 000 = _____	$\frac{1}{2}$ of 660 = _____	$\frac{1}{2}$ of 4 200 = _____

Write the numbers given in the problems into the bar models.

7. Dad used half of his paycheque to pay the rent, electricity and water. After that he had $806 left. How much was his paycheque?



8. A fisherman sold half of his 800-kg catch to one market, and then another 350 kg to another market. How many kilograms of fish does he have left now?



9. Emma spent half of her money to buy a phone. Then she spent $12 on some food. Now she has $15 left. How much did she have in the beginning? (You can draw a bar model to help you.)

10. Estimate. Round the dividend (the first number) so that you can easily divide mentally.

a. 352 ÷ 5 ≈ _____ ÷ 5 = _____	**b.** 198 ÷ 4 ≈ _____ ÷ 4 = _____	**c.** 403 ÷ 8 ≈ _____ ÷ 8 = _____

11. Estimate. This time round *both* the dividend and the divisor to the nearest ten.

a. 802 ÷ 21 ≈ _____ ÷ _____ = ____	**b.** 356 ÷ 61 ≈ _____ ÷ _____ = ____	**c.** 596 ÷ 32 ≈ _____ ÷ _____ = ____

12. Estimate each division result by rounding either the dividend or the divisor.

a. 80 ÷ 21 ≈ 120 ÷ 59 ≈ 2 000 ÷ 512 ≈	**b.** 46 ÷ 5 ≈ 16 235 ÷ 400 ≈ 297 ÷ 30 ≈

13. Write a division problem with a divisor of 5 and a quotient of 90.

14. Write a division problem that is impossible to solve.

15. Find what number the unknown stands for.

a. $y \times 8 = 64\,000$ $y =$ _____	**b.** $s \div 6 = 700$ $s =$ _____	**c.** $2\,400 \div w = 80$ $w =$ _____

16. Solve. Notice the patterns. Use the top two problems to help you solve the *third* problem in each set. Then write two more problems for each set, continuing the pattern.

a. 500 ÷ 5 = _____ 505 ÷ 5 = _____ 510 ÷ 5 = _____ _____ ÷ 5 = _____ _____ ÷ 5 = _____	**b.** 466 ÷ 2 = _____ 468 ÷ 2 = _____ 470 ÷ 2 = _____ _____ ÷ 2 = _____ _____ ÷ 2 = _____	**c.** 366 ÷ 3 = _____ 369 ÷ 3 = _____ 372 ÷ 3 = _____ _____ ÷ 3 = _____ _____ ÷ 3 = _____

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Order of Operations and Division

1. Do operations within ( ) first.

2. Then multiply and divide, from left to right.

3. Then add and subtract, from left to right.

1. Solve. When there are many multiplications and divisions, do them from left to right.

$24 \div 3 \times 2 \div 4$ $= 8 \times 2 \div 4$ $= 16 \div 4 = 4$	**a.** $18 \div 2 \div 3$	**b.** $160 \div 4 \times 20 \div 8$
	c. $60 \times 20 \div 10$	**d.** $5 \times 80 \div 4 \times 20$

2. Solve. Do multiplications and divisions first.

$36 \div 3 + 20 \div 4$ $= 12 + 5$ $= 17$	**a.** $12 \times 5 + 6 \div 3$	**b.** $16 \times 2 + 15 \times 8$
	c. $80 \times 30 - 4\,000 \div 10$	**d.** $400 \div 50 + 400 \div 40$

3. Solve what is within the parentheses first.

$(36 + 4) \div (5 + 5)$ $= 40 \div 10$ $= 4$	**a.** $(100 - 1) \div (5 + 6)$	**b.** $(140 + 17 + 13) \div 10$
	c. $2 \times (700 \div 7)$	**d.** $150 \div (13 + 7 + 10)$

4. Solve. Compare.

a.	**b.**	**c.**
$24 \div 2 + 10 =$ _____	$18 + 30 \div 2 =$ _____	$40 - 40 \div 8 =$ _____
$24 \div (2 + 10) =$ _____	$(18 + 30) \div 2 =$ _____	$(40 - 40) \div 8 =$ _____

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Long Division 1

Divide hundreds, tens, and ones separately.

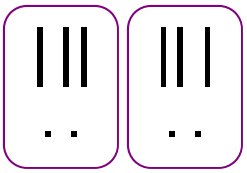
Write the dividend inside the long division "corner", and the quotient on top.

64 ÷ 2 = ?

Divide tens and ones separately:

6 tens ÷ 2 = 3 tens (t)

4 ones ÷ 2 = 2 ones (o)



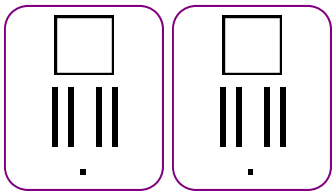
	t	o
2)	3	2
	6	4

282 ÷ 2 = ?

2 hundreds ÷ 2 = 1 hundred (h)

8 tens ÷ 2 = 4 tens (t)

2 ÷ 2 = 1 (o)



	h	t	o
2)	2	8	2
	1	4	1

1. Make groups. Divide. Write the dividend inside the "corner" if it is missing.

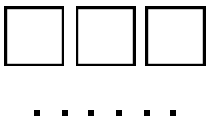
a. Make 2 groups

2) 6 2

b. Make 3 groups

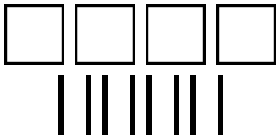
3)

c. Make 3 groups



3)

d. Make 4 groups



4)

2. Divide thousands, hundreds, tens, and ones separately.

a. 4) 8 4

b. 3) 3 9 3

c. 3) 6 6 0

d. 4) 8 0 4 0

e. 3) 6 6

f. 2) 6 0 4 2

g. 3) 3 3 0

h. 4) 4 8 0 4

h t o 0 4) 2 4 8 h t o 0 6 2 4) 2 4 8	th h t o 0 5) 3 5 0 5 th h t o 0 7 0 1 5) 3 5 0 5
Four does not go into 2. You can put zero in the quotient in the hundreds place or omit it. Four does go into 24, six times. Put 6 in the quotient.	Five does not go into 3. You can put zero in the quotient. Five does go into 35, seven times.
Explanation: The 2 of 248 is 200 in reality. If you divided 200 by 4, the result would be less than 100, so that is why the quotient will not have any whole hundreds. Then you combine the 2 hundreds with the 4 tens. That makes 24 tens, and you CAN divide 24 tens by 4. The result, 6 tens goes as part of the quotient. Check the final answer: 4 × 62 = 248.	**Explanation:** 3 000 ÷ 5 will not give any whole thousands to the quotient because the answer is less than 1 000. But 3 thousands and 5 hundreds make 35 hundreds together. You can divide 3 500 ÷ 5 = 700, and place 7 as part of the quotient in the hundreds place. Check the final answer: 5 × 701 = 3 505.
If the divisor does not "go into" the first digit of the dividend, look at the first two digits of the dividend.	

3. Divide. Check your answer by multiplying the quotient and the divisor.

a. 0 4 / 3) 1 2 3

b. 4) 2 8 4

c. 6) 3 6 0

d. 8) 2 4 8

e. 2) 1 8 4

f. 7) 4 2 7

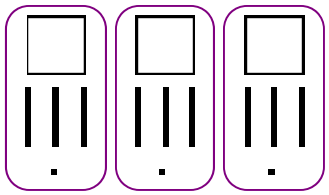
g. 3) 1 8 3 3

h. 4) 2 4 0 4

i. 7) 4 9 7 0

j. 5) 4 5 0 5

The ones division is not even. There is a remainder.



395 ÷ 3 = 131 R2

$$\begin{array}{r} \text{h}\ \ \text{t}\ \ \text{o} \\ 1\ 3 \\ \hline 3\,)\,3\ 9\ 5 \end{array}$$

3 goes into 3 one time.
3 goes into 9 three times.

$$\begin{array}{r} \text{h}\ \ \text{t}\ \ \text{o} \\ 1\ 3\ 1\,\text{R}2 \\ \hline 3\,)\,3\ 9\ 5 \end{array}$$

3 goes into 5 one time, but not evenly.
Write the remainder 2 after the quotient.

$$\begin{array}{r} \text{h}\ \ \text{t}\ \ \text{o} \\ 0\ 4\ 1\,\text{R}1 \\ \hline 4\,)\,1\ 6\ 5 \end{array}$$

Four does not go into 1 (hundred).
So combine the 1 hundred with the 6 tens (160).

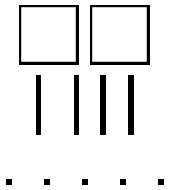
Four goes into 16 four times.
Four goes into 5 once, with a remainder of 1.

$$\begin{array}{r} \text{th}\ \ \text{h}\ \ \text{t}\ \ \text{o} \\ 0\ 4\ 0\ 0\,\text{R}7 \\ \hline 8\,)\,3\ 2\ 0\ 7 \end{array}$$

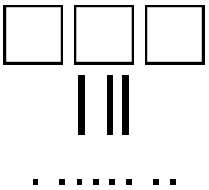
Eight does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3 200).
Eight goes into 32 four times (3 200 ÷ 8 = 400)
Eight goes into 0 zero times (tens).
Eight goes into 7 zero times, with a remainder of 7.

4. Divide into groups. Find the remainder.

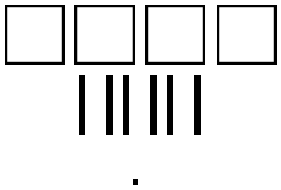
a. $2\,)\,6\ 3$



b. $2\,)$



c. $3\,)$



d. $2\,)$

5. Divide. Indicate the remainder if any.

a. $4\,)\,8\ 4\ 7$ **b.** $2\,)\,6\ 9$ **c.** $3\,)\,3\ 6\ 7$ **d.** $4\,)\,8\ 9$

e. $2\,)\,1\ 2\ 1$ **f.** $6\,)\,1\ 8\ 0\ 5$ **g.** $7\,)\,2\ 1\ 5$ **h.** $8\,)\,2\ 4\ 8\ 2$

In the problems before, you just wrote down the remainder of the ones. Usually, we write down the subtraction that actually finds the remainder. Look carefully:

h t o 0 6 1 4) 2 4 7 − 4 3	th h t o 0 4 0 2 4) 1 6 0 9 − 8 1
When dividing the ones, 4 goes into 7 one time. Multiply 1 × 4 = 4, write that four under the 7, and subtract. This finds us the remainder of 3. Check: 4 × 61 + 3 = 247	When dividing the ones, 4 goes into 9 two times. Multiply 2 × 4 = 8, write that eight under the 9, and subtract. This finds us the remainder of 1. Check: 4 × 402 + 1 = 1 609

6. Practise some more. Subtract to find the remainder in the ones. Check your answer by multiplying the divisor times the quotient, and then adding the remainder. You should get the dividend.

a. $3\overline{)128}$

b. $3\overline{)95}$

c. $6\overline{)4267}$

d. $4\overline{)2845}$

e. $5\overline{)5507}$

f. $2\overline{)8063}$

7. Divide these numbers mentally. Remember, you can always check by multiplying!

a. 440 ÷ 4 = 820 ÷ 2 =	**b.** 3 600 ÷ 400 = 369 ÷ 3 =	**c.** 824 ÷ 2 = 560 ÷ 90 =

Chapter 6: Geometry
Introduction

We start our study of geometry by revising the third grade concepts of area and the perimeter of rectangles. Students also apply these concepts in various problems, including problems where they write simple equations and a problem where they explore all possible perimeters for a given area.

Note: Students will need a ruler and a protractor throughout the chapter.

The focus of the chapter is angles. Students learn about lines, rays, and angles; and about acute, right, obtuse, and straight angles. They learn how to measure and draw angles with a protractor. We also study angle problems where students write simple equations. The lesson *Estimating Angles* has an optional section on turning in an angle, which can be challenging, so feel free to omit it if you wish.

The lesson *Parallel and Perpendicular Lines* also ties in with the topic of angles, because two lines are perpendicular if they form a right angle. After that, we study parallelograms and other quadrilaterals in more detail, paying attention to their angles and lengths of sides.

We also study triangles and classify them according to their angles (acute, obtuse, or right triangles). Classifying triangles according to their sides (equilateral, isosceles, or scalene) will be studied in 5th grade. The last (and easy) topic in this chapter is line symmetry.

The lessons include quite a few drawing exercises which can be done on blank paper, in a notebook, or in the worktext (for most). Please stress to the student to always use a ruler and other proper tools, such as a protractor or a triangular ruler, so the drawings will be as accurate as possible. Some exercises may mention to only sketch something, in which case it is okay to not use any drawing tools.

Geometry is full of strange-sounding words. I suggest that student(s) keep a geometry notebook, where they draw picture(s) and text to explain every new concept or term. This will help them to remember those terms. They can also do the drawing exercises in the notebook. Encourage the students to be creative so that the notebook becomes their own special work. You can even give them credit for it.

The Lessons in Chapter 6

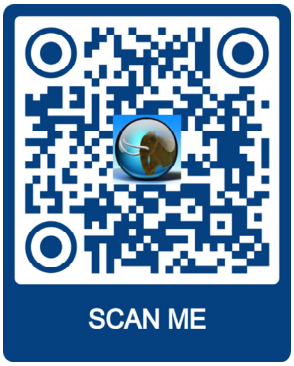
Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better, and even just for some fun. Enjoy!

https://links.mathmammoth.com/gr4ch6



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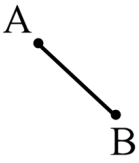
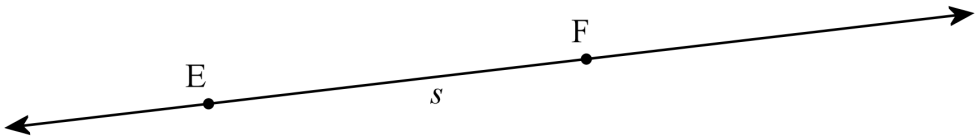
Lines, Rays, and Angles

This is point A. (Points are named with capital letters.)

This is a **line segment**. We write this as line segment AB or line segment $\overline{AB}$.

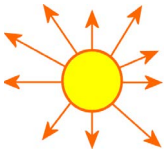


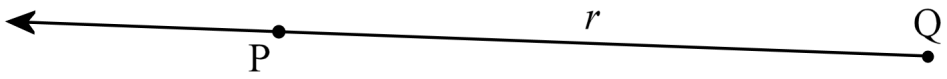
A line has no beginning point or end point. Imagine it continuing indefinitely in both directions. We can illustrate that by little arrows on both ends.



A line is named using two points on it. This is line EF or line $\overleftrightarrow{EF}$ (note the arrows on both ends). Or, we can name a line using a lowercase letter: this is line *s*.

A ray starts out at a point but continues on indefinitely, without ending. We can show that by drawing an arrow at one end of the ray. Think of the sun's rays!

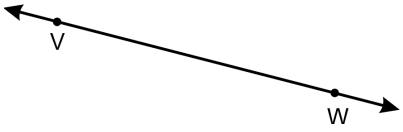




A ray is named using its starting point and one other point on the ray: this is ray QP or ray $\overrightarrow{QP}$ (note the one arrow). Or, we can name a ray using a lowercase letter: this is ray *r*.

1. Write the name if each figure is a line, ray, or a line segment.

a. _____



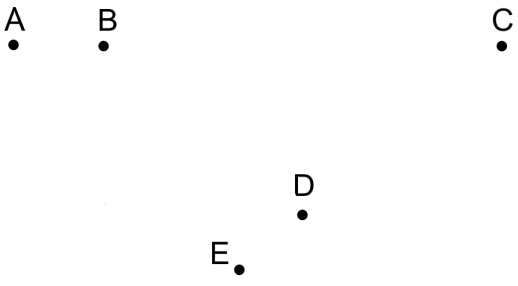
b. _____

c. _____

2. **a.** Draw the ray BD.

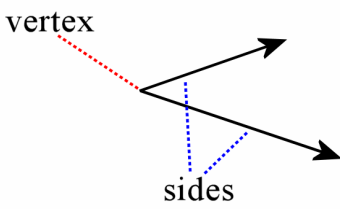
b. Draw the line AB.

c. Draw the line segment ED.

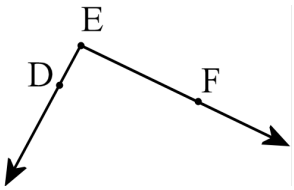


What is an angle? Many people think that an angle is some kind of slanted line. However, in geometry, **an angle** is made up of **TWO RAYS that have the same beginning point**.

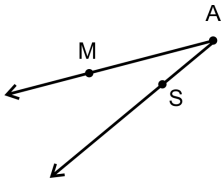
That point is called the **vertex**, and the two rays are called the **sides** of the angle.



To name an angle, we use three points, listing the vertex in the middle. This is angle DEF or ∠DEF. We can use the symbol ∠ for angle.

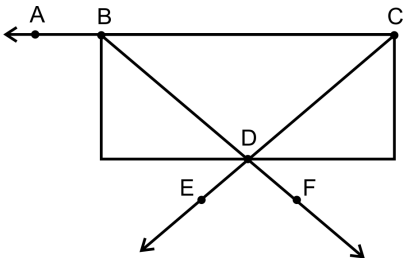


3. Name the angle.



4. **a.** Find the angle formed by the rays DE and DF. How do we name it?

b. Find the angle formed by the rays CA and CE. How do we name it?



5. **a.** Draw two points, D and E. Then draw line DE.

b. Draw point *Q not* on the line.

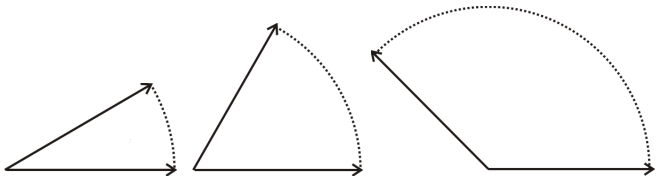
c. Draw rays DQ and EQ.

d. Find angles EDQ and DEQ in your drawing.

Angles "open up"

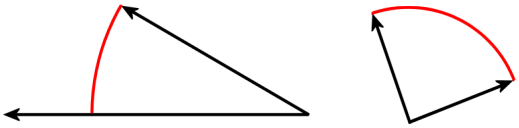
Take two pencils to illustrate the two rays which make the two sides of an angle. Set them side by side to show a zero angle! ⟹

Then start "opening up" the angle. Keep one pencil stationary while you rotate the other. As the angle opens up, an imaginary arc of a circle is drawn.



Telling the size of angles

Which of these two angles is bigger?

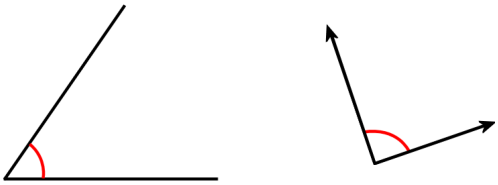


Do not look at how LONG the sides of the angle are. Remember, the sides of an angle are rays, and rays go on indefinitely (even if they are drawn short).

Instead, look at how much each angle has *opened*, or how big a part of a circle the sides have drawn. The second angle (on the right) is bigger.

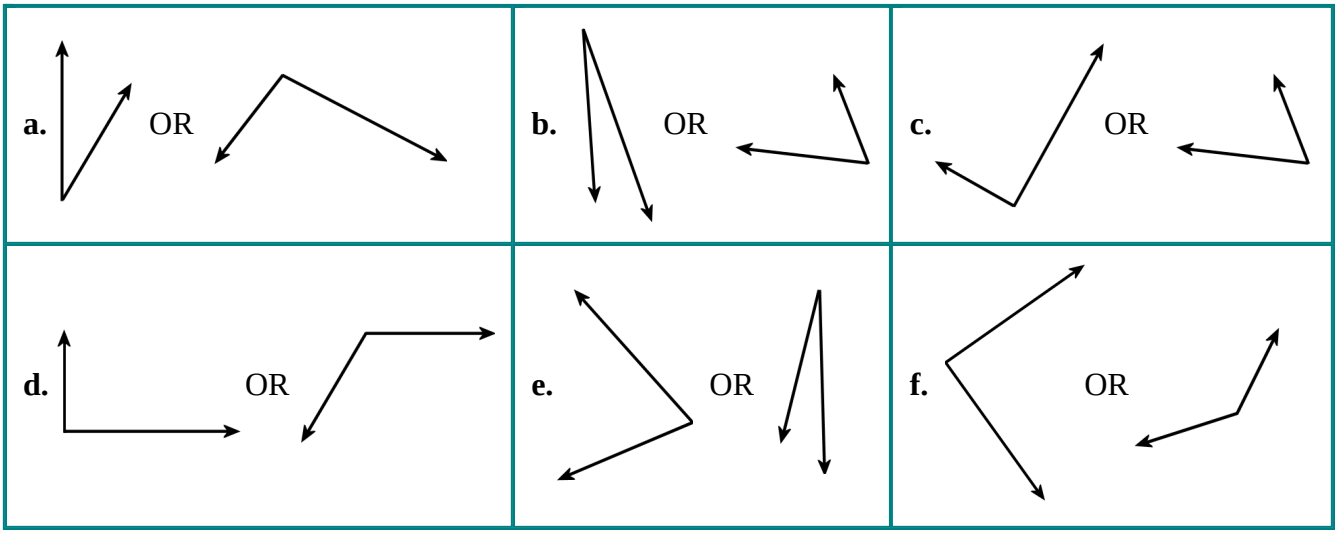
The **size of an angle** is determined by **how much it has opened as compared to the whole circle**.

Many times the arrows are omitted from the rays, and the arc of the circle is drawn as a tiny arc near the vertex. Even that is not necessary.



Which of these is the bigger angle?
Again, the second one is bigger.

6. Which angle is bigger? You can use pencils to help!



New Terms & Symbols	• *line segment* • *ray* • *line* • *angle*

Note: In the next lesson, students will need a *protractor* to measure angles.

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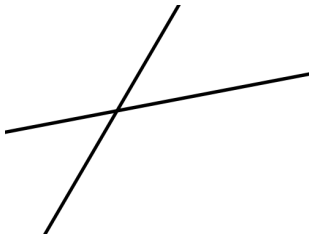
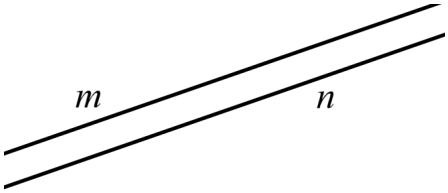
Parallel and Perpendicular Lines

Two lines or line segments can either **intersect** (cross each other) or be **parallel.**

Think of two parallel lines as never meeting each other, no matter how much you would continue them in both directions.



These lines intersect.

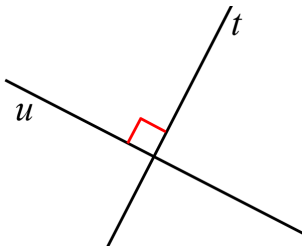


Lines m and n are parallel.
We write $m \parallel n$.

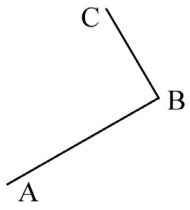
Two lines or line segments are **perpendicular** if they **form a right angle**.

We use the symbol $\perp$ to mean "is perpendicular to".

In an image, we can mark a right angle with a little corner like this: ⌝.



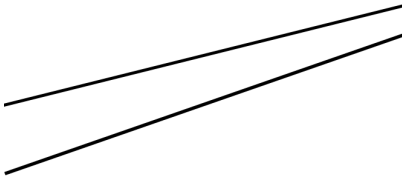
How many right angles do the lines u and t form?



The line segments $\overline{AB}$ and $\overline{BC}$ are perpendicular.

We write $\overline{AB} \perp \overline{BC}$.

1. Do these lines intersect or are they parallel? Continue the lines with your ruler.



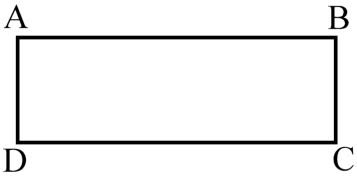
2. Which line segments in these figures are parallel? Which are perpendicular?

a. Line segments AB and BC

are _____.

Line segments AD and BC

are _____.

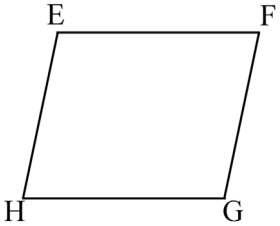


b. Line segments EF and GH

are _____.

Line segments EH and FG

are _____.



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Chapter 7: Fractions
Introduction

In third grade, students have studied equivalent fractions and compared some easy fractions. In fourth grade, it is time to expand their knowledge of fraction topics. We study:

- mixed numbers
- adding and subtracting like fractions and mixed numbers with like fractional parts (sums where the denominators are the same, such as 5/6 + 3/6 or 1 2/3 + 2 1/3)
- equivalent fractions (for example, 2/3 = 8/12)
- comparing fractions
- multiplying a fraction by a whole number (for example 5 × ½)

Then in fifth grade, students tackle *all* four operations with fractions. This chapter is laying groundwork for that. The lessons here are important also because they are the basis for understanding decimal numbers, which is the topic of the next chapter.

In this grade, we continue studying fractions and their operations with the help of visual models. In addition to the visuals in the lessons, you can optionally also use fraction manipulatives, but they are not required.

Visual models help children build a strong conceptual understanding of fraction operations. While we do study some actual rules of fraction arithmetic in this chapter, we also want to avoid presenting fraction math as a list of computational rules to be learned by rote memory. If students only memorize these rules, then they will also easily confuse them (eventually), because there are so many of them. The rules become *shortcuts* for ideas that are already understood, but we don't want to start with them. The goal is to let the ideas and concepts "sink in" first, and then study the shortcuts.

A friendly reminder: don't automatically assign all the exercises. As always, use your judgment.

The Lessons in Chapter 7

Helpful Resources on the Internet

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- **online practice** for concepts;
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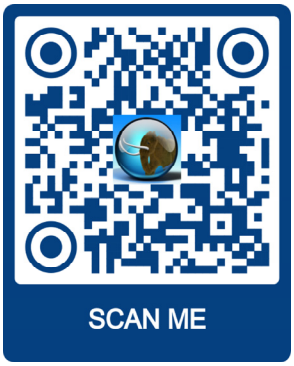


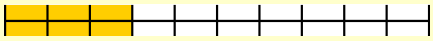
One Whole and Its Fractional Parts

A fraction always relates to some kind of *one whole*. Study the examples below:

Let's say the one whole is this square. It is divided into 12 parts.

Each part is $\frac{1}{12}$ of the whole. Also, we can write $1 = \frac{12}{12}$.

Maybe the one whole is this line, and $\frac{3}{10}$ of it is coloured.

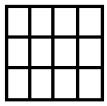


Maybe the one whole is Daddy's salary. To find 5/6 of it, imagine dividing the salary into 6 parts, and taking five of those parts. All six parts form the one whole, or $\frac{6}{6} = 1$

$\frac{7}{12}$ The top number is the **numerator**. It *numerates* or counts *how many pieces* there are.
The bottom number is the **denominator**. It *denominates* or *names* what kind of parts they are.

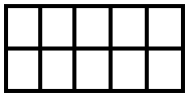
1. Colour parts. Write the coloured part *and* the white (uncoloured) part as a fraction.

a. Colour 1 part.



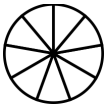
$\frac{1}{12}$ and ___

b. Colour 5 parts.



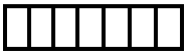
and

c. Colour 8 parts.



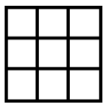
and

d. Colour 3 parts.



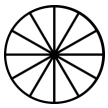
and

2. Colour and write one whole as a fraction.

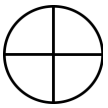


a. 1 = ___

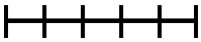
b. 1 = ___



c. 1 = ___



d. 1 = ___



e. 1 = ___

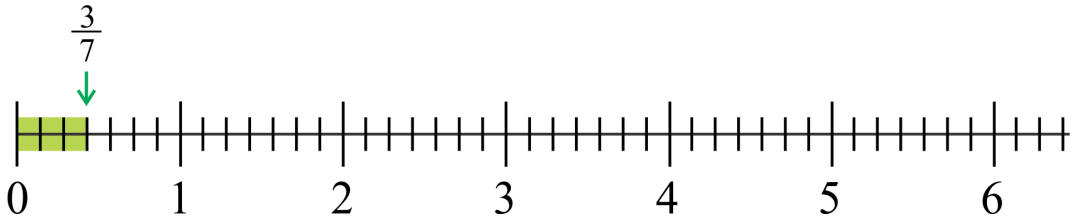
3. Solve.

a. The Jacksons ate $\frac{3}{4}$ of the pie.
How much is left?

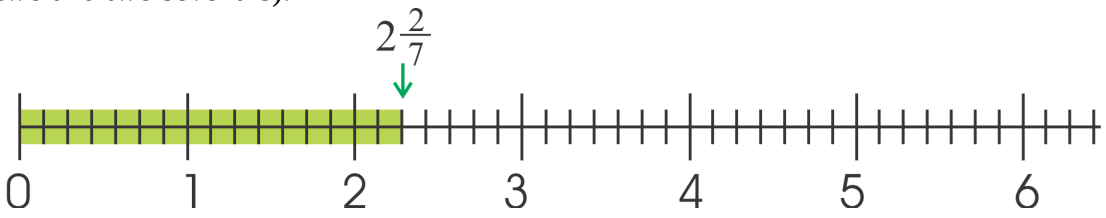
b. Jerry ate $\frac{1}{6}$ of the pizza.
How much is left?

c. Five boys shared a chocolate bar equally. Each one got ___ of the bar.

To show 3/7 on a number line, each whole-number interval (from 0 to 1, from 1 to 2, from 2 to 3, and so on) is divided into seven parts. Three of those parts are coloured to show 3/7.

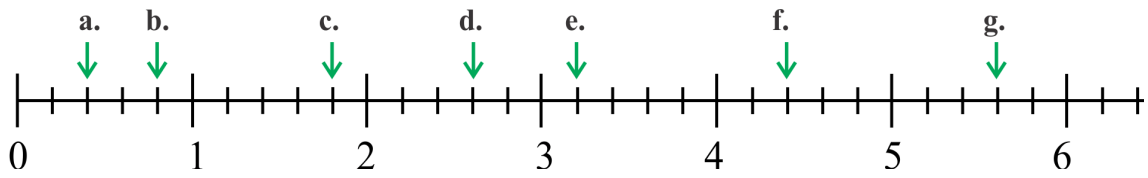


In a **mixed number**, we have a whole number and a fraction. The number line below shows 2 2/7 (two and two sevenths).

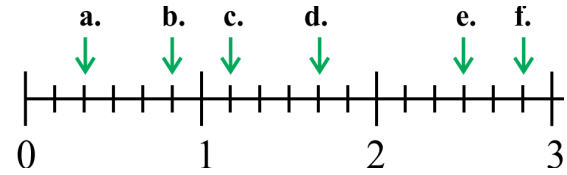


In problems 4 - 6, write the fractions and mixed numbers that the arrows mark on the number line.

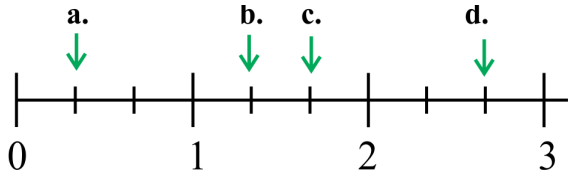
4.



5.



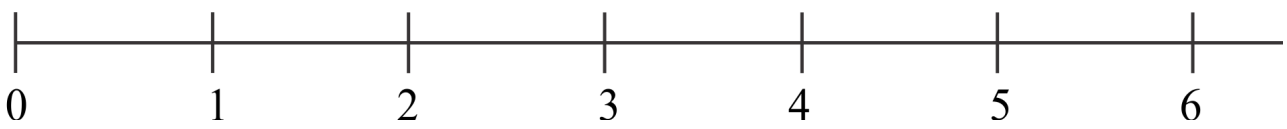
6.



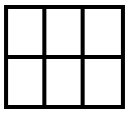
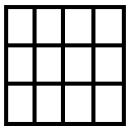
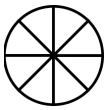
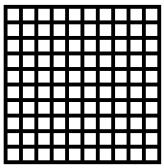
7. Mark the mixed numbers on the number line:

a. 1 2/4 **b.** 3/4 **c.** 4 1/4 **d.** 5 1/2 **e.** 3 1/4 **f.** 2 3/4

Hint: First divide each whole-number interval into four parts (using three tick marks).



8. Colour. Then write an addition, adding the coloured and white parts. Notice what sum you get.

a. Colour 1 part.	**b.** Colour 10 parts.	**c.** Colour 3 parts.	**d.** Colour 15 parts.
 $\frac{1}{6} + \frac{\quad}{\quad} = 1$			

9. Find what fraction is missing from one whole.

a. $\frac{3}{4} + \frac{\quad}{\quad} = 1$ **b.** $\frac{6}{7} + \frac{\quad}{\quad} = 1$ **c.** $\frac{1}{8} + \frac{\quad}{\quad} = 1$ **d.** $\frac{11}{12} + \frac{\quad}{\quad} = 1$

10. **a.** Mary drank $\frac{1}{4}$ litre of juice from a 1-litre pitcher, and her brother drank another $\frac{1}{4}$ litre. How much juice is left in the pitcher?

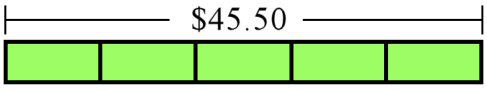
b. A loaf of bread was cut into 20 slices. Jack and John ate three slices each. What fractional part of the bread is left?

11. Let's revise how to find a fractional part using division.

a. Remember division? Find 1/10 of 90 km.

Then find 4/10 of 90 km.

b. A restaurant bill was $45.50. It was divided so that Cindy paid 2/5 of it and Sandy paid 3/5 of it.



How many dollars did Cindy pay?

How many dollars did Sandy pay?

c. Dad used 2/9 of his $2 700 paycheque.

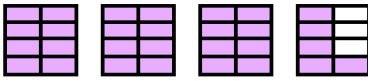
What fractional part is left of his paycheque?

How many dollars are left of his paycheque?

Mixed Numbers

Mixed numbers have two parts: a whole-number part and a fractional part.

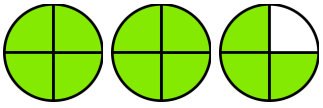
This picture illustrates $3\frac{5}{8}$: *three and five eighths*.



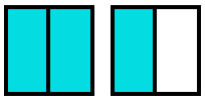
Notice: the coloured portion is $3\frac{5}{8}$. The uncoloured part is $\frac{3}{8}$.

If we add the coloured and uncoloured parts, we get four wholes: $3\frac{5}{8} + \frac{3}{8} = 4$.

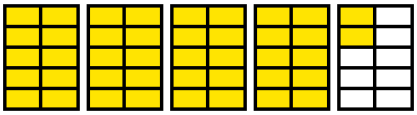
1. Write the mixed numbers these pictures illustrate.



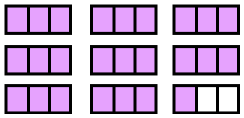
a.



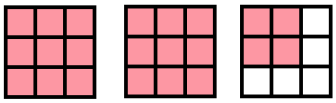
b.



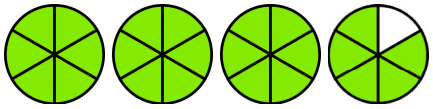
c.



d.



e.

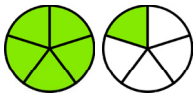


f.

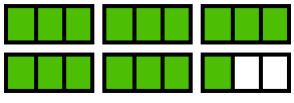
2. Write an addition sentence, adding what is coloured and what is not. Look at the example.

a. $2\frac{2}{4} + \frac{2}{4} = 3$

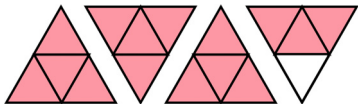
b.



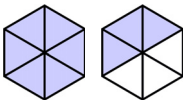
c.



d.



e.



f.

3. How much is missing from the next whole number?

a. $1\frac{1}{4} + \square = 2$

b. $3\frac{2}{10} + \square = 4$

c. $8\frac{4}{9} + \square = 9$

d. $5\frac{1}{8} + \square = 6$

Chapter 8: Decimals
Introduction

In fourth grade, students learn about decimal numbers that have one or two decimal digits, and they learn to add and subtract them. It is important to grasp these simple topics well because we are laying a groundwork for fifth and sixth grades where decimal operations take "centre stage."

The focus is, first of all, on understanding that decimals are simply fractions with a denominator of 10 or 100. Then with that in mind, we study comparing, adding, and subtracting them.

Take note of this common misconception that students have. Many students add 0.5 + 0.9 = 0.14. The correct way to view 0.5 + 0.9 is as 5 tenths plus 9 tenths, which is 14 tenths = 1.4.

An example of another misconception is when a student adds 0.5 + 0.11 = 0.16. This student is thinking of the decimal parts as if they were "whole numbers" and adding 5 + 11 = 16. To solve 0.5 + 0.11 correctly, students can rewrite 0.5 as 0.50, and then the problem becomes 0.50 + 0.11 = 0.61.

In the lesson *Using Decimals with Measuring Units*, students encounter decimals in connection with metric units, such as 0.1 km or 2.4 kg, and they also convert between the units, such as writing 0.5 km as 500 m. This topic will be studied further in 5th grade.

The Lessons in Chapter 8

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

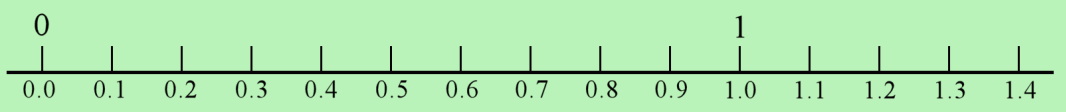
- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

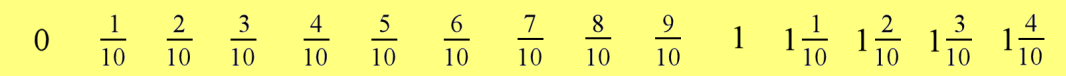
We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better, and even just for some fun. Enjoy!

https://links.mathmammoth.com/gr4ch8

Decimal Numbers—Tenths

The number line between 0 and 1 is divided into ten parts. Each of these ten parts is $\frac{1}{10}$, a **tenth**.





Under the tick marks, you see **decimal numbers** such as 0.1, 0.2, 0.3, and so on.

These are the same numbers as the fractions $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, and so on.

The digit right after the decimal point (such as the digit 3 in 0.3) tells us **how many tenths** the number has. That digit is in the tenths place. So, 0.3 means—and is read as—three tenths.

0.6 means six tenths, or $\frac{6}{10}$.

1.5 means 1 whole and 5 tenths, or $1\frac{5}{10}$.

Note: $\frac{1}{8}$ is ***not*** 0.8. Instead, 0.8 is eight tenths, or $\frac{8}{10}$. The denominator is always 10!

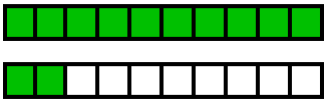
1. Write the fractions as decimals and vice versa.

a. $\frac{7}{10}$	**b.** $2\frac{4}{10}$	**c.** $10\frac{9}{10}$	**d.** 0.9	**e.** 29.3

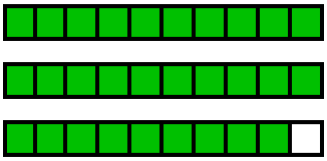
2. Write the decimal and the fraction that each picture shows.

a.

b.



c.



3. Shade parts to show the decimals.

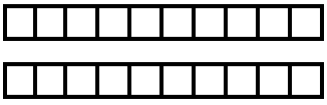
a.

0.4

b.

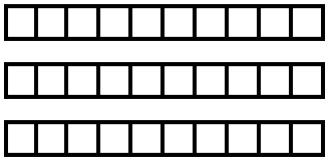
0.1

c.



1.6

d.

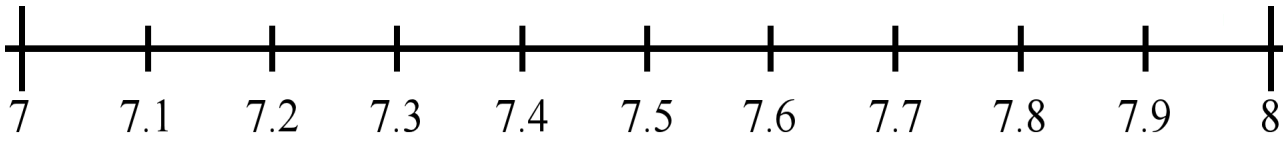


2.8

"**Decimal**" comes from the Latin word *decem*, which simply means "**ten**." The way we write numbers is a **decimal number system**, because it is based on number ten: we use ten different digits (from 0 to 9) and write digits in places such as the ones place, tens place, hundreds place, and so on, each of those places having a value that is ten times the value of the previous place.

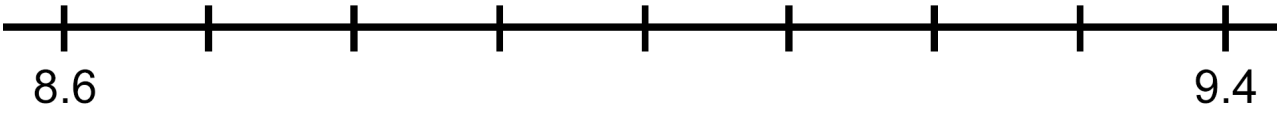
In common language, the word "decimal number" has come to mean numbers which have digits after the decimal point, such as 5.8 or 9.302. In reality, any number within the decimal number system could be termed as a decimal number, including whole numbers such as 12 or 381.

4. Write a mixed number under each decimal number.

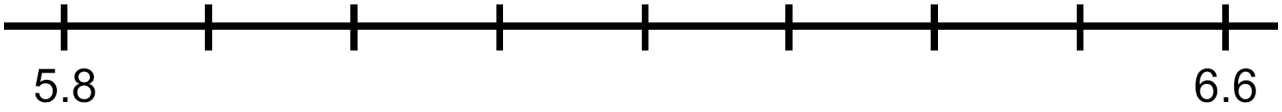


5. Label the tick marks with decimal numbers.

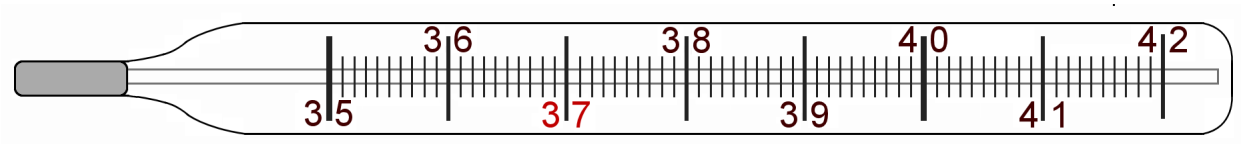
a.



b.



6. **a.** Mark these temperatures with dots on the thermometer:
37.4°C, 36.2°C, 38.7°C, 41.8°C, 40.5°C



b. Which temperatures would indicate a fever?

7. Compare. Write $<$, $>$, or $=$ between the numbers.

a. 0.5 ☐ 0.9	**b.** 1.3 ☐ 0.3	**c.** 5.1 ☐ 4.9	**d.** 0.4 ☐ $\frac{1}{2}$	**e.** 16.0 ☐ 16

8. Write in order from the smallest to the largest number:

1.2 0.9 2.6 0.1 $2\frac{1}{2}$ 2.3 3.0 $\frac{1}{2}$

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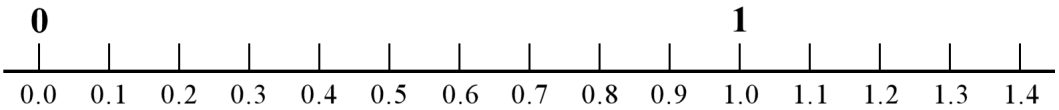
Adding and Subtracting with Tenths

You already know how to add or subtract decimals that have tenths, such as 0.8 + 0.5. They are just fractions with a denominator of 10. Compare the two additions in each box. One of them is written with decimals and the other with fractions.	$0.1 + 0.5 = 0.6$ $\frac{1}{10} + \frac{5}{10} = \frac{6}{10}$	$8.4 - 2.3 = 6.1$ $8\frac{4}{10} - 2\frac{3}{10} = 6\frac{1}{10}$
There is one tricky thing: 0.6 + 0.7 *is* **NOT** 0.13! To see why, add the corresponding fractions. Notice that six-tenths and seven-tenths makes thirteen-tenths, which is more than one!	$0.6 + 0.7 = 1.3$ $\frac{6}{10} + \frac{7}{10} = \frac{13}{10} = 1\frac{3}{10}$	$1.5 + 0.9 = 2.4$ $1\frac{5}{10} + \frac{9}{10} = 2\frac{4}{10}$

1. Write an addition *or* subtraction sentence for each "number-line jump.



a. You are at 0.7, and you jump *five tenths* to the right. _____

b. You are at 0.6, and you jump *eight tenths* to the right. _____

c. You are at 1.1, and you jump *eight tenths* to the left. _____

d. You are at 1.3, and you jump *four tenths* to the left. _____

e. You are at 0.2, and you jump *eleven tenths* to the right. _____

2. Solve the fraction additions, and then write them using decimals.

a. $\frac{2}{10} + \frac{7}{10} =$ *0.2 +*	**b.** $\frac{5}{10} + \frac{6}{10} =$	**c.** $\frac{9}{10} + \frac{8}{10} =$

3. Add or subtract.

a.	**b.**	**c.**	**d.**
0.9 + 0.2 = _____	0.5 + 0.7 = _____	0.8 + 0.7 = _____	1.8 − 0.9 = _____
1.9 + 0.2 = _____	3.5 + 0.7 = _____	0.8 + 2.7 = _____	5.8 − 0.9 = _____

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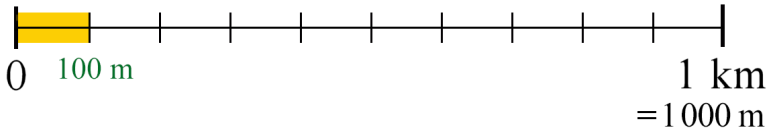
Using Decimals with Measuring Units

Since metric units are based on the number **ten**, it is easy to find one tenth (0.1) of any metric unit.

Distance

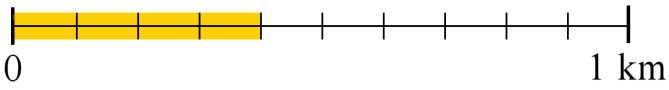
One kilometre is 1 000 metres. The illustration shows 1 km divided into ten equal parts. Each part is one-tenth of a kilometre, and is 100 metres.



Using decimals, we can write 0.1 km = 100 m. Similarly, 0.8 km (8 tenths of a km) is 800 m.

1. Write the distance shown on the number line both in metres and in kilometres (using decimals).

a.



_____ m = _____ km

b.

_____ m = _____ km

2. Fill in the missing parts. The number lines above can help.

a. 500 m = _____ km	**b.** 900 m = _____ km	**c.** _____ m = 0.2 km

3. Convert between the units. Use decimals when writing the distances in kilometres.

a. 0.6 km = _____ m 1.1 km = _____ m	**b.** 700 m = _____ km 1 800 m = _____ km	**c.** 10.9 km = _____ m 24 600 m = _____ km

4. Julie lives 1.2 km away from a college she goes to. Her friend Amanda lives only 300 m from the college. They both walk from home to the college and back each day.

   a. What distance does Amanda walk in one day, in *kilometres*?

   b. How many more *kilometres* does Julie walk than Amanda in a day?

5. Jack ran 2 040 metres, and Andrew ran 2.4 km.
   Who ran a longer distance?

   How much longer (in metres)?